Injection

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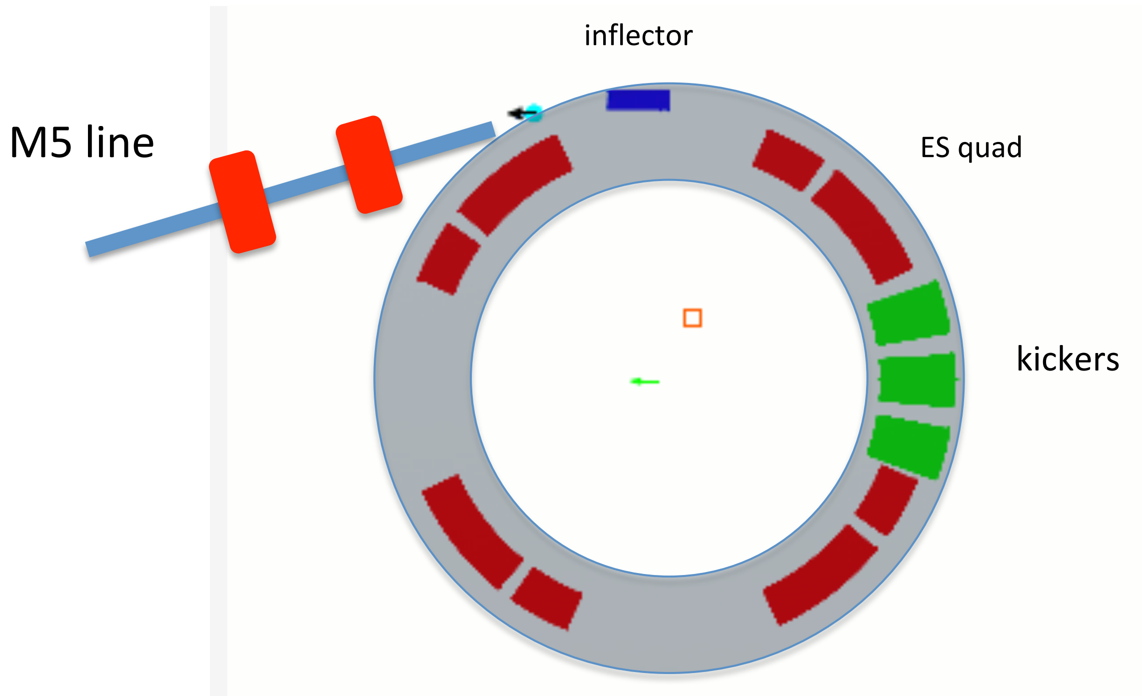
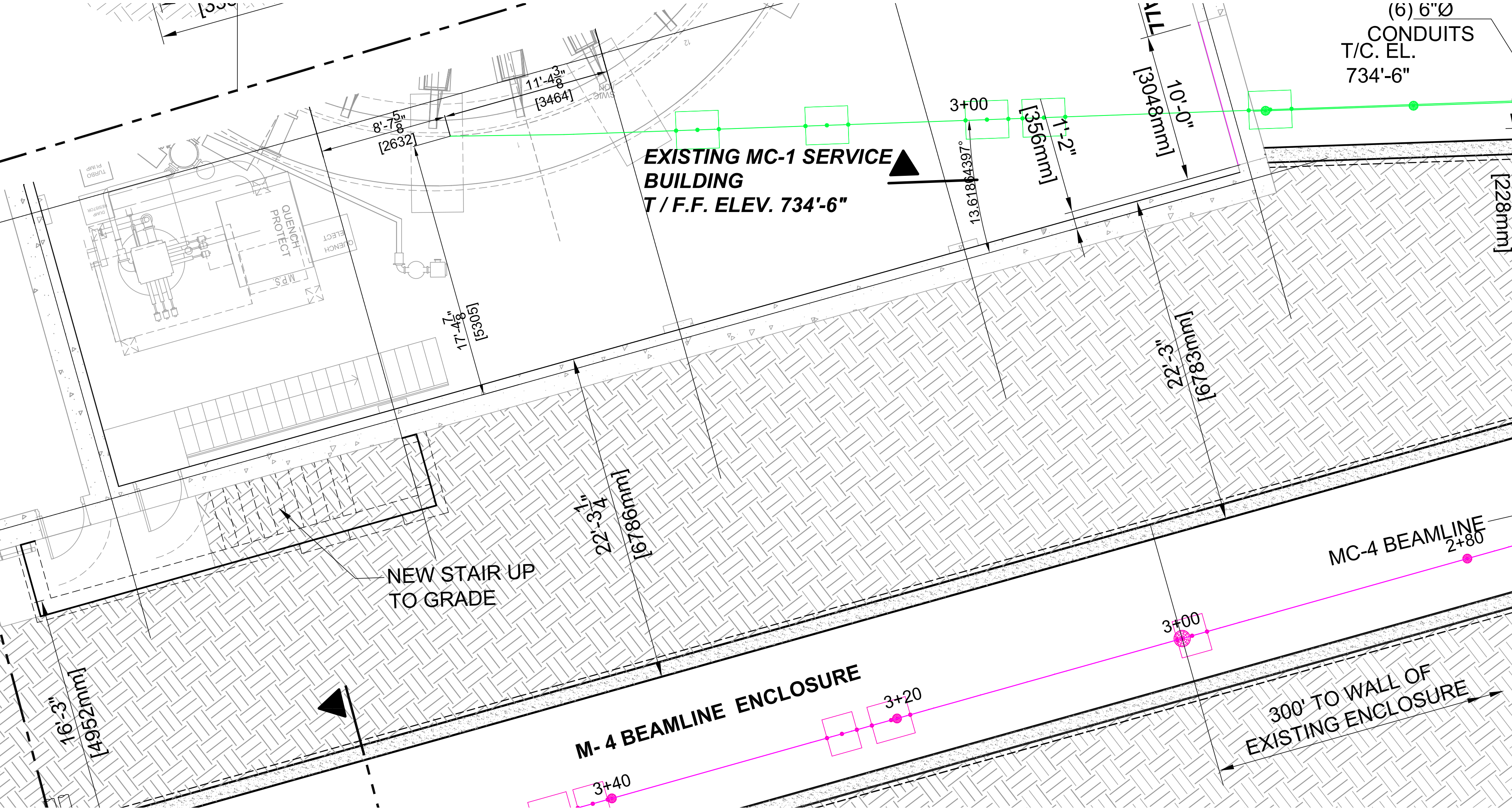
USPAS
2019

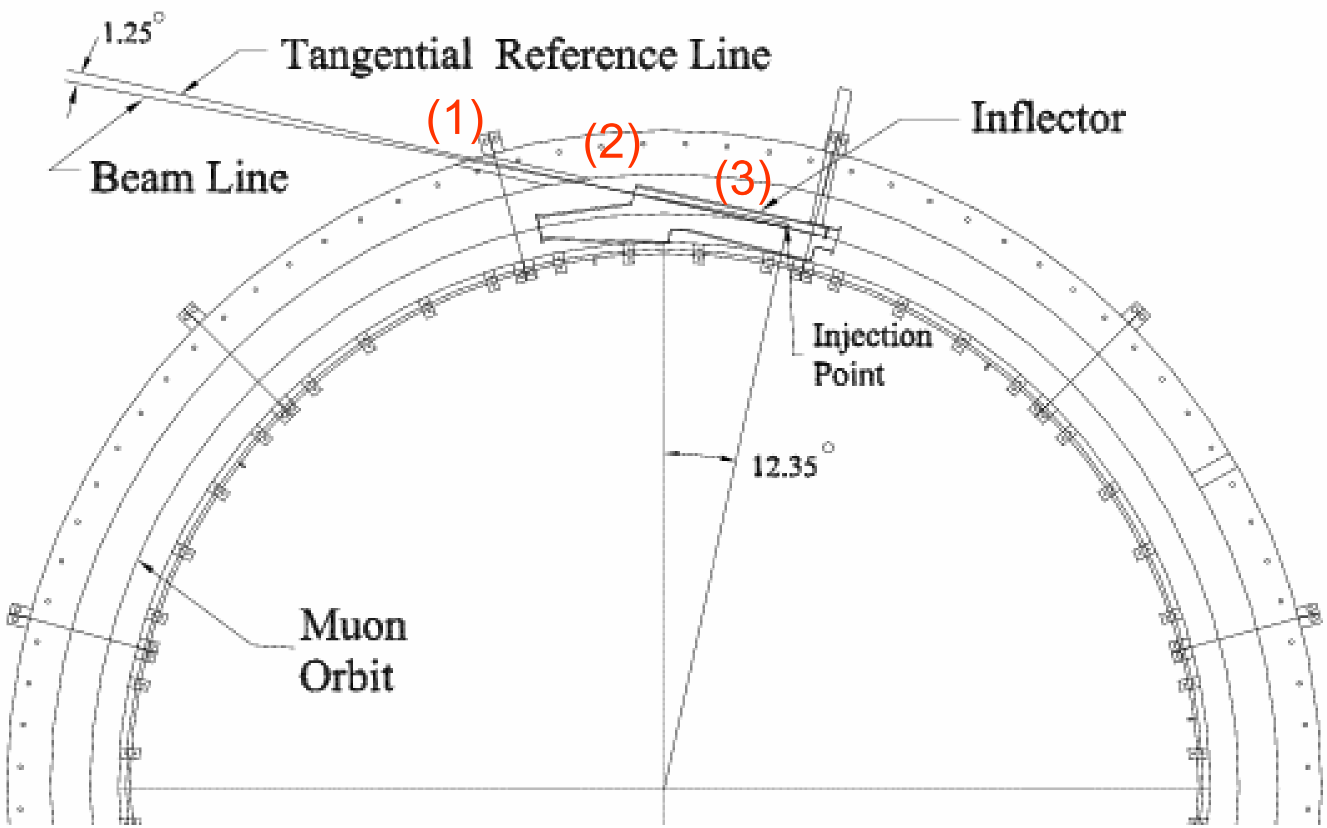
Muon Storage Ring



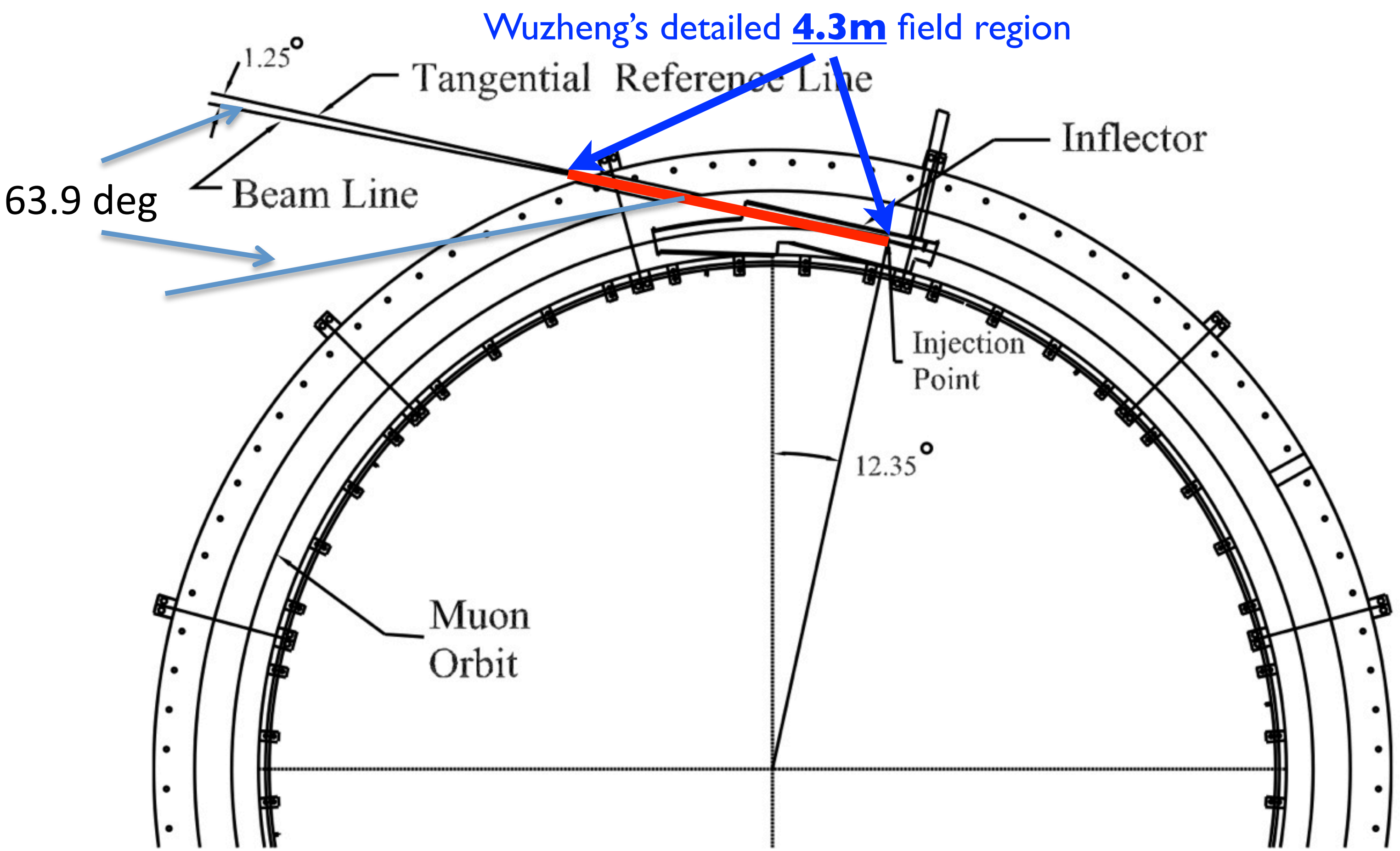


(6) 6"Ø
CONDUITS
T/C. EL.
734'-6"
10'-0"
[3048mm]
1'-2"
[356mm]
3+00
EXISTING MC-1 SERVICE
BUILDING
T / F.F. ELEV. 734'-6"
11'-4 3/8"
[3464]
8'-7 5/8"
[2632]
17'-4 7/8"
[5305]
QUENCH PROTECT
22'-3"
[6783mm]
[228mm]
22'-3 1/4"
[6786mm]
NEW STAIR UP
TO GRADE
MC-4 BEAMLINE
2+80
3+00
3+20
3+40
M- 4 BEAMLINE ENCLOSURE
300' TO WALL OF
EXISTING ENCLOSURE
16'-3"
[4952mm]

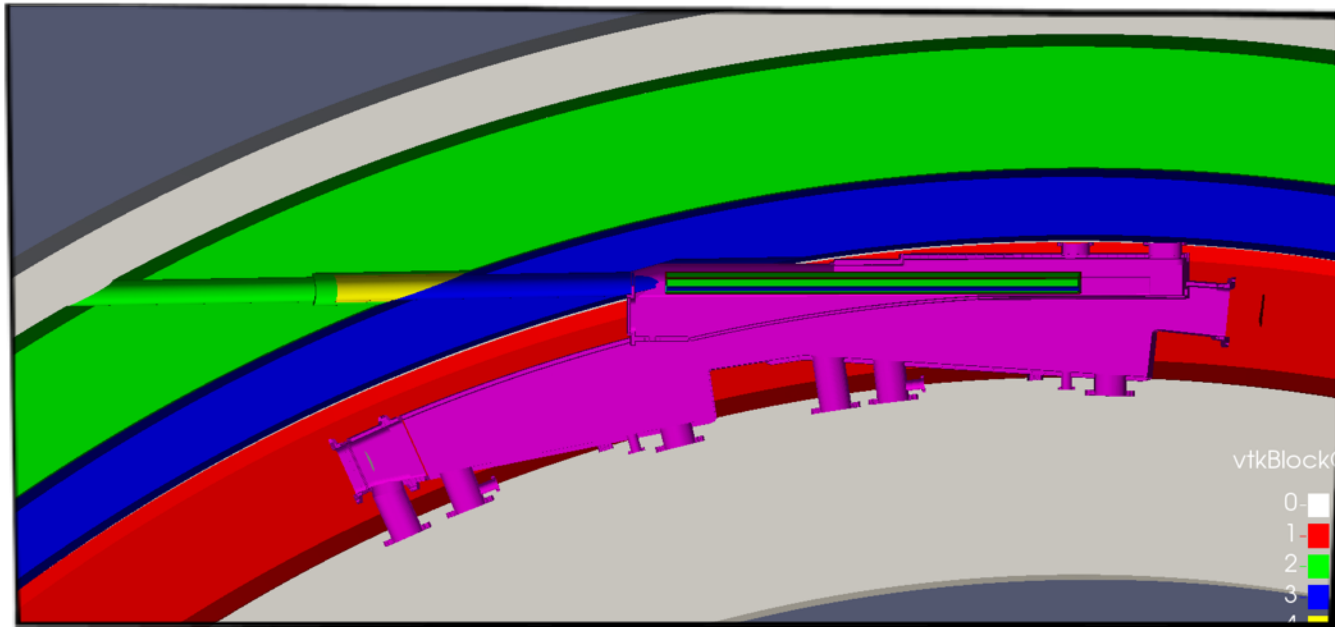
How do twiss parameters propagate through iron, cryostat, inflector into ring



(1) hole in back leg, (2) storage ring fringe field, (3) inflector channel



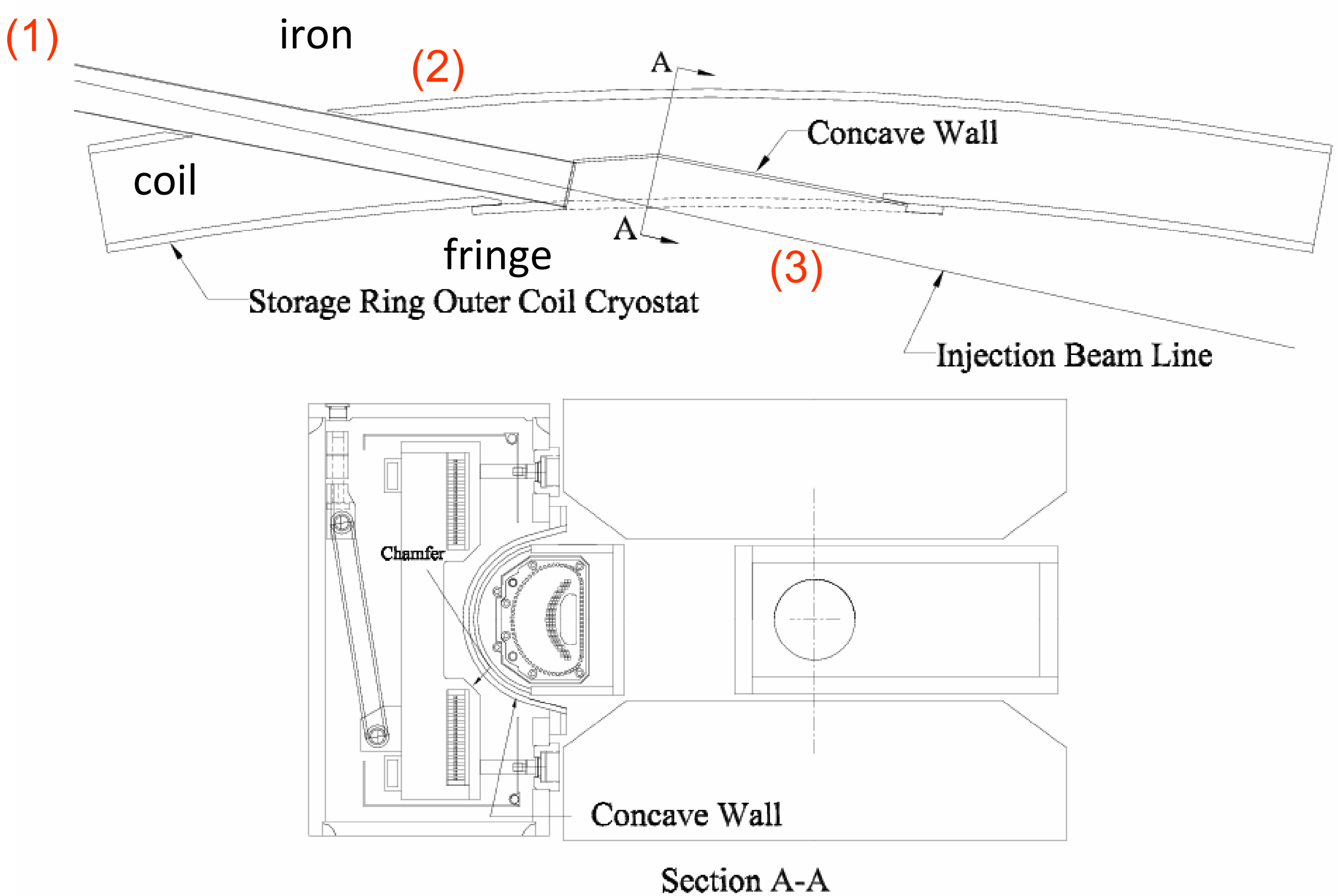
Effective gradient $G \approx (1+\cos(63.9))/2\ (\Delta B_y/\Delta R)$



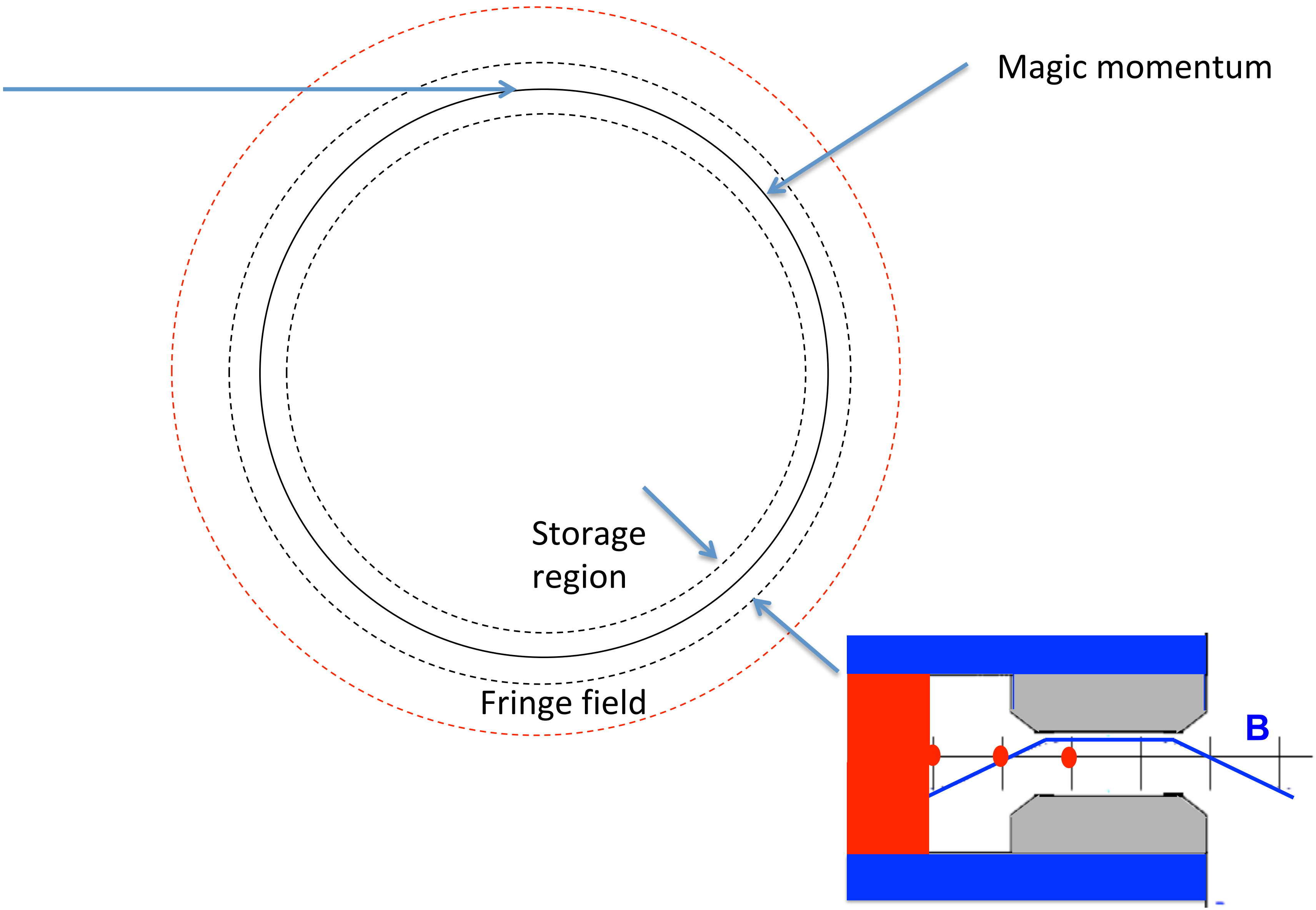
vtkBlock(
0
1
2
3

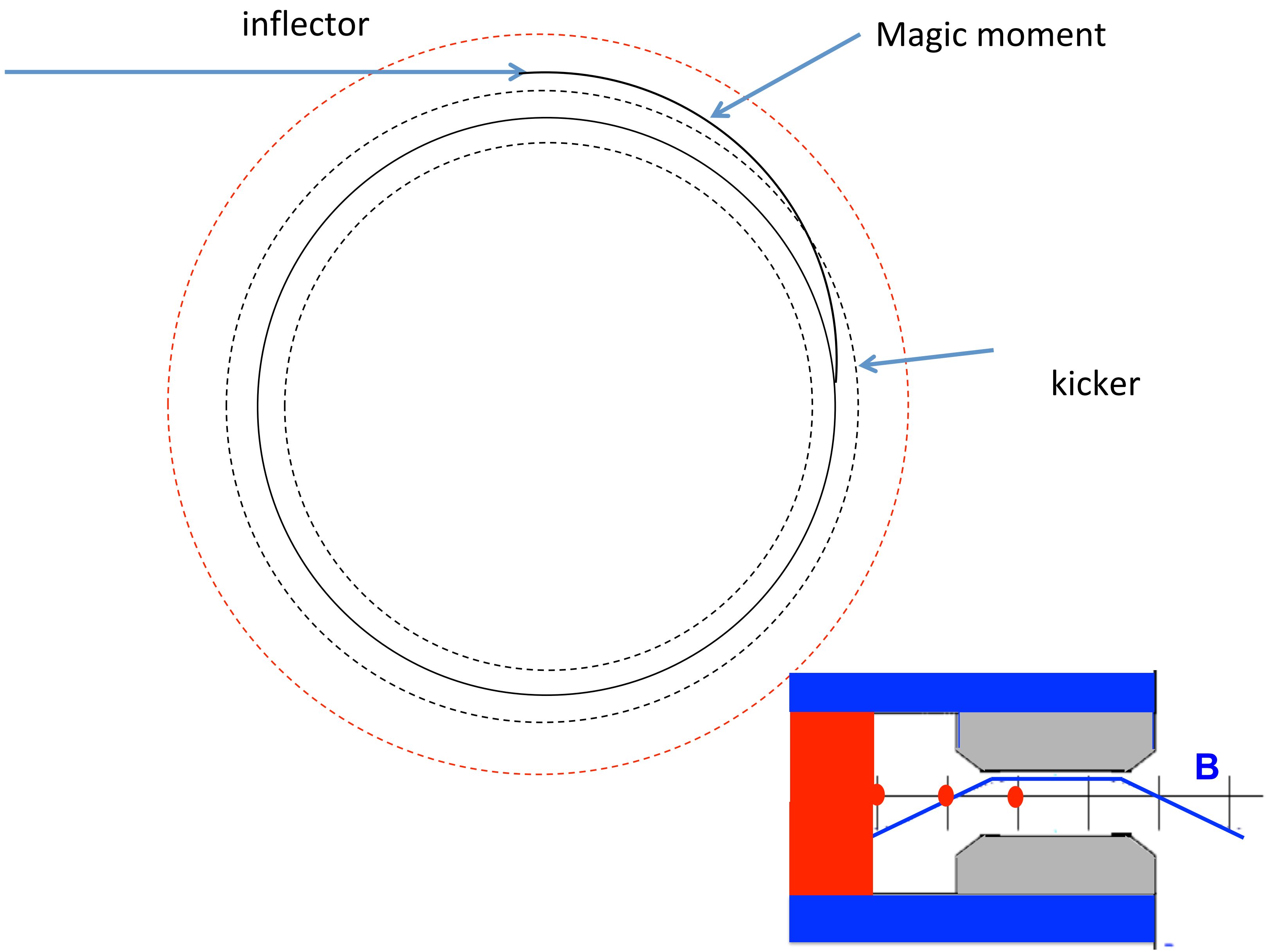
What are the fields along the route of injected beam ?

Another View



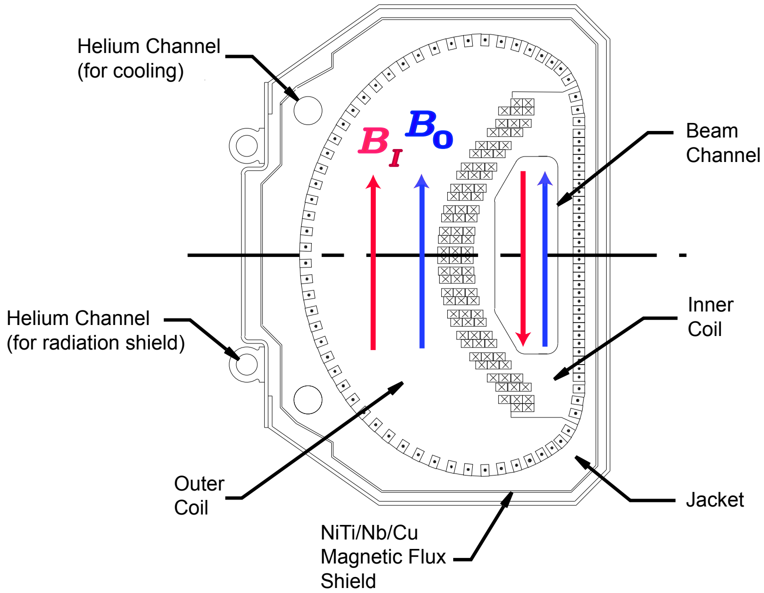
From W. Meng, June 2008 collaboration meeting

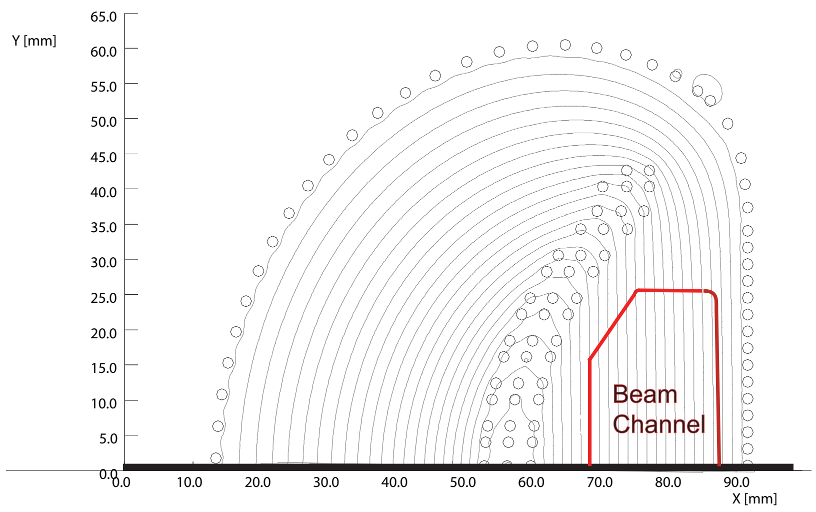

Magic momentum
Storage
region
Fringe field
B



inflector
Magic moment
kicker
B

Inflector


Helium Channel
(for cooling)
Beam
Channel
Helium Channel
(for radiation shield)
Inner
Coil
Outer
Coil
Jacket
NiTi/Nb/Cu
Magnetic Flux
Shield

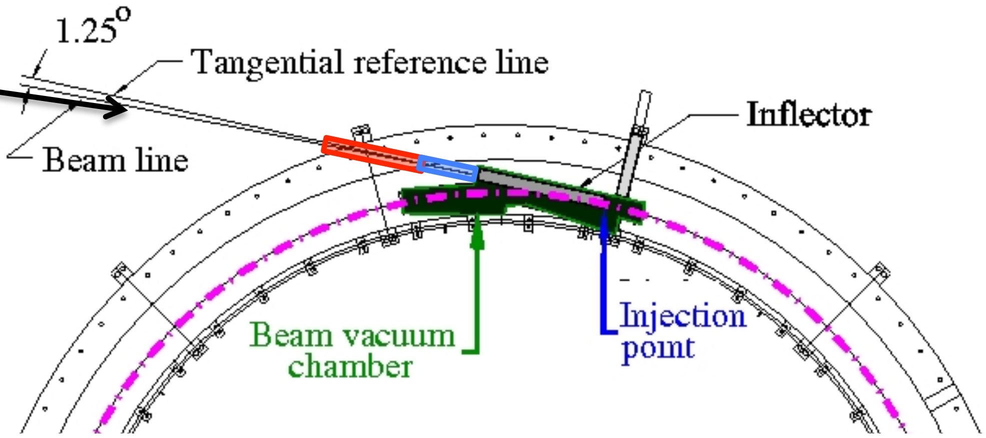

Y [mm]
65.0
60.0
55.0
50.0
45.0
40.0
35.0
30.0
25.0
20.0
15.0
10.0
5.0
0.0
0.0
10.0
20.0
30.0
40.0
50.0
60.0
70.0
80.0
90.0
X [mm]
Beam
Channel

Injection Channel

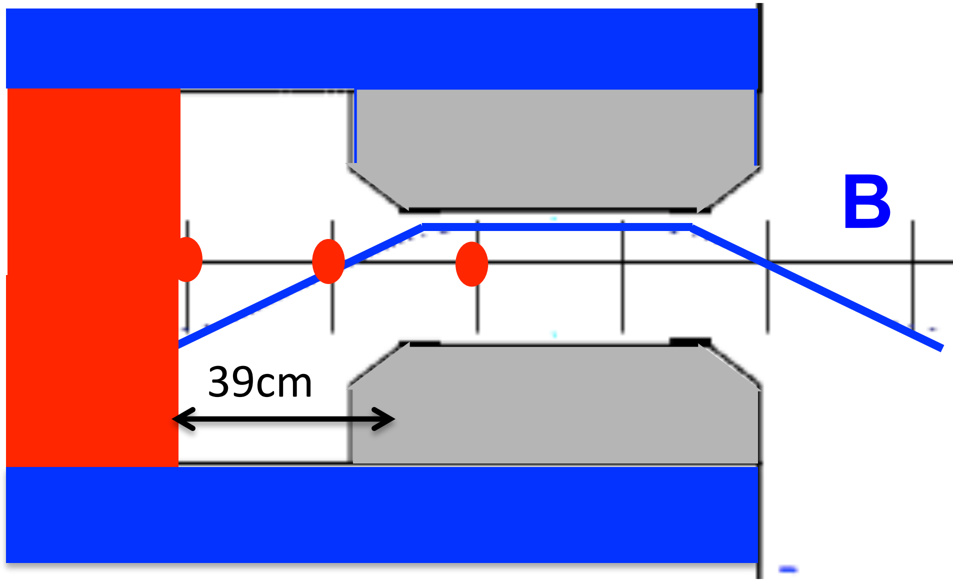
Muons come to the end of the M5 line and then propagate through:

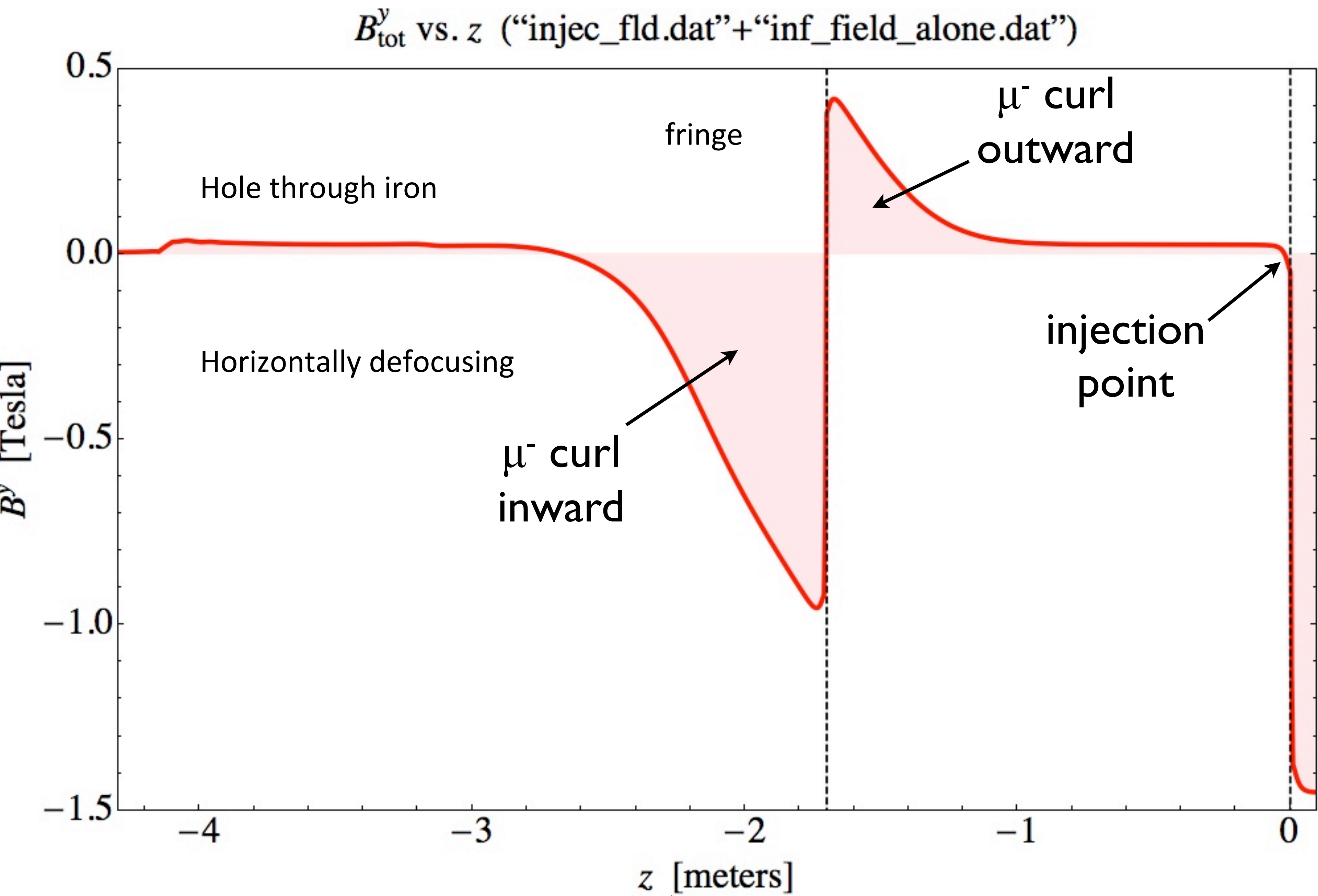
- Hole in magnet yoke
- Dipole fringe field
- Inflector

And exit the inflector 77 mm from the center of the dipole aperture

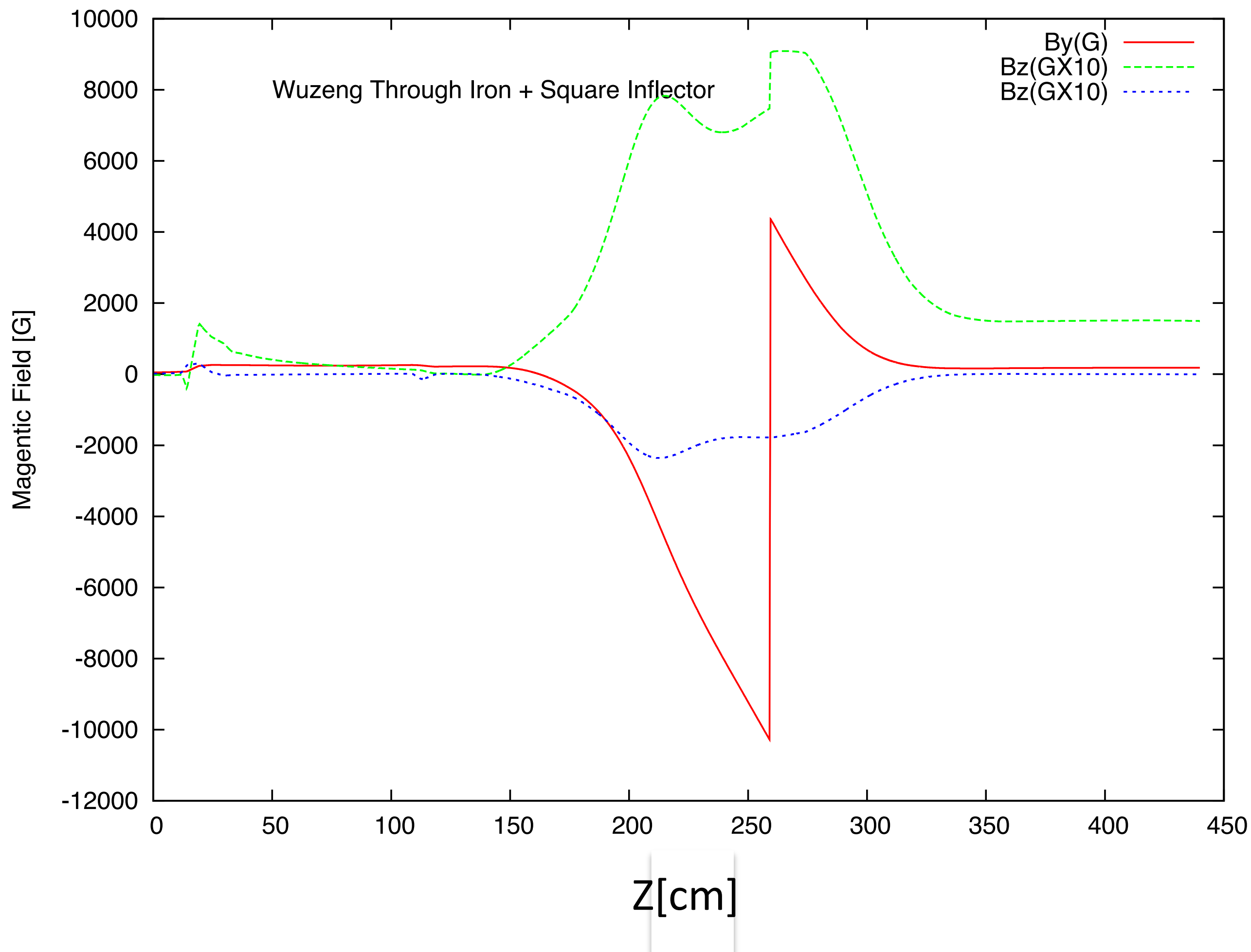


The magnetic field is near zero at the inner surface of the yoke, and rises to 1.45T between the magnet poles, over a distance of ~39cm

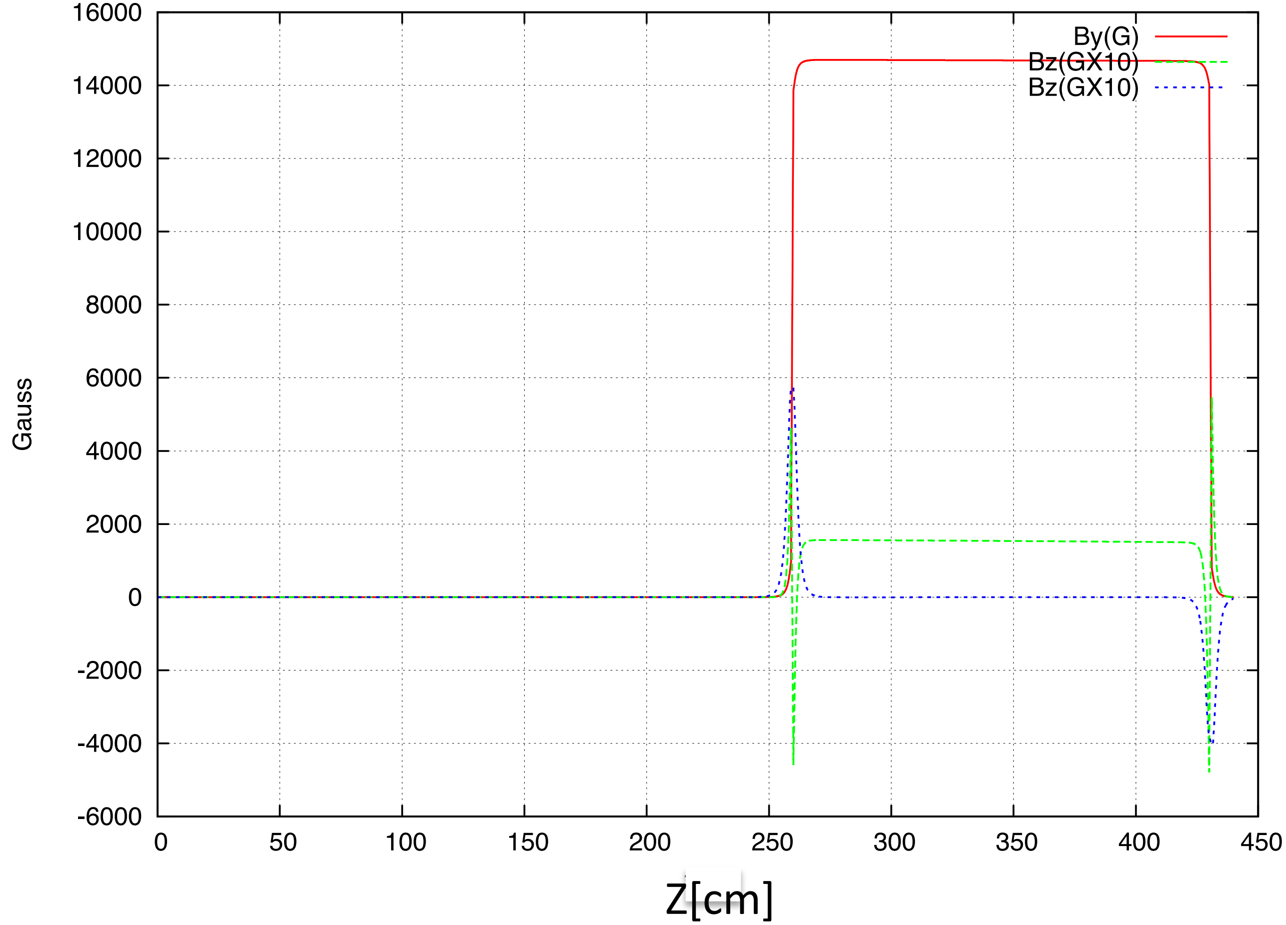



B^y_{tot} vs. z ("injec_fld.dat"+"inf_field_alone.dat")
0.5
0.0
−0.5
−1.0
−1.5
B^y [Tesla]
−4
−3
−2
−1
0
z [meters]
fringe
μ⁻ curl
outward
Hole through iron
injection
point
Horizontally defocusing
μ⁻ curl
inward

Inflector and main dipole



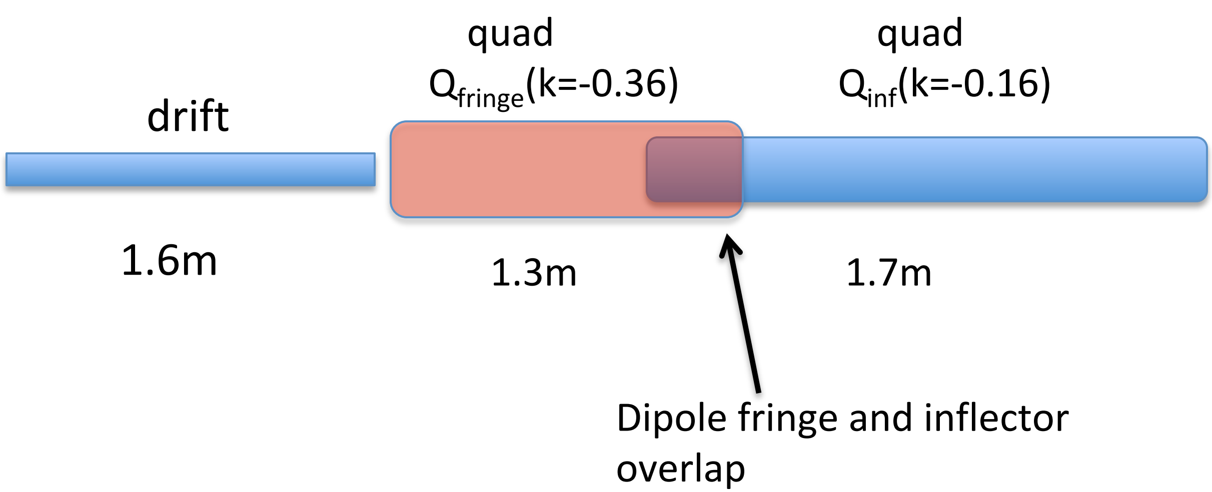
Inflector



By(G)
Bz(GX10)
Bz(GX10)
16000
14000
12000
10000
8000
6000
4000
2000
0
-2000
-4000
-6000
Gauss
0
50
100
150
200
250
300
350
400
450
Z[cm]

Injection channel defocusing

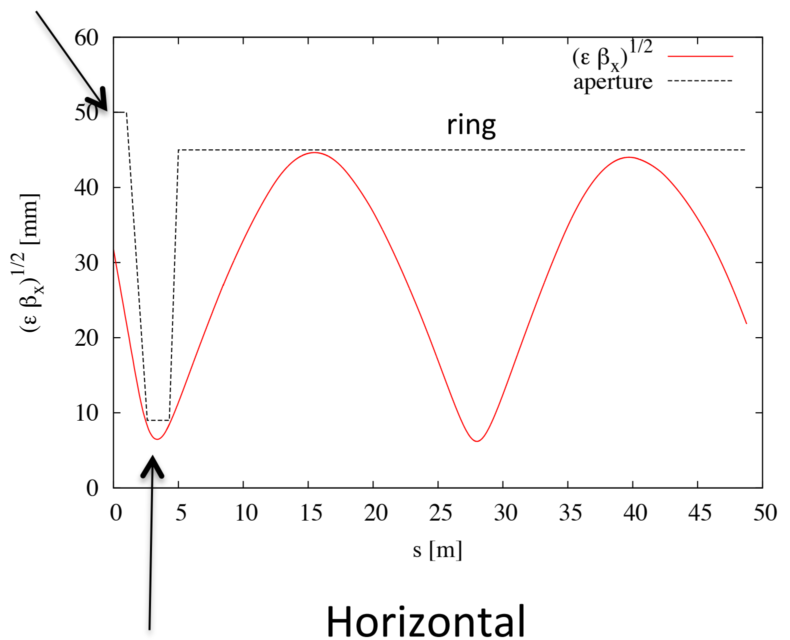
The 4.3m injection channel can be modeled as follows



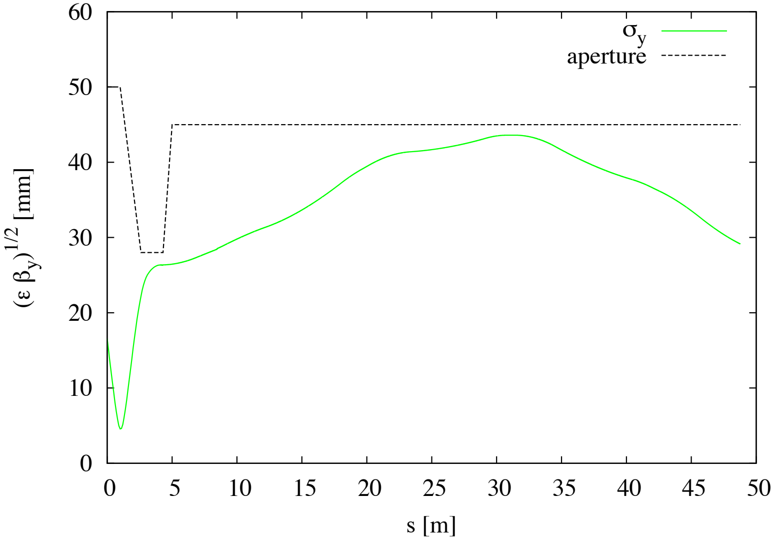
Or we can compute the transfer maps numerically by tracking

Acceptance

β,α,η at end of M5 line (entrance to iron) are chosen so that inflector and ring have equivalent acceptance



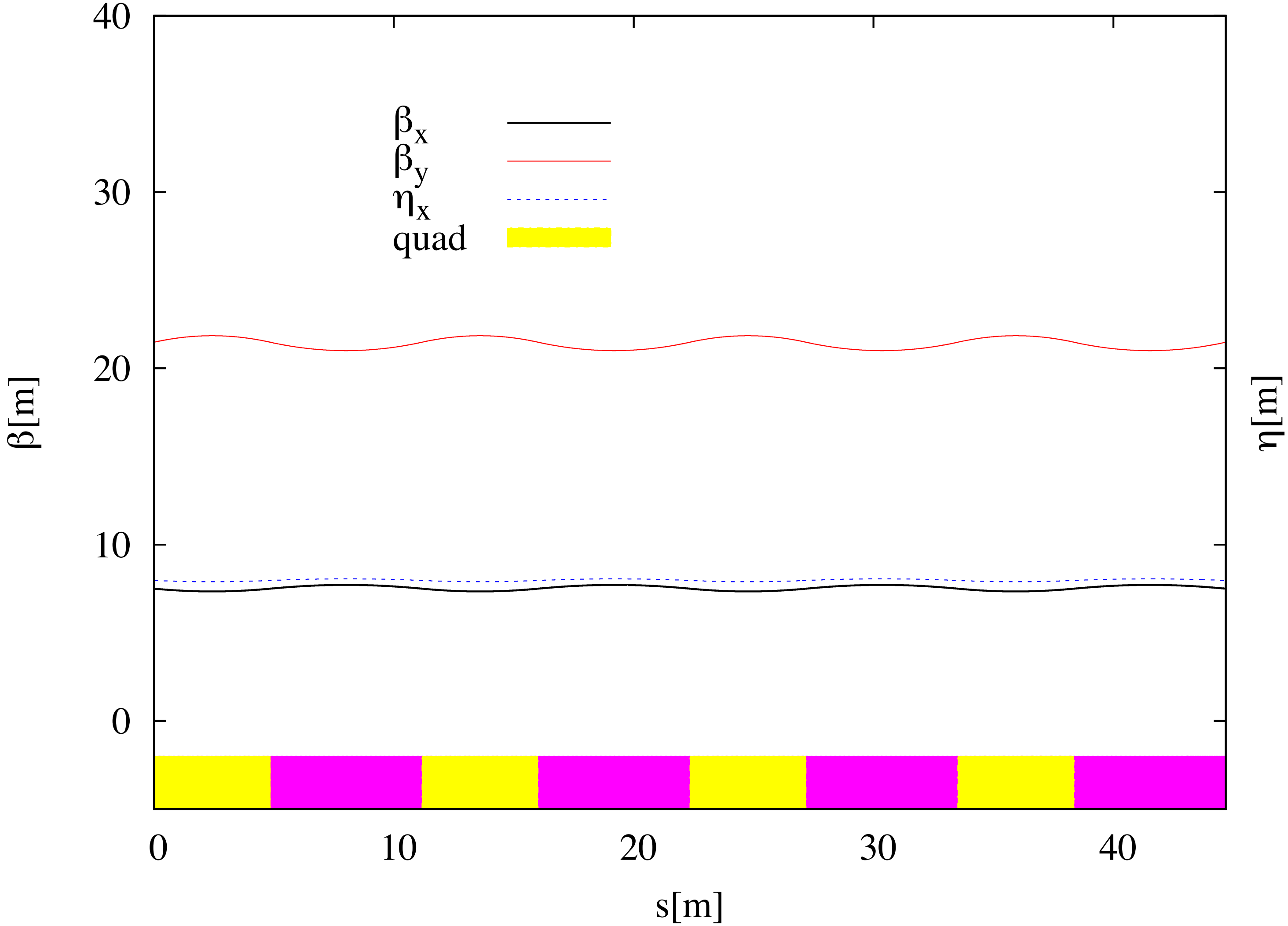
Horizontal



vertical

Aperture and beam size

Twiss parameters (A-matrix) is chosen to be single valued in ring



Acceptance

We maximize transmission and capture by focusing the beam so that inflector and ring have equivalent acceptance

$$\sqrt{\frac{\beta_{\rm inf}}{\beta_{\rm max-ring}}} = \frac{A_{\rm inf}}{A_{\rm ring}} \rightarrow \beta_{\rm inf} = \left(\frac{A_{\rm inf}}{A_{\rm ring}}\right)^2 \beta_{\rm max-ring}$$

The maximum β in the ring depends on the matching according to

$$\beta_{\rm max-ring} \sim \frac{\beta^2_{\rm ring}}{\beta_{\rm inf}}$$

$$\begin{pmatrix} \beta^x_{\rm ring} = & 8{\rm m} \\ \beta^y_{\rm ring} = & 16{\rm m} \end{pmatrix}$$

Then $\beta_{\rm inf} = \frac{A_{\rm inf}}{A_{\rm ring}} \beta_{\rm ring}$

We find that inflector and ring have equivalent acceptance if

$$\beta^{\rm x}_{\rm inf} = 1.6{\rm m}, \quad \beta^{\rm y}_{\rm inf} = 10.0{\rm m}$$

Dispersion

Energy acceptance of inflector and ring are equivalent if

$$\eta_{\rm inf} = \frac{A_{\rm inf}}{A_{\rm ring}}\eta_{\rm max-ring}$$ and since $$\eta_{\rm max-ring} = 2\eta_{\rm ring} - \eta_{\rm inf}$$

=> $\eta_{\rm inf} \sim 2.5{\rm m}$

(Tracking simulations indicate optimal capture for $\eta \sim 0$

The energy acceptance follows from

$$\sigma = \sqrt{\beta_x\epsilon + \eta^2_{\rm max-ring}\delta^2}$$

=> $\delta \sim 0.1\%$

$$\begin{pmatrix} \beta^x_{\rm max-ring} = & 40{\rm m} \\ \eta_{\rm max-ring} = & 17{\rm m} \\ \epsilon_x = & 40\mu{\rm m} \end{pmatrix}$$

Dispersion

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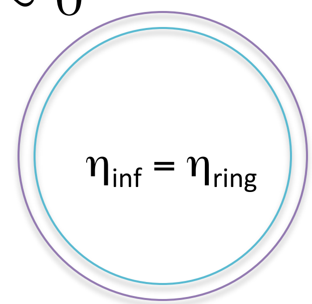

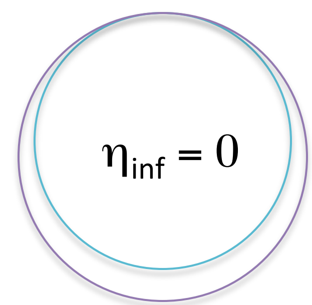

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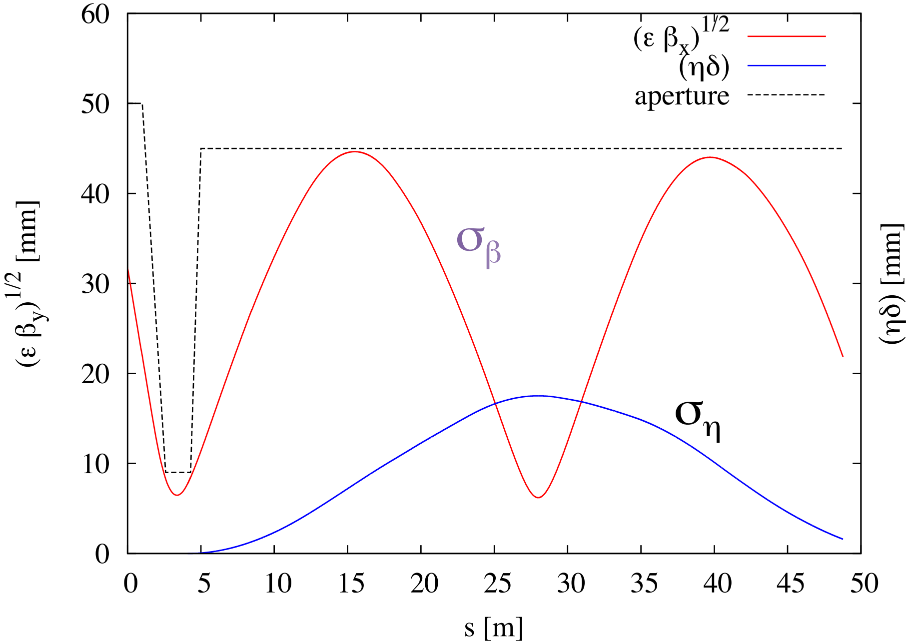
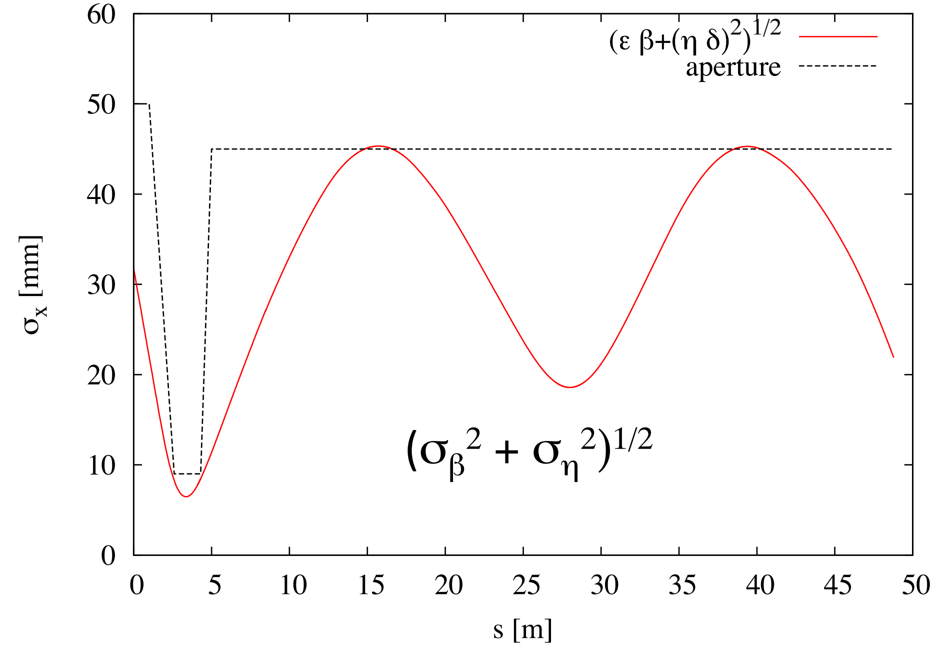
Dispersion

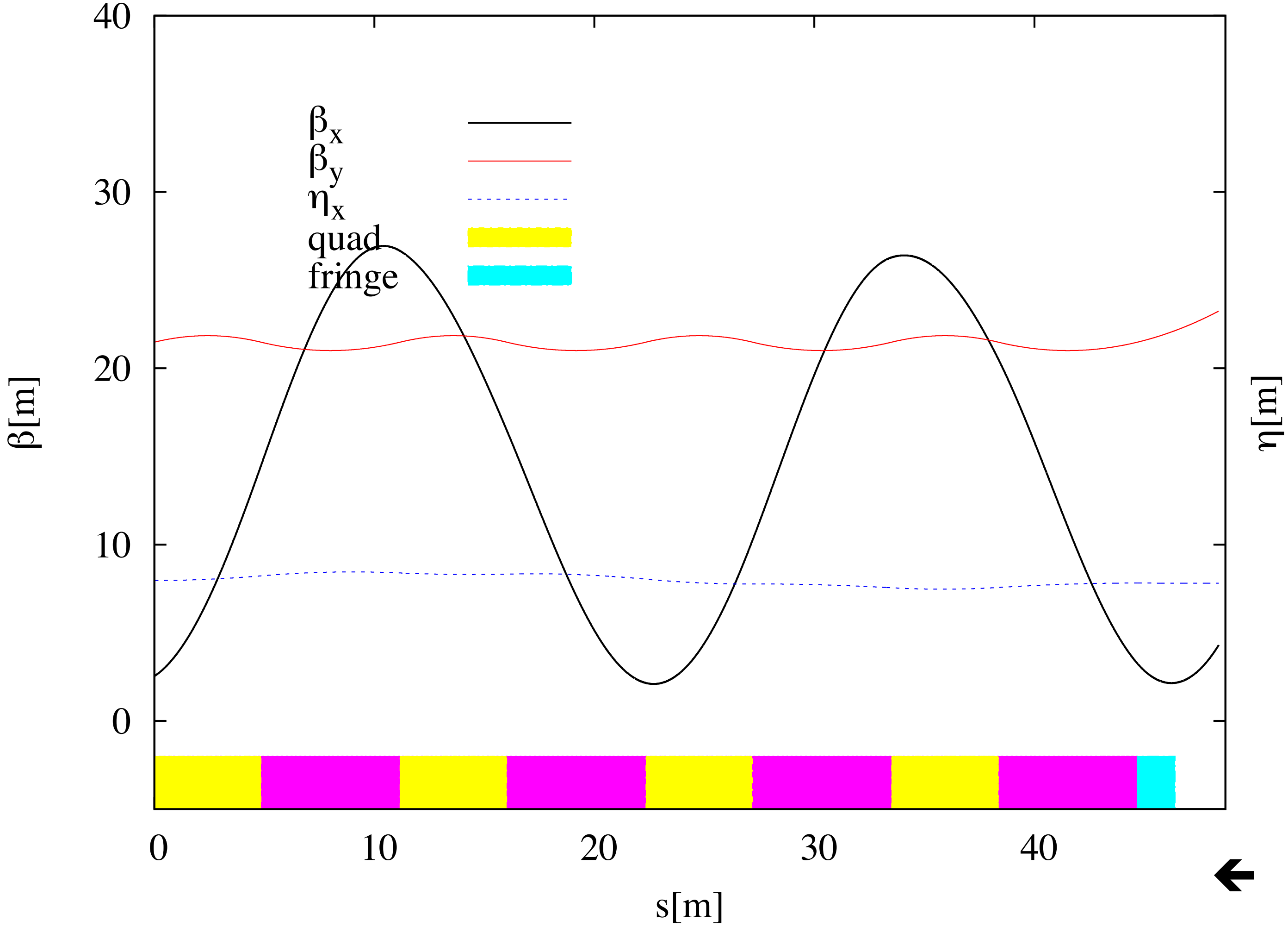
Contribution to beam size from betatron motion and energy spread





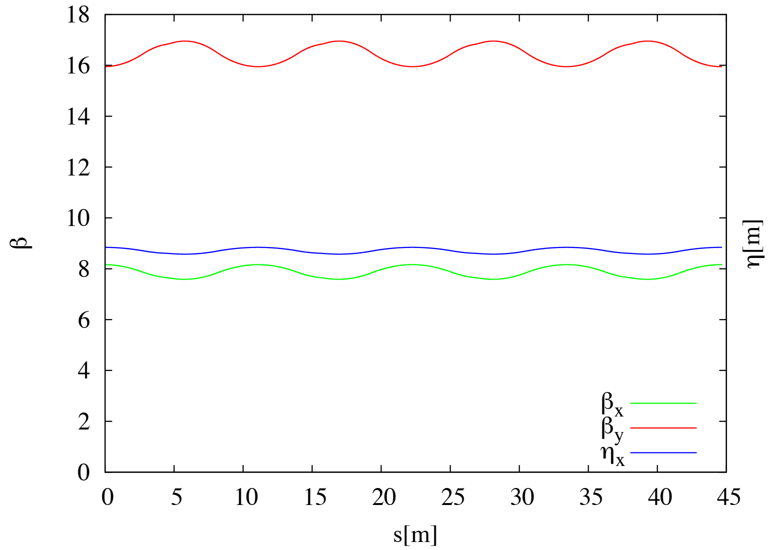
Horizontal aperture and beam size
Including 0.1% energy spread

Suppose we choose β_x β_y at upstream end of inflector so that 40 mm-mrad so that most of the beam fits through the inflector aperture
(Assuming *ideal* inflector: zero field, opened ends)



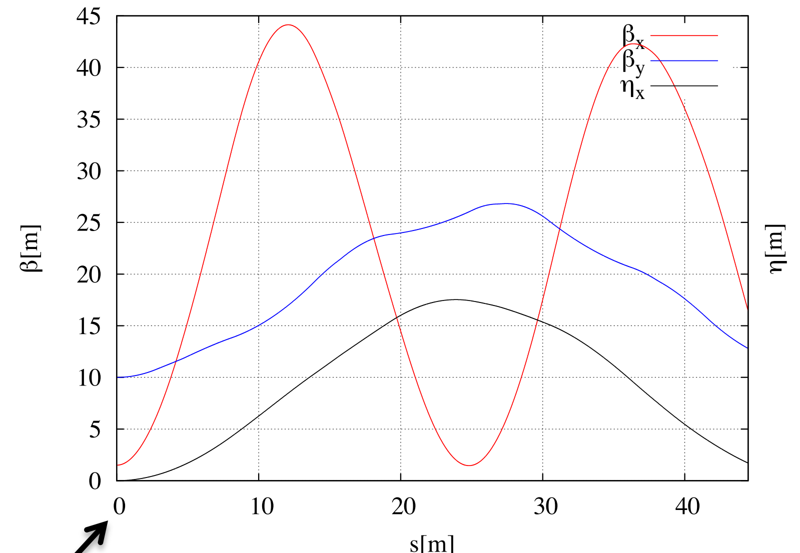
Mismatch

- Emittance of the muon beam is large and the number of captured muons is limited by apertures
- Inflector aperture << ring aperture
- Mismatch of twiss parameters at inflector-ring interface => peak β & η in the ring are always greater than closed ring values: $\beta_{\mathrm{max-ring}} \sim \frac{\beta^2_{\mathrm{ring}}}{\beta_{\mathrm{inf}}}$



Closed ring

$$\beta^x_{\mathrm{inf}} = 1.5\mathrm{m}$$
$$\beta^y_{\mathrm{inf}} = 10.0\mathrm{m}$$ =>
$$\eta_{inf} = 0$$



Inflector exit

Mismatch

How do we propagate

- twiss parameters
- Beam size
- Phase space distribution

How do we determine M?

Invariant

$$a = \gamma x^2 + 2\alpha x x' + \beta {x'}^2$$

Define

$$\vec{\gamma} = \begin{pmatrix} \gamma \\ 2\alpha \\ \beta \end{pmatrix} \qquad \vec{X} = \begin{pmatrix} x^2 \\ xx' \\ {x'}^2 \end{pmatrix} \qquad a = \vec{\gamma} \cdot \vec{X}$$

$$x_f = M x_i$$

$$\vec{X}_f = \mathcal{M} \vec{X}_i = \begin{pmatrix} (M_{11} x_i + M_{12} x_i')^2 \\ (M_{11} x_i + M_{12} x_i')(M_{21} x_i + M_{22} x_i') \\ (M_{21} x_i + M_{22} x_i')^2 \end{pmatrix}$$

$$\mathcal{M} = \begin{pmatrix} M_{11}^2 & 2M_{11}M_{12} & M_{12}^2 \\ M_{11}M_{21} & (M_{11}M_{22} + M_{12}M_{21}) & M_{12}M_{22} \\ M_{21}^2 & 2M_{21}M_{22} & M_{22}^2 \end{pmatrix}$$

$$\begin{aligned} \vec{\gamma} \cdot \vec{X} &= \gamma^T X = \gamma^T \mathcal{M}^{-1} \mathcal{M} X \\ &= (\mathcal{M}^{-1})^T \vec{\gamma} \cdot \mathcal{M} \vec{X} \\ \rightarrow (\mathcal{M}^{-1})^T \vec{\gamma}_i &= \vec{\gamma}_f \end{aligned}$$

If we know M we can construct $\mathcal{M}$ and $\mathcal{N}$ to propagate twiss parameters

To construct $\mathcal{N}$ replace the elements of M with M^{-1} in $\mathcal{M}$ and transpose

$$\mathcal{N}^T = \begin{pmatrix} M_{22}^2 & -2M_{22}M_{12} & M_{12}^2 \\ -M_{22}M_{21} & (M_{11}M_{22}+M_{12}M_{21}) & -M_{12}M_{11} \\ M_{21}^2 & -2M_{21}M_{11} & M_{11}^2 \end{pmatrix}$$

$$\mathcal{N} = \begin{pmatrix} M_{22}^2 & -M_{22}M_{21} & M_{21}^2 \\ -2M_{22}M_{12} & (M_{11}M_{22}+M_{12}M_{21}) & -2M_{21}M_{11} \\ M_{12}^2 & -M_{12}M_{11} & M_{11}^2 \end{pmatrix}$$

In a drift $M = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$

$$\mathcal{N} = \begin{pmatrix} 1 & 0 & 0 \\ -2L & 1 & 0 \\ L^2 & -L & 1 \end{pmatrix}$$

If at L=0, β is at a minimum then

$$\vec{\gamma}(L) = \mathcal{N} \begin{pmatrix} 1/\beta_0 \\ 0 \\ \beta_0 \end{pmatrix} = \begin{pmatrix} 1/\beta_0 \\ -\frac{2L}{\beta_0} \\ \frac{L^2}{\beta_0} + \beta_0 \end{pmatrix}$$

$$\beta(L) = \beta_0 + \frac{L^2}{\beta_0}$$

β increases rapidly away from a minimum

Now we know how to propagate twiss parameters along a beam line

Extract twiss parameters from particle distribution ?

$$x = \sqrt{2\beta\epsilon_x}\cos\phi_x + \eta_x\delta$$

$$\beta_x\epsilon_x = \langle xx\rangle - \frac{\langle x\delta\rangle^2}{\langle\delta\delta\rangle}$$

Notation

$$\beta_x\,\epsilon_x = \sigma(1,1) - \frac{\sigma^2(1,6)}{\sigma(6,6)}$$

Betatron dist

$$\sigma_\beta(i,j) \equiv \sigma(i,j) - \frac{\sigma(i,6)\,\sigma(j,6)}{\sigma(6,6)}$$

$$\epsilon_x^2 = \sigma_\beta(1,1)\,\sigma_\beta(2,2) - \sigma_\beta^2(1,2)$$

We can compute $\sigma(i,j)$ for the distribution and extract twiss

$$\epsilon_x \begin{pmatrix} \beta_x & -\alpha_x \\ -\alpha_x & \gamma_x \end{pmatrix} = \begin{pmatrix} \sigma_\beta(1,1) & \sigma_\beta(1,2) \\ \sigma_\beta(1,2) & \sigma_\beta(2,2) \end{pmatrix}$$

If we know M we can propagate twiss
And if we have a distribution we can compute twiss

What if we know twiss, how do we determine M?

M transports particles from point 1 with $\vec{\gamma}_1$ to point 2 with $\vec{\gamma}_2$

Write M in terms of $\vec{\gamma}_1, \vec{\gamma}_2$

$$x_1 = a\sqrt{\beta_1}\cos\phi_1, \quad x_1' = -a\left(\frac{\alpha_1}{\sqrt{\beta_1}}\cos\phi_1 + \frac{1}{\sqrt{\beta_1}}\sin\phi_1\right)$$

$$\begin{pmatrix} a\sqrt{\beta_2}\cos\phi_2 \\ -a(\frac{\alpha_2}{\sqrt{\beta_2}}\cos\phi_2 + \frac{1}{\sqrt{\beta_2}}\sin\phi_2) \end{pmatrix} = M\begin{pmatrix} a\sqrt{\beta_1}\cos\phi_1 \\ -a(\frac{\alpha_1}{\sqrt{\beta_1}}\cos\phi_1 + \frac{1}{\sqrt{\beta_1}}\sin\phi_1) \end{pmatrix}$$

Define $$G = \begin{pmatrix} \sqrt{\beta_i} & 0 \\ -\frac{\alpha_i}{\sqrt{\beta_i}} & -\frac{1}{\sqrt{\beta_i}} \end{pmatrix}$$

So that $$\begin{pmatrix} x \\ x' \end{pmatrix} = G\begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix} = G \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix}$$

$$G_2 a \begin{pmatrix} \cos\phi_2 \\ \sin\phi_2 \end{pmatrix} = M G_1 a \begin{pmatrix} \cos\phi_1 \\ \sin\phi_1 \end{pmatrix}$$

$$G_2^{-1} M G_1 = \begin{pmatrix} \cos\Delta\phi & -\sin\Delta\phi \\ \sin\Delta\phi_1 & \cos\Delta\phi \end{pmatrix}$$

$$M = G_2 \begin{pmatrix} \cos\Delta\phi & -\sin\Delta\phi \\ \sin\Delta\phi_1 & \cos\Delta\phi \end{pmatrix} G_1^{-1}$$

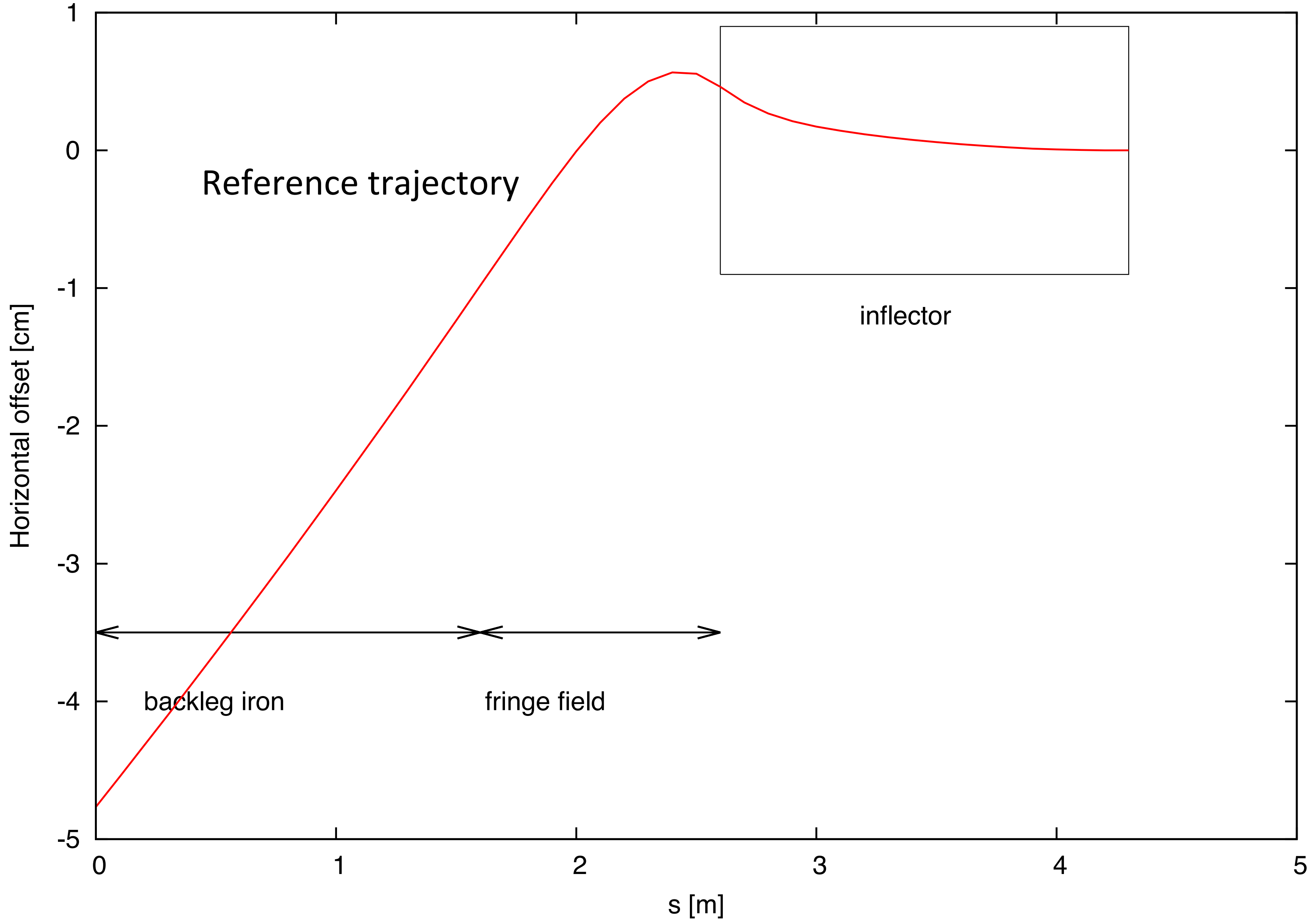
$$M = \begin{pmatrix} \sqrt{\frac{\beta_2}{\beta_1}}[\cos\Delta\phi + \alpha_1 \sin\Delta\phi] & \sqrt{\beta_2\beta_1}\sin\Delta\phi \\ -\frac{1}{\sqrt{\beta_1\beta_2}}[(\alpha_1 - \alpha_2)\cos\Delta\phi - (1 + \alpha_2\alpha_1)\sin\Delta\phi] & \sqrt{\frac{\beta_1}{\beta_2}}[\cos\Delta\phi - \alpha_2 \sin\Delta\phi] \end{pmatrix}$$

If we know M we can propagate twiss
And if we have a distribution we can compute twiss
From twiss we can determine M (to a phase advance)

What if all we have is a field map?
How do we compute M

M is the Jacobian of the mapping from 1 to 2
We need to compute derivatives by tracking
Derivatives are a change with respect to a reference

$$M = \begin{pmatrix} \frac{\partial x_1^f}{\partial x_1^i} & \frac{\partial x_1^f}{\partial x_2^i} \\ \\ \frac{\partial x_2^f}{\partial x_1^i} & \frac{\partial x_2^f}{\partial x_2^i} \end{pmatrix}$$


Reference trajectory
inflector
backleg iron
fringe field
Horizontal offset [cm]
s [m]
1
0
-1
-2
-3
-4
-5
0
1
2
3
4
5

To compute the transfer matrix with respect to a reference trajectory
track from 2 sets of initial coordinates near the reference

$$\vec{x}_i \rightarrow \vec{x}_f$$

Offset from reference by $\vec{\Delta}^j$

$$M\vec{x}_i^1 = \vec{x}_f^1$$

Or for two non degenerate trajectories

$$M\vec{x}_i^1 x_i^2 = \vec{x}_f^1 \vec{x}_f^2$$

$$MX_1 = X_2$$

$$M = X_2 {X_1}^{-1}$$

Closed orbit reference

$$M(\vec{x}_i - \vec{x}_c) = \vec{x}_f - \vec{x}_c$$

$$M\vec{x}_i - (M\vec{x}_c - \vec{x}_c) = \vec{x}_f$$

$$M\vec{x}_i - \vec{v} = \vec{x}_f$$

Define $$P = \begin{pmatrix} M & \begin{pmatrix} -v \\ -v' \end{pmatrix} \\ 0 & 1 \end{pmatrix} \qquad PX_i = X_f \qquad X_i = \begin{pmatrix} x_i \\ x_i' \\ 1 \end{pmatrix}$$

$$Y_i = X_i^1 X_i^2 X_i^3 \qquad PY_i = Y_f \qquad P = Y_f Y_i^{-1}$$

$$\rightarrow M, \vec{v} \qquad \vec{x}_c = (M - I)^{-1}\vec{v}$$

Conclusion: track 3 non degenerate orbits through 1 full turn to find closed orbit and M
Track 7 non degenerate orbits to find M for full 6-d phase space

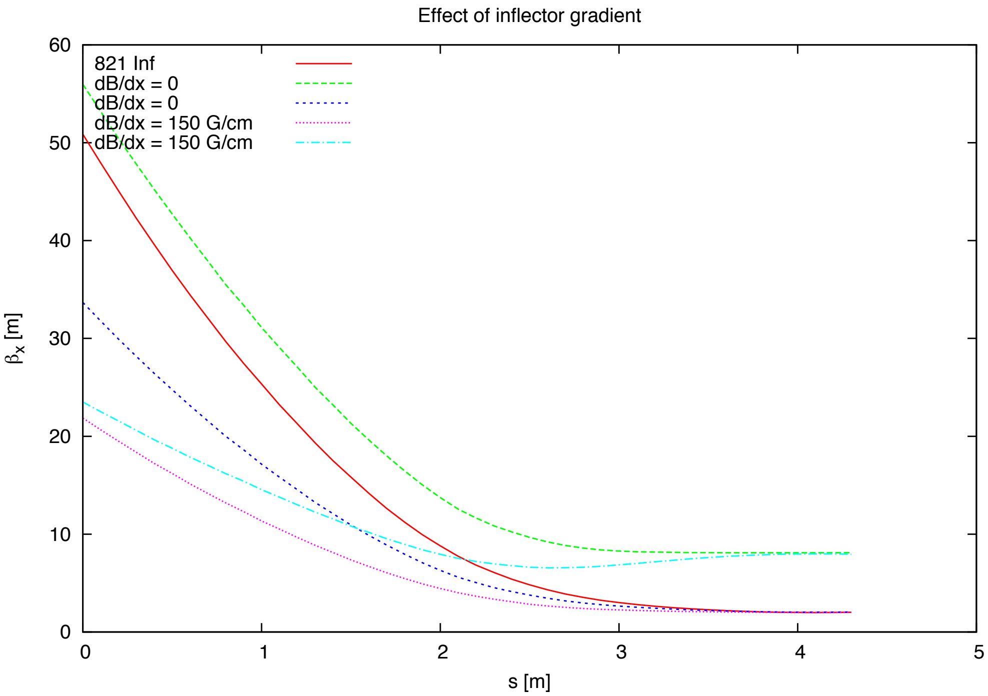
Suppose we want to study the dependence of capture on beta in the inflector

- Beta in the inflector is determined by quads in M5 line.
- Changing Beta in the inflector => change beta in M5

The distribution to be tracked is defined at the end of M5 with unknown twiss

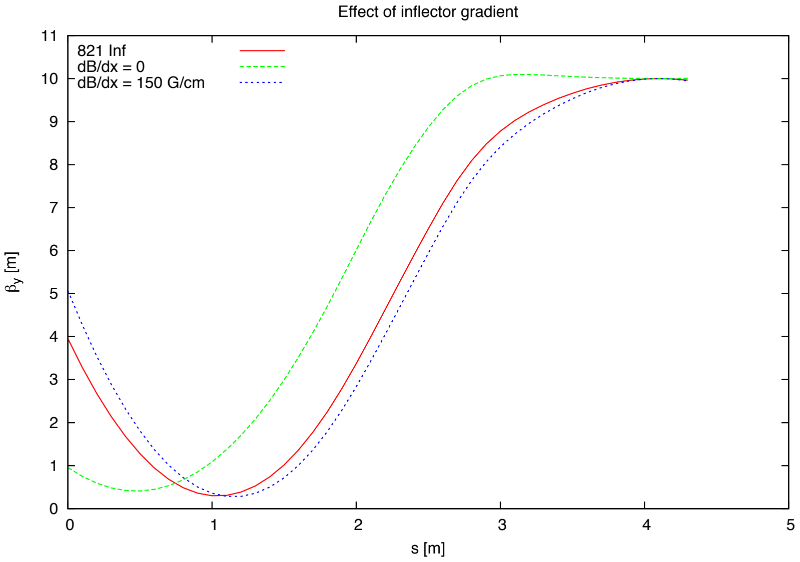
1. Construct M through the injection channel by tracking through the field maps
2. Use M for injection channel to propagate twiss in inflector $\vec{\gamma}_I$ back to end of M5 $\vec{\gamma}_{M5}$
3. Compute twiss $\vec{\gamma}_D$ for Diktys's distribution, (which is defined at end of M5)
4. Construct $M(\vec{\gamma}_D, \vec{\gamma}_{M5})$ and use it to transform the distribution to the target twiss
5. The transformed distribution will be characterized by $\vec{\gamma}_{M5}$ at the start of tracking and by $\vec{\gamma}_I$ when tracked (using M) to the inflector

$dB_y/dx = 150G/cm$



Horizontal beta

Vertical beta

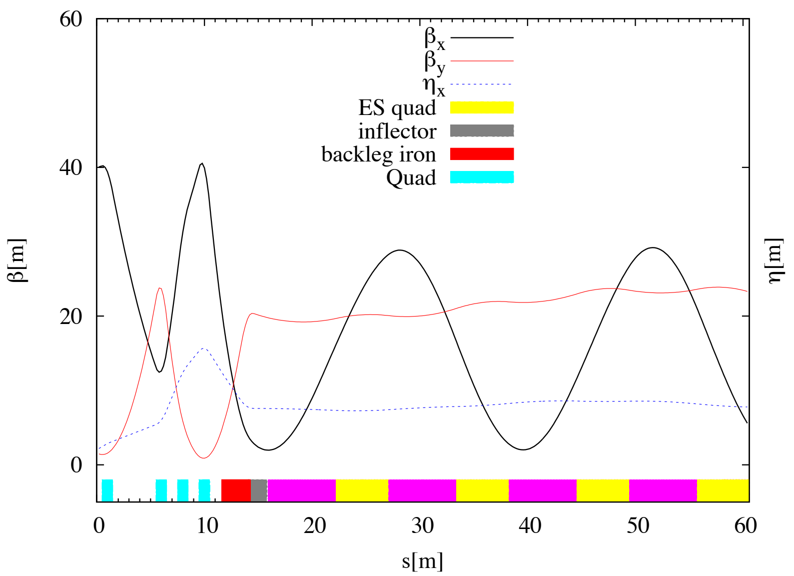


Example

Phase space matching

- 40 mm-mrad clears inflector if at inflector midpoint β_x=2m, β_y=19m
- Transfer map for propagation of twiss parameters determined by tracking through Wuzeng field map

$$T = \begin{pmatrix} 1.61951 & 5.68287 & -0.10265 & -0.12099 & 0.00032 & 0.02890 \\ 0.29750 & 1.65985 & -0.03583 & -0.05500 & 0.00025 & 0.00047 \\ -0.08510 & -0.05582 & 0.44649 & 3.19115 & -0.00014 & 0.00884 \\ -0.02370 & -0.04136 & -0.25378 & 0.40416 & -0.00010 & -0.00031 \\ 0.00776 & 0.04529 & -0.00296 & 0.00323 & 1.00000 & 0.00046 \\ 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 1.00000 \end{pmatrix}$$



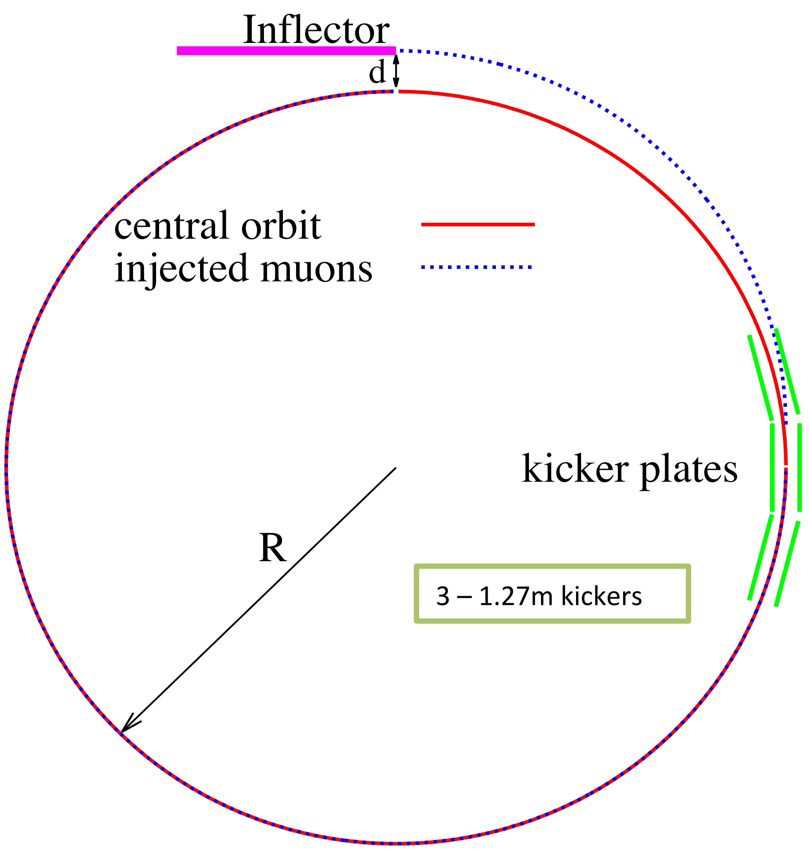
Large aperture inflector - Kickers

Kicker fields scale with displacement *d*

With larger aperture inflector
  d = 77mm => 91mm

Kick angle ~ 10.8 to 12.8 mrad
Integrated B-field ~ 1.11 to 1.31 kG-m

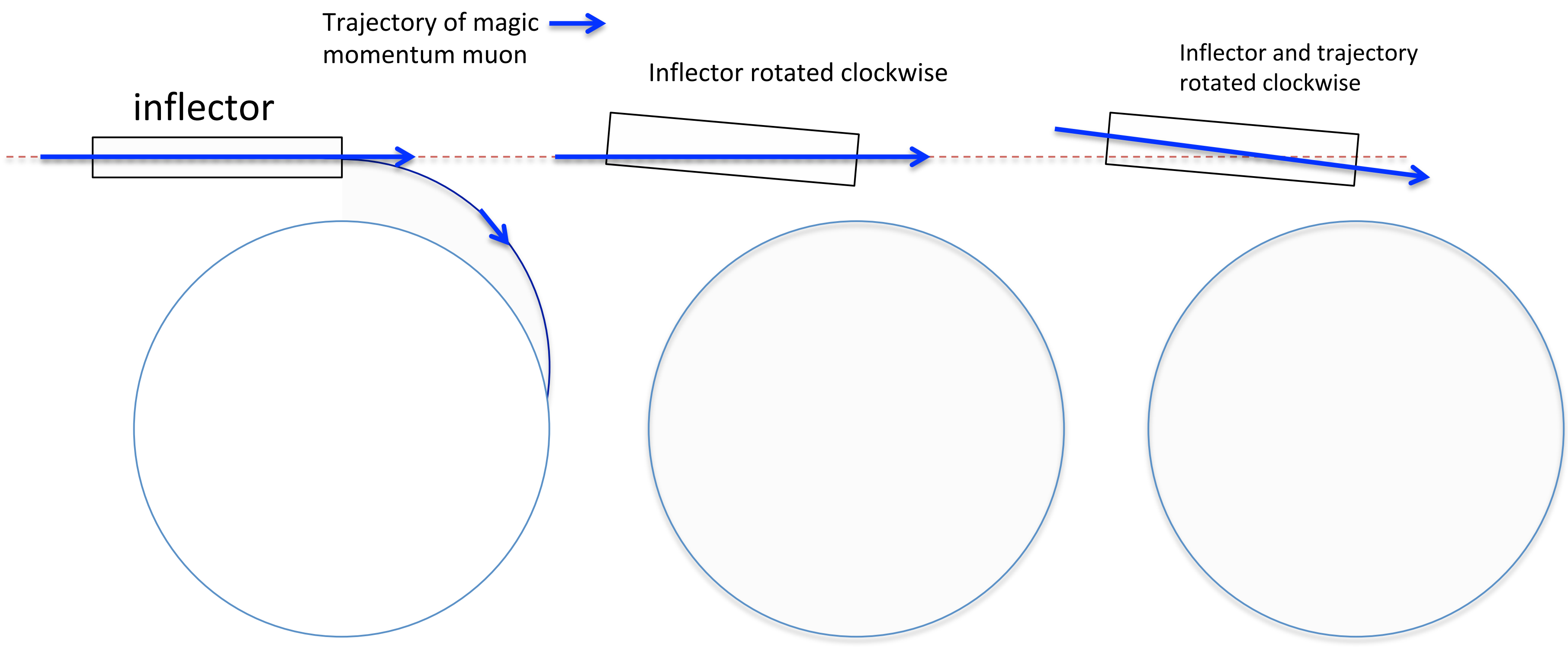
Kick Angle = d/R
$R = R_0(1+\delta)$
$\theta = \theta_0(1-\delta)$



τ_{rev} = 149 ns

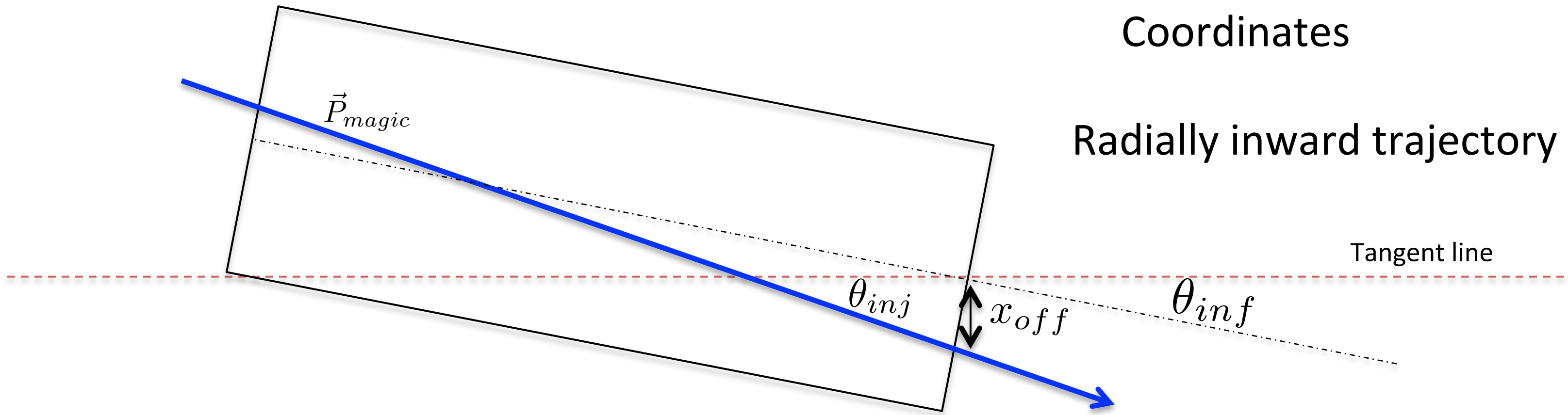
Orientation

Injection channel

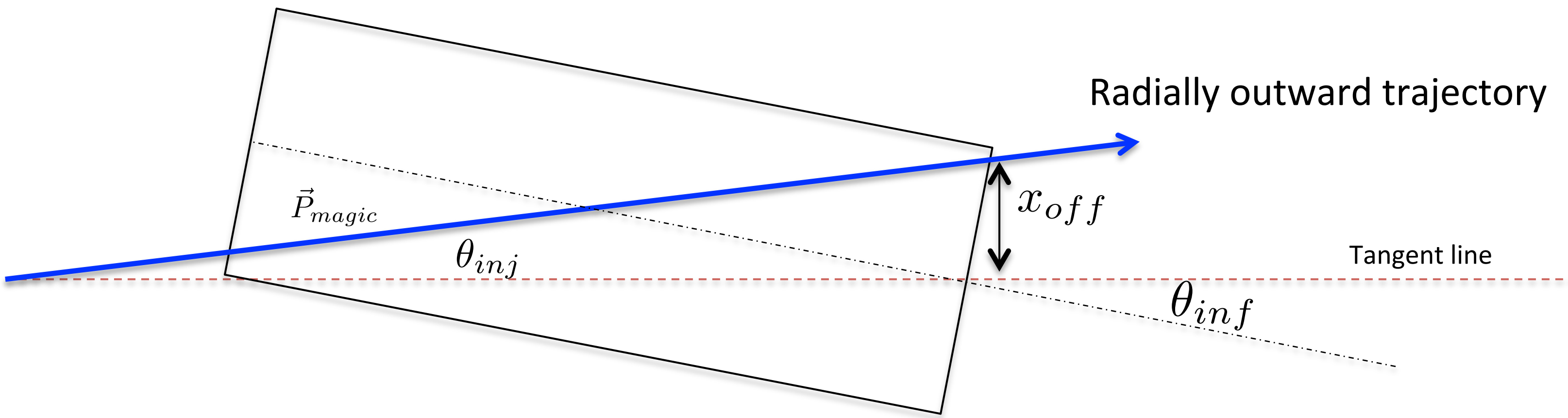


Coordinates

Radially inward trajectory $(\theta_{inj} < 0)$



Radially outward trajectory $(\theta_{inj} > 0)$



Angles and offset of magic momentum trajectory

Trajectory of injected particle

$$x_{inf} = x_{\beta 0} + x_{\delta 0}$$
$$= x_{\beta 0} + \eta\delta$$

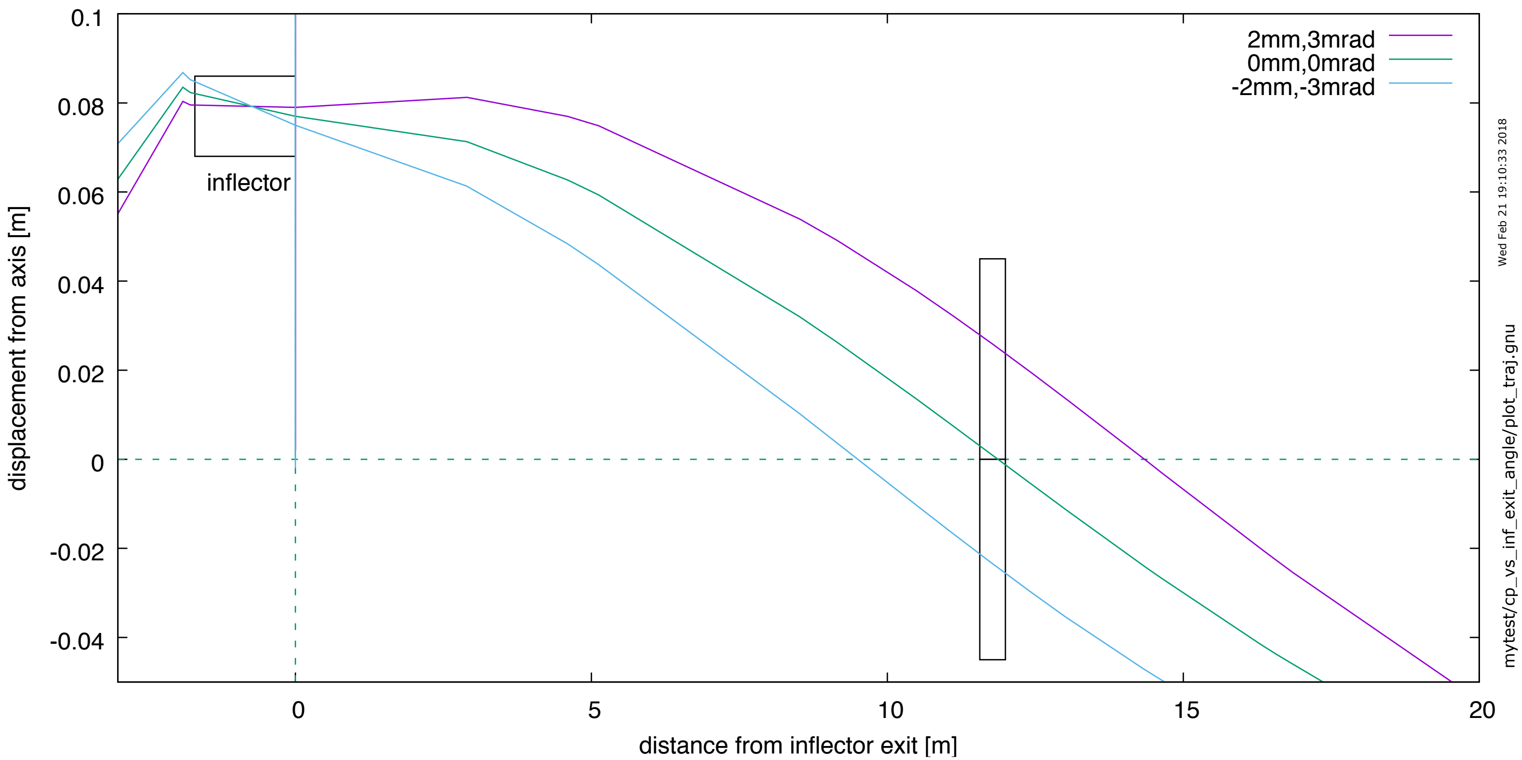
$$x_{\beta 0} = x_{inf} - \eta(0)\delta$$

Trajectory of injected particle

$$x_{inf} = x_{\beta 0} + x_{\delta 0}$$
$$= x_{\beta 0} + \eta\delta$$

$$x_{\beta 0} = x_{inf} - \eta(0)\delta$$

$$x(s) = x_{\beta 0}\sqrt{\frac{\beta(s)}{\beta(0)}}\cos(\phi(s)) + \eta(s)\delta$$



2mm,3mrad
0mm,0mrad
-2mm,-3mrad
inflector
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
0
5
10
15
20
displacement from axis [m]
distance from inflector exit [m]
Wed Feb 21 19:10:33 2018
mytest/cp_vs_inf_exit_angle/plot_traj.gnu

Trajectory of injected particle

$$x_{inf} = x_{\beta 0} + x_{\delta 0}$$
$$= x_{\beta 0} + \eta\delta$$

$$x_{\beta 0} = x_{inf} - \eta(0)\delta$$

$$x(s) = x_{\beta 0}\sqrt{\frac{\beta(s)}{\beta(0)}}\cos(\phi(s)) + \eta(s)\delta$$

$$\Delta x(s) = k\sqrt{\beta_{kick}\beta(s)}\sin(\phi(s) - \pi/2)$$ Change due to kicker

Trajectory of injected particle

$$x_{inf} = x_{\beta 0} + x_{\delta 0}$$
$$= x_{\beta 0} + \eta\delta$$

$$x_{\beta 0} = x_{inf} - \eta(0)\delta$$

$$x(s) = x_{\beta 0}\sqrt{\frac{\beta(s)}{\beta(0)}}\cos(\phi(s)) + \eta(s)\delta$$

$$\Delta x(s) = k\sqrt{\beta_{kick}\beta(s)}\sin(\phi(s) - \pi/2)$$

Change due to kicker

$$x(s) = (x_{inf} - \delta\eta_0 - k\beta_0)\cos(\phi(s)) + \eta_0\delta$$

All together

Trajectory of injected particle

$$x_{inf} = x_{\beta 0} + x_{\delta 0}$$
$$= x_{\beta 0} + \eta\delta$$

$$x_{\beta 0} = x_{inf} - \eta(0)\delta$$

$$x(s) = x_{\beta 0}\sqrt{\frac{\beta(s)}{\beta(0)}}\cos(\phi(s)) + \eta(s)\delta$$

$$\Delta x(s) = k\sqrt{\beta_{kick}\beta(s)}\sin(\phi(s) - \pi/2)$$

Change due to kicker

$$x(s) = (x_{inf} - \delta\eta_0 - k\beta_0)\cos(\phi(s)) + \eta_0\delta$$

All together

$$\begin{aligned} x_{min} &= x_{inf} - k\beta_0 \\ x_{max} &= -x_{inf} + k\beta_0 + 2\eta_0\delta \end{aligned}$$

Perfect kick

$$k\beta_0 = x_{inf}$$

If the particle exits the inflector with some angle x'_{inj} (with respect to the tangent line)

Then before it reaches the kicker

$$x(s) = x_{\beta 0} \cos\phi(s) + x'_{xinf}\beta_0 \sin\phi(s) + \eta_0\delta$$

If the particle exits the inflector with some angle x'_{inj} (with respect to the tangent line)

Then before it reaches the kicker

$$x(s) = x_{\beta 0} \cos\phi(s) + x'_{xinf}\beta_0 \sin\phi(s) + \eta_0\delta$$

And beyond the kicker

$$\begin{aligned} x(s) &= (x_{inf} - \delta\eta_0)\cos(\phi(s)) + \eta_0\delta - k\beta_0\cos(\phi(s)) + x'_{inf}\beta_0 \sin\phi(s) \\ &= A\cos(\phi(s) + \phi_0) + \eta_0\delta \end{aligned}$$

where

$$A = \pm\sqrt{(x_{inf} - \delta\eta_0 - k\beta_0)^2 + (x'_{inf}\beta_0)^2}$$ (CBO amplitude)

and

$$\tan\phi_0 = \frac{x'_{inf}\beta_0}{x_{inf} - \delta\eta_0 - k\beta_0}$$ (CBO phase)

The extremes of displacement are

$$x_{ext} = \pm|A| + \eta_0\delta$$ (CBO envelope)

$$A = \pm\sqrt{(x_{inf} - \delta\eta_0 - k\beta_0)^2 + (x'_{inf}\beta_0)^2}$$

$$\rightarrow |A| \geq x'_{inf}\beta_0 \sim (3 \text{ mrad})(7 \text{ m}) \sim 21 \text{ mm}$$

The kicker can not compensate a nonzero injection angle

$$A = \pm\sqrt{(x_{inf} - \delta\eta_0 - k\beta_0)^2 + (x'_{inf}\beta_0)^2}$$

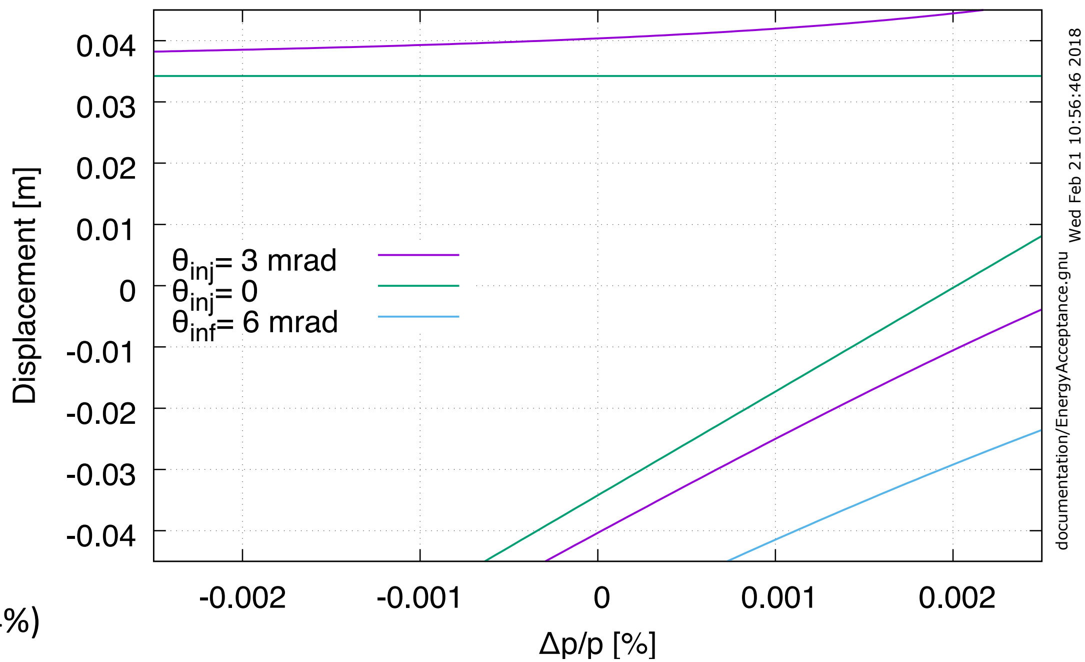
$$\rightarrow |A| \geq x'_{inf}\beta_0 \sim (3 \text{ mrad})(7 \text{ m}) \sim 21 \text{ mm}$$

The kicker can not compensate a nonzero injection angle

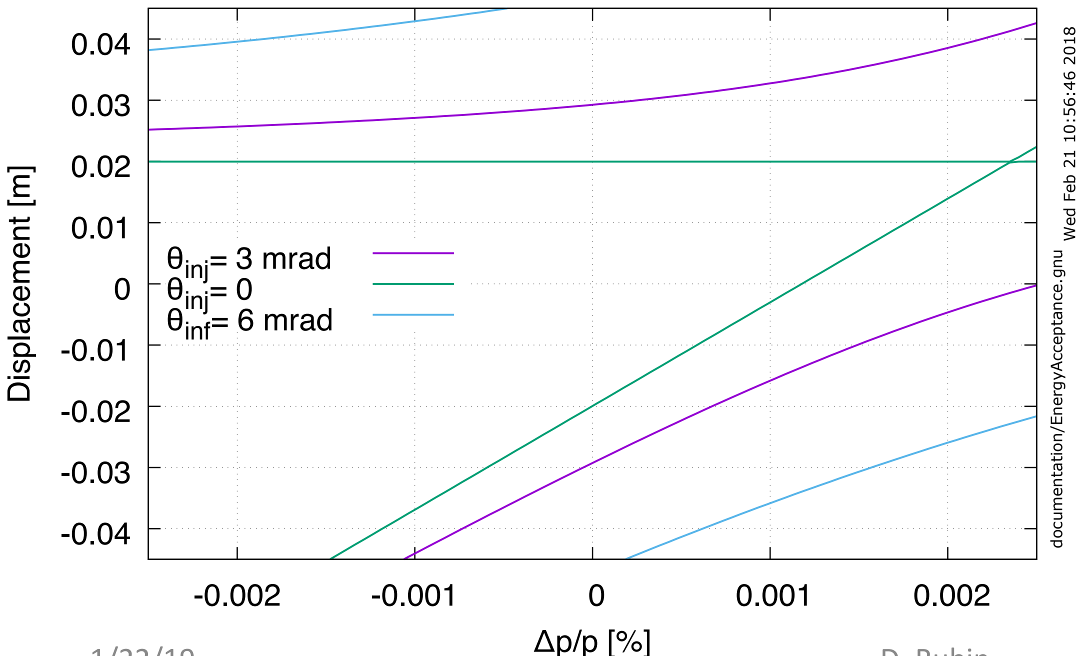
Set $\left\{ \begin{array}{l} x_{inf} = 77 \text{ mm} \\ \eta_0 \sim 8 \text{ m} \\ \beta_0 \sim 7 \text{ m} \end{array} \right.$ And vary kick *k*, momentum δ, θ_{inj}

$$A = \pm\sqrt{(x_{inf} - \delta\eta_0 - k\beta_0)^2 + (x'_{inf}\beta_0)^2}$$

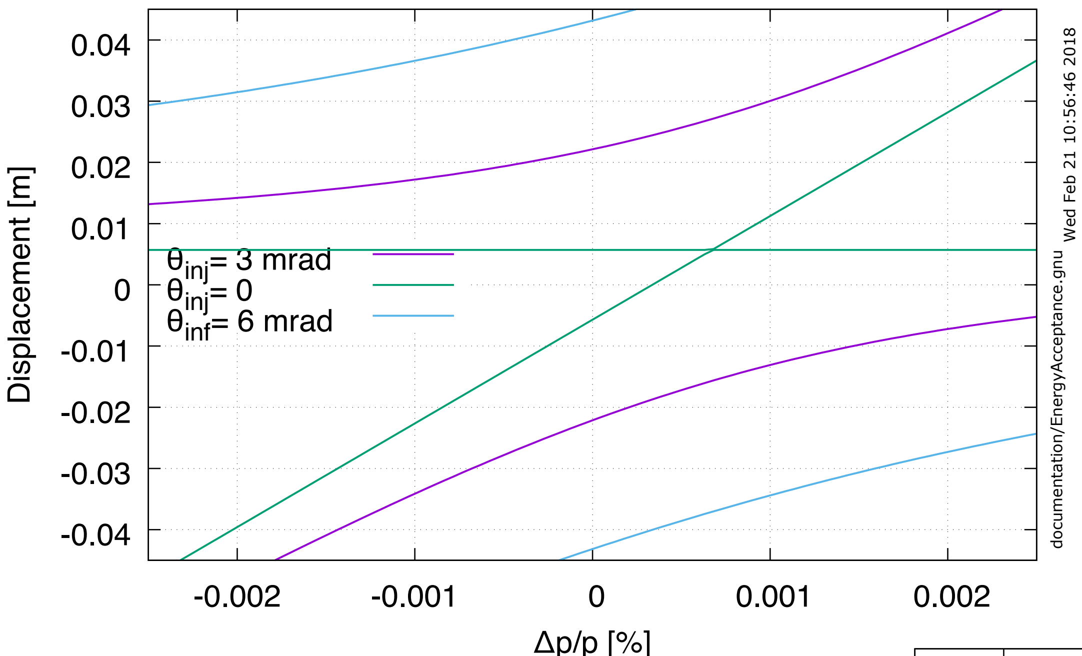
kick angle = 6 mrad (56%)



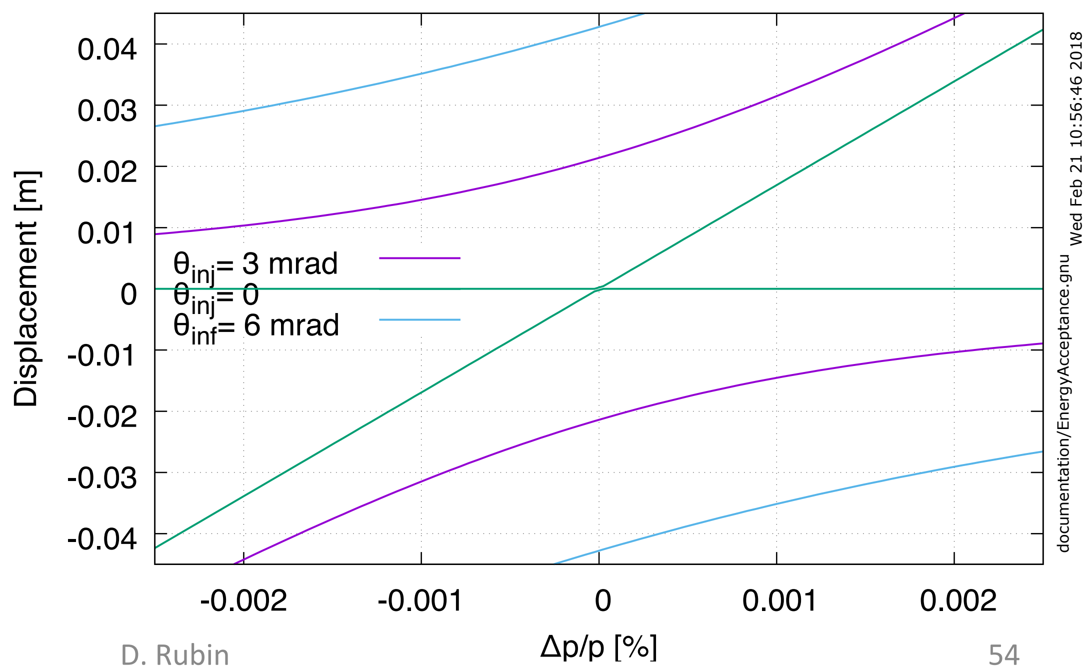
kick angle = 8 mrad (74%)

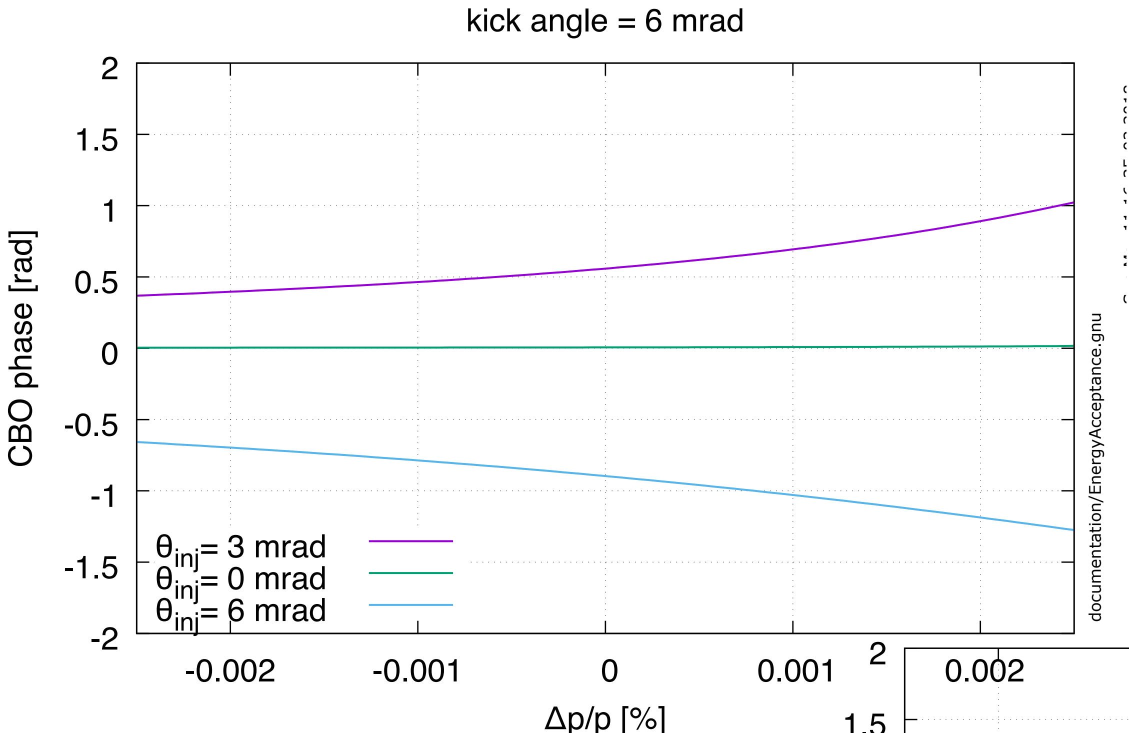


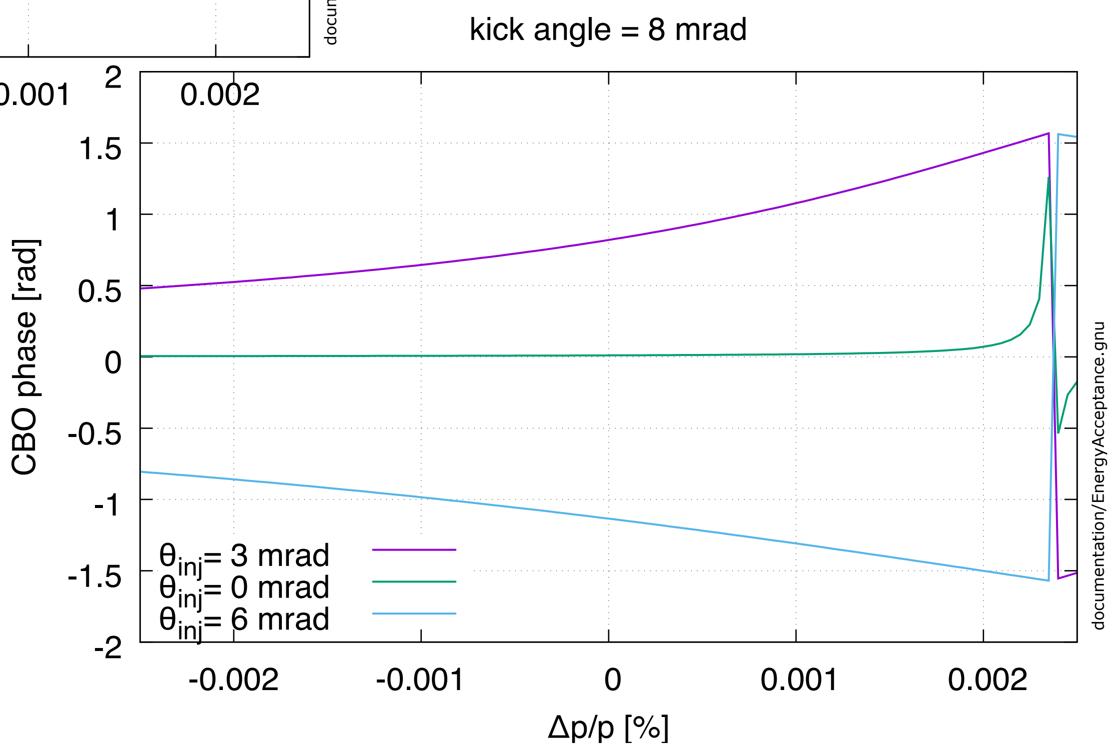
kick angle = 10 mrad (93%)

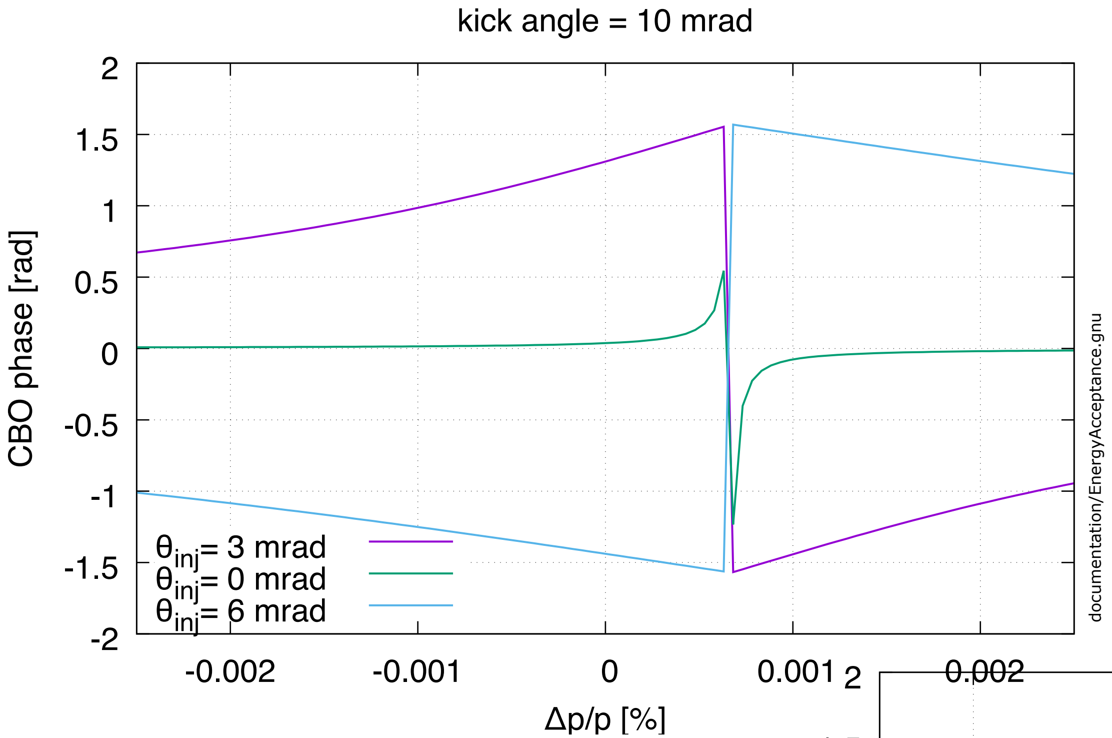


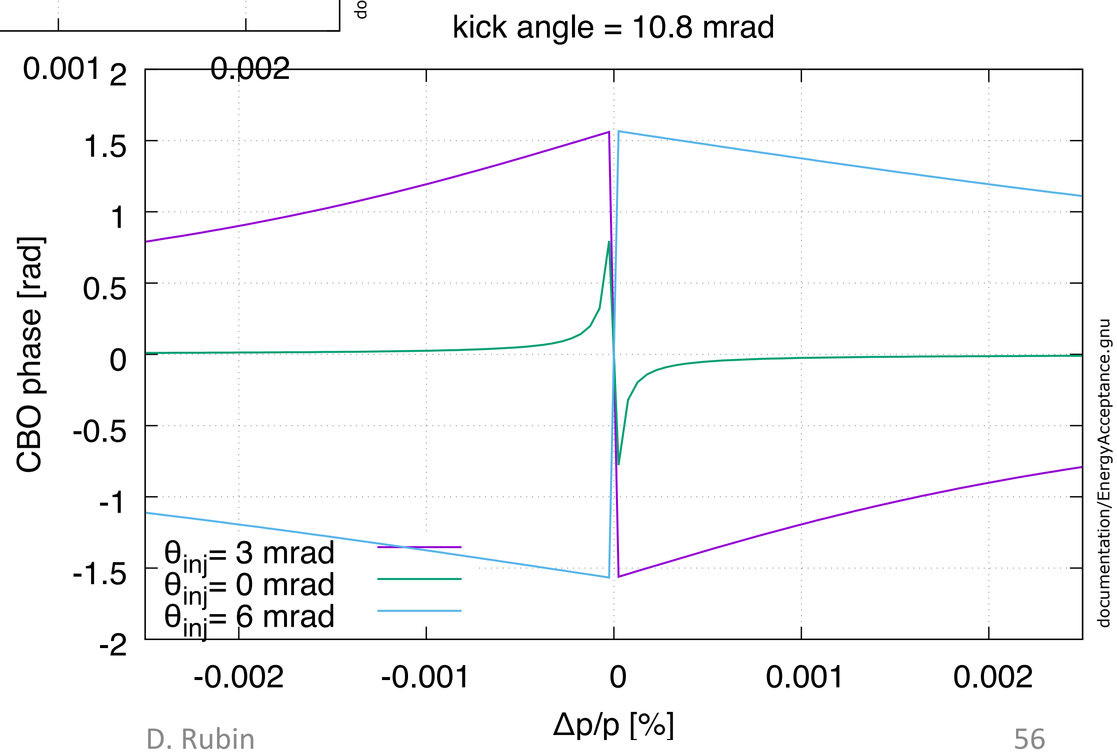
kick angle = 10.8 mrad (100%)




kick angle = 6 mrad
CBO phase [rad]
Δp/p [%]
θinj= 3 mrad
θinj= 0 mrad
θinj= 6 mrad
documentation/EnergyAcceptance.gnu


kick angle = 8 mrad
CBO phase [rad]
Δp/p [%]
θinj= 3 mrad
θinj= 0 mrad
θinj= 6 mrad
documentation/EnergyAcceptance.gnu


kick angle = 10 mrad
CBO phase [rad]
Δp/p [%]
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-0.002
-0.001
0
0.001
0.002
θinj= 3 mrad
θinj= 0 mrad
θinj= 6 mrad
documentation/EnergyAcceptance.gnu


kick angle = 10.8 mrad
CBO phase [rad]
Δp/p [%]
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-0.002
-0.001
0
0.001
0.002
θinj= 3 mrad
θinj= 0 mrad
θinj= 6 mrad
documentation/EnergyAcceptance.gnu

Suppose there is finite dispersion at the inflector exit

$$A = \pm\sqrt{(x_{inf} - \delta(\eta_0 - \eta_{inf}) - k\beta_0)^2 + ((x'_{inf} + \eta'_{inf}\delta)\beta_0)^2}$$

and

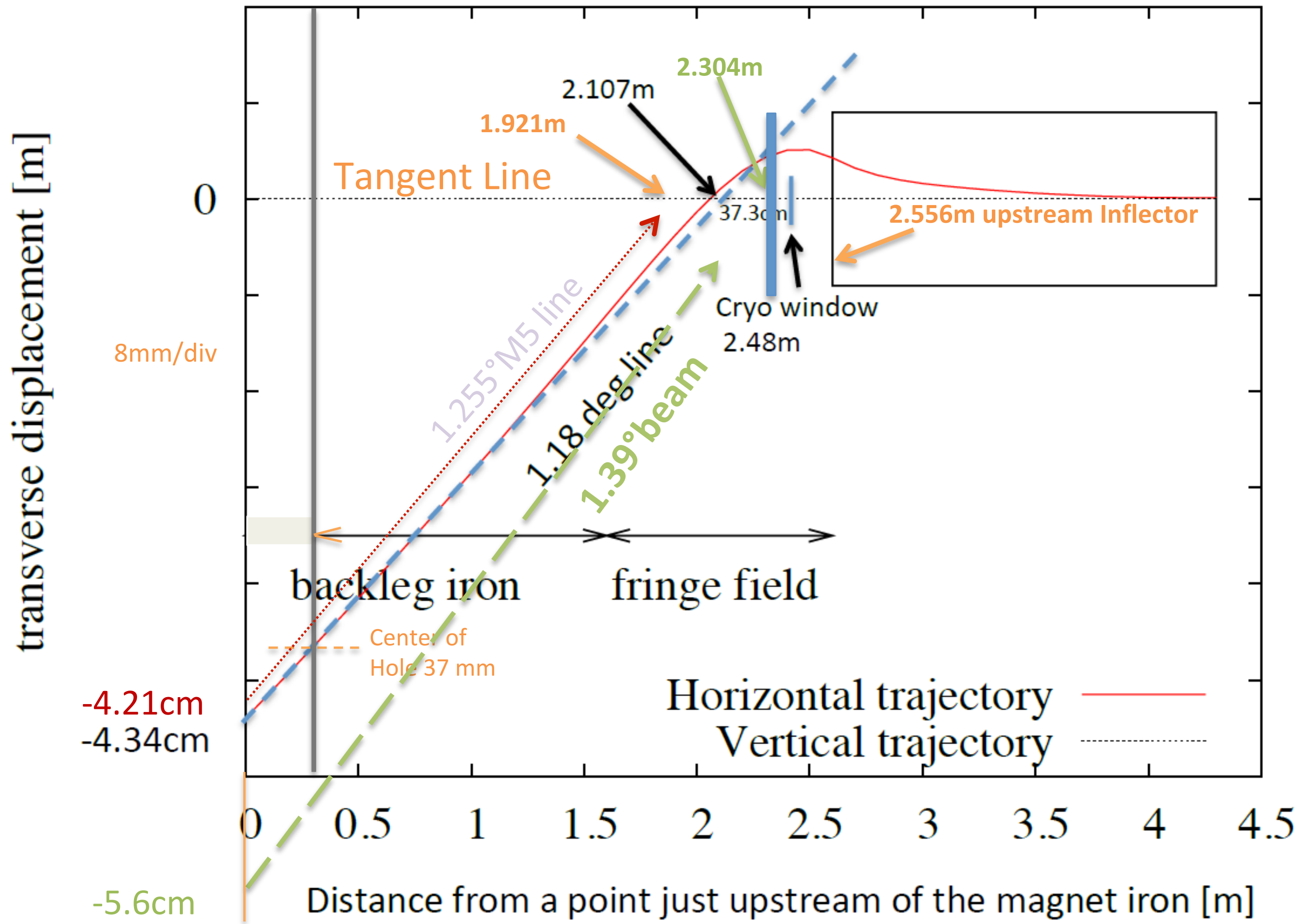
$$\tan\phi_0 = \frac{x_{inf} - \delta(\eta_0 - \eta_{inf}) - k\beta_0}{(x'_{inf} + \eta'_{inf})\beta_0}$$

The extremes of displacement become

$$x_{ext} = \pm|A| + (\eta_0 - \eta_{inf})\delta$$

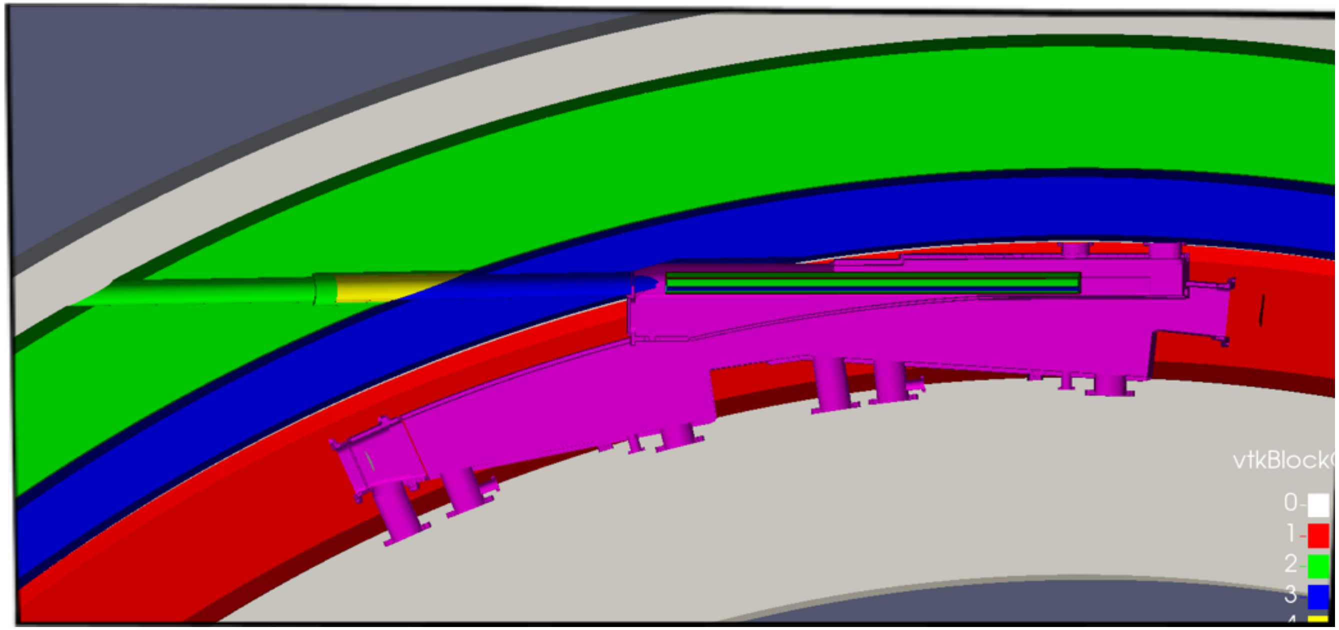
Some Facts

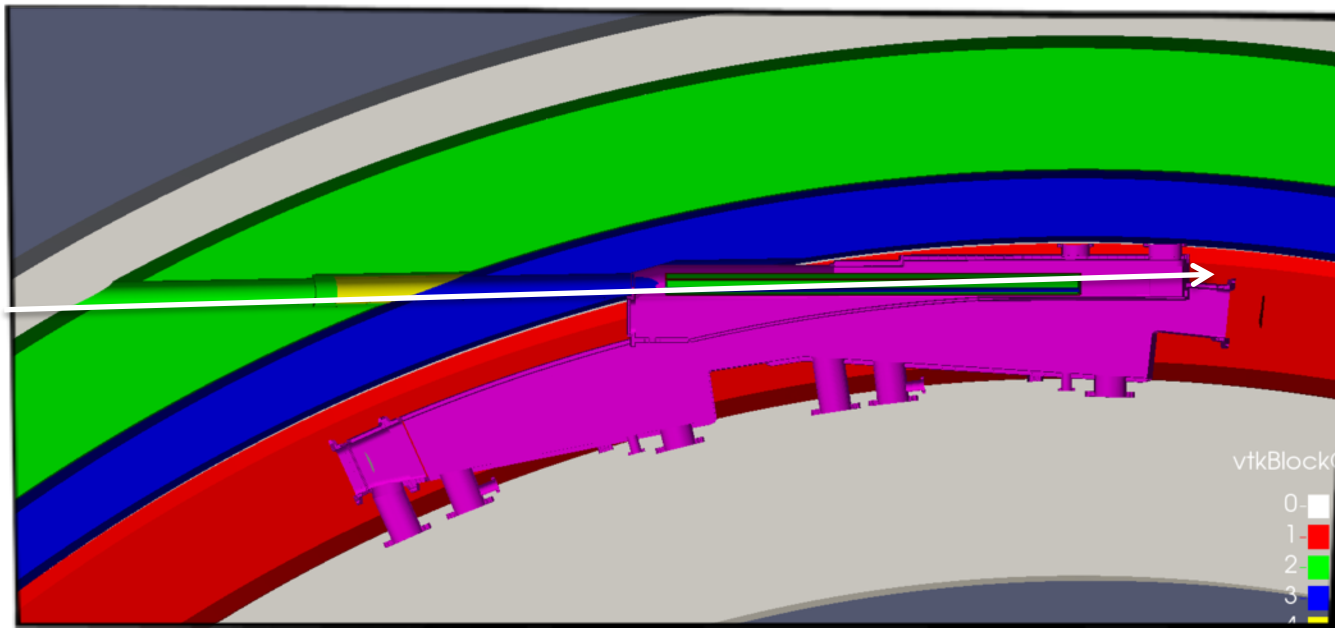
- Storage fraction is optimal if beam enters inflector with outward going angle (+3.5 mrad)
- Inflector is rotated about downstream end by -2.3 mrad (pointing radially inward)
- Storage optimized with B_{inf} ~ 1.5% high
- Storage fraction increases rapidly with kicker voltage (with no plateau in sight)
- CBO amplitude (~ 28mm peak-peak), depends weakly on kicker voltage and inflector field
- Closed orbit displaced 6-10 mm outward.
  - Displacement depends weakly on kicker voltage and inflector field
- De-coherence time is long; ~ 300 us => narrow momentum acceptance
- De-coherence time increases with kicker voltage
  - More muons are stored, but in a narrower momentum band?



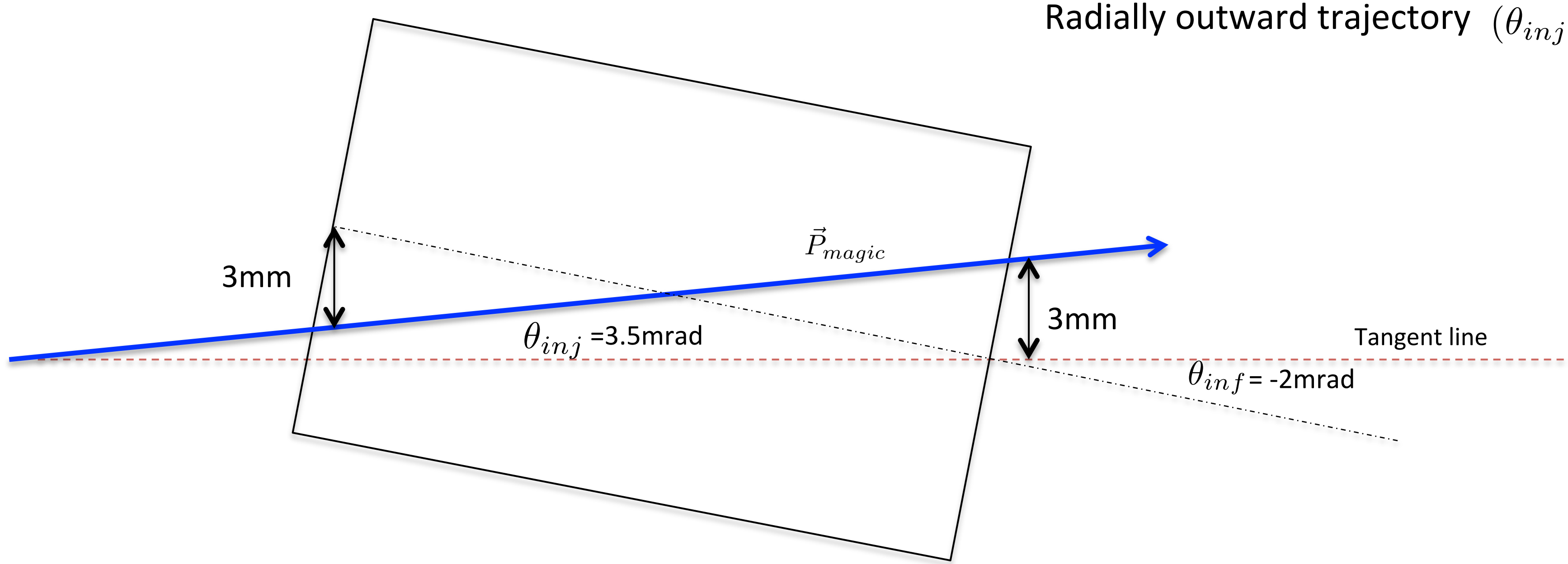
2.304m
2.107m
1.921m
Tangent Line
0
37.3cm
2.556m upstream Inflector
Cryo window
2.48m
8mm/div
1.255°M5 line
1.18 deg line
1.39°beam
transverse displacement [m]
backleg iron
fringe field
Center of
Hole 37 mm
Horizontal trajectory
Vertical trajectory
-4.21cm
-4.34cm
-5.6cm
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
Distance from a point just upstream of the magnet iron [m]

Jim Morgan

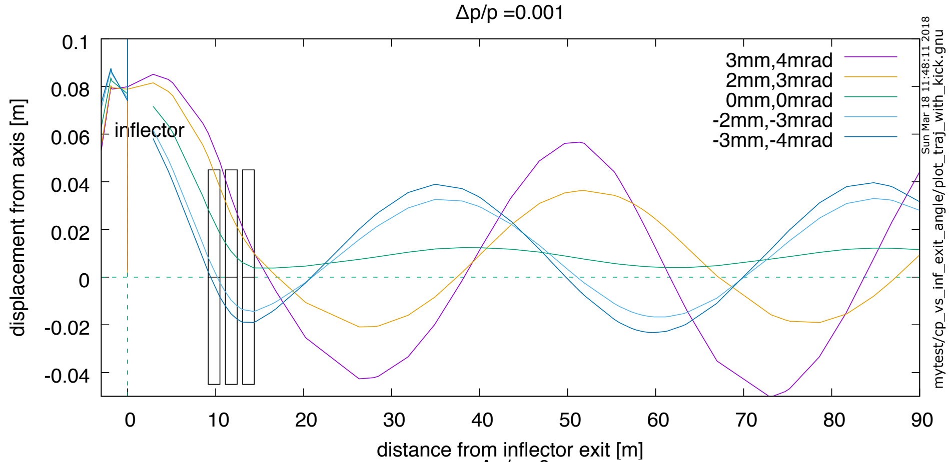

vtkBlock(
0
1
2
3
4


vtkBlock(
0
1
2
3

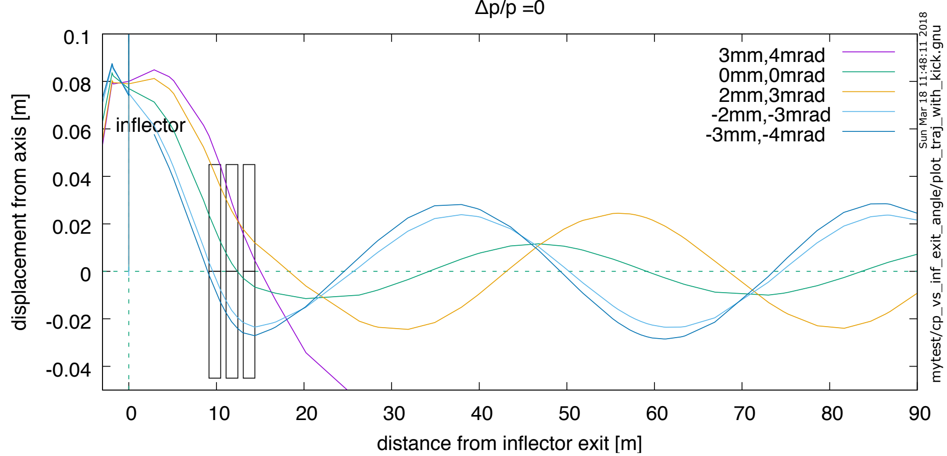
Radially outward trajectory $(\theta_{inj} > 0)$



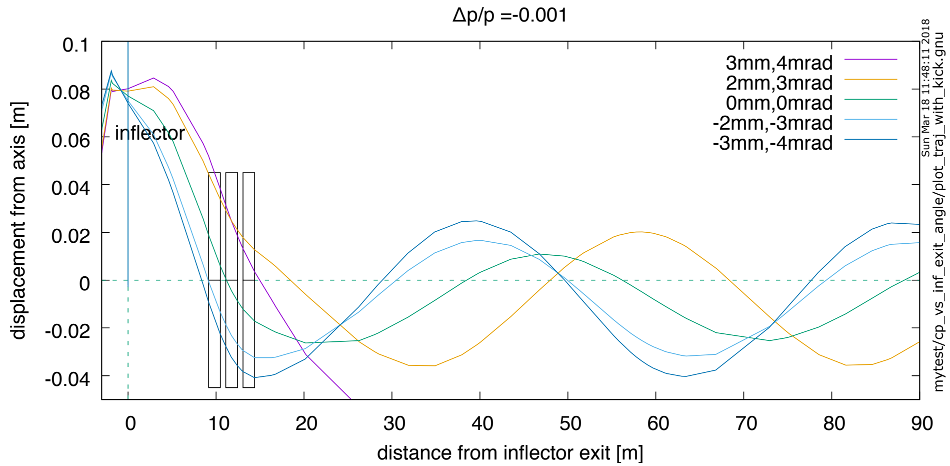
$$\Delta p/p = +0.001$$



Magic momentum



$$\Delta p/p = -0.001$$



Summary

What we know (or suspect)

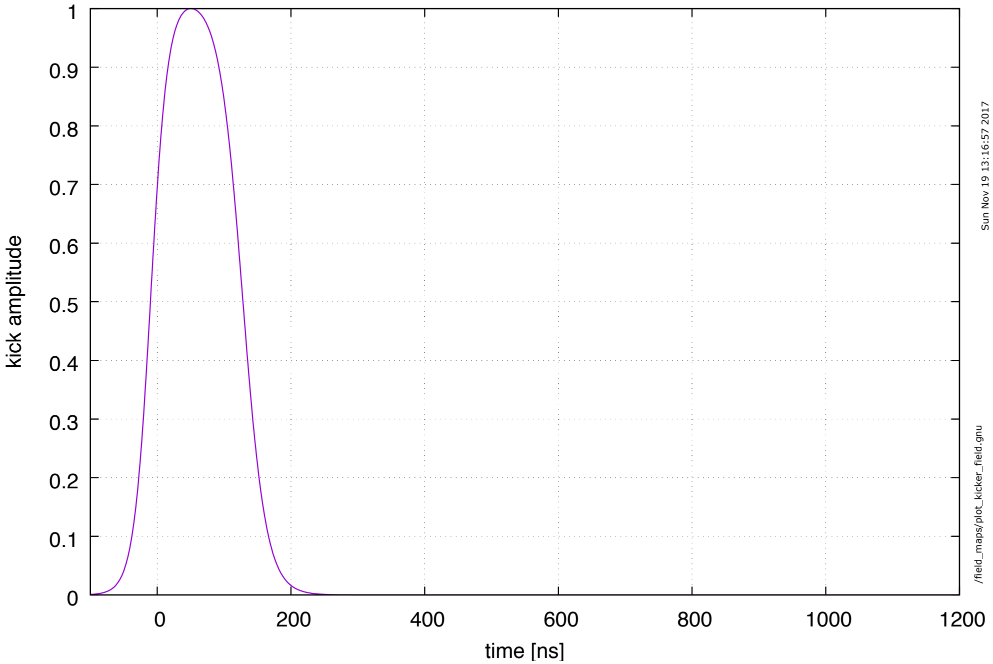
- Inflector is tilted clockwise about exit (-2.3 mrad)
- Beam trajectory from end of M5 and into backleg and inflector is directed radially outwards by a few mrad
    - ➢ Beam exiting inflector is likewise directed radially outwards
- 'Large' angle at inflector exit with respect to tangent line
    - ➢ CBO amplitude depends weakly on kick
    - ➢ Momentum acceptance skewed high
    - ➢ Optimal kick is 'high'
- Optimal inflector current (inflector exit angle) is high
- Kicker field is ?

There are alternative scenarios (parameter sets) that are equally compelling;

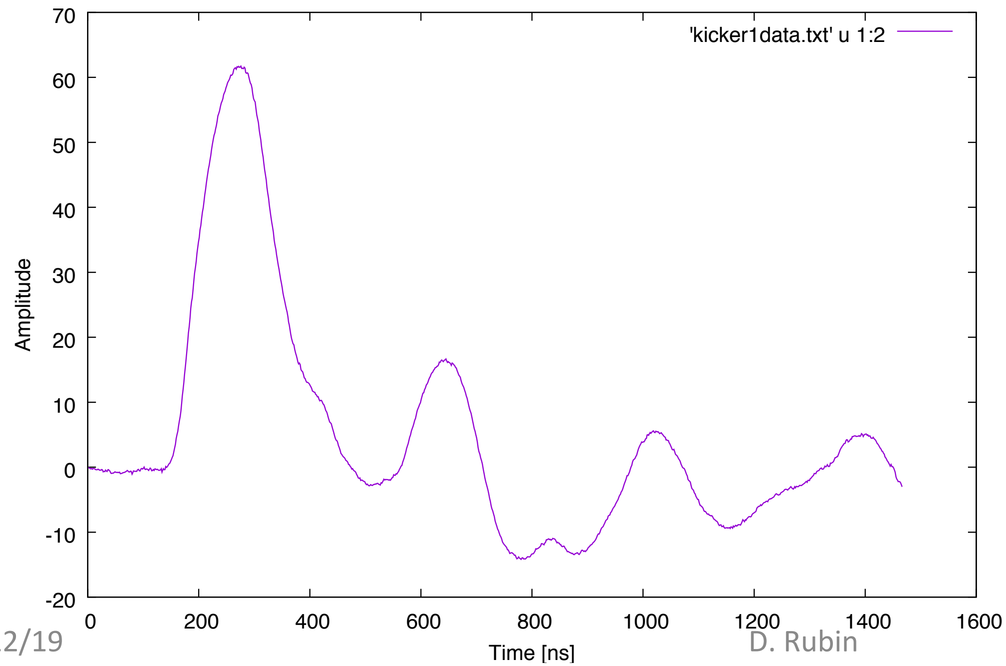
- *trajectory exiting inflector is radially inward and kicker field is low*
- *Trajectory is straight ahead and kicker field is very low*

With our now stable conditions

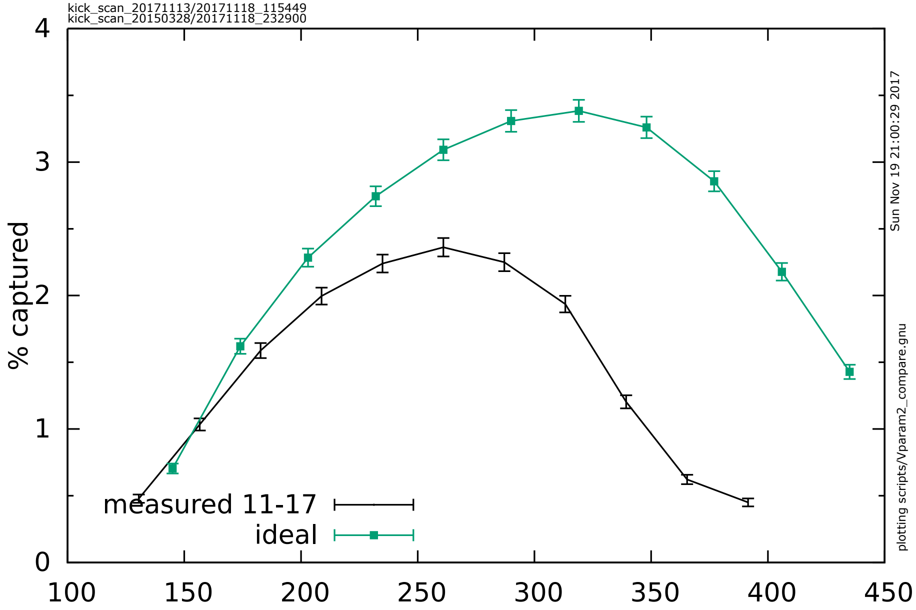
Data from dedicated scans of inflector field (exit angle) and kicker voltage, with fiber harps would help to constrain the phase space of solutions



‘Ideal’ kicker pulse

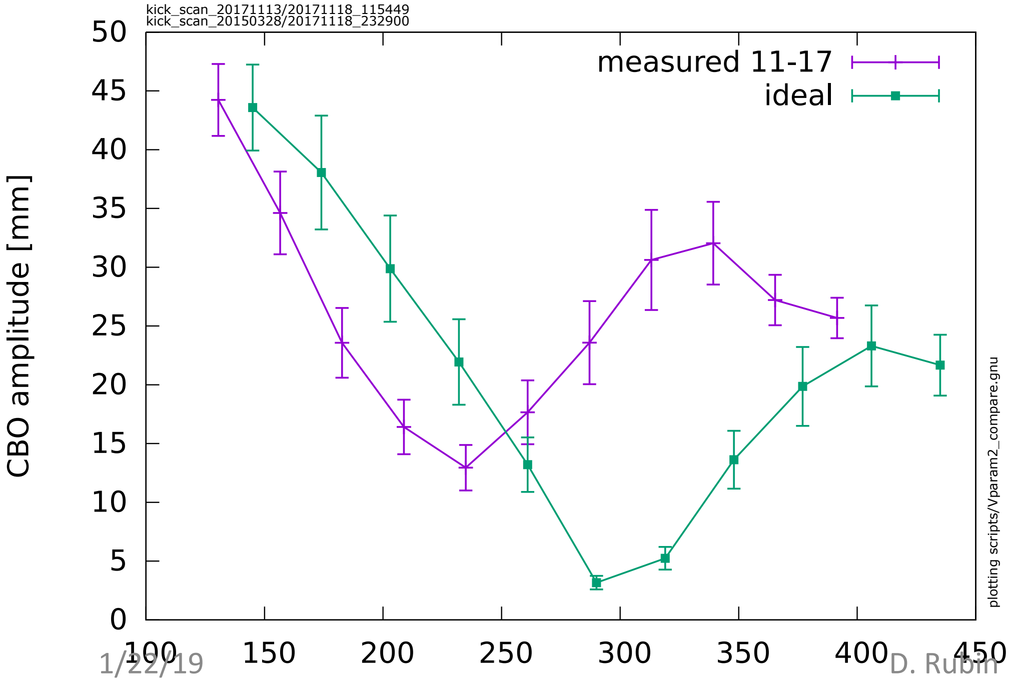


Magnetometer measurement of kicker 1

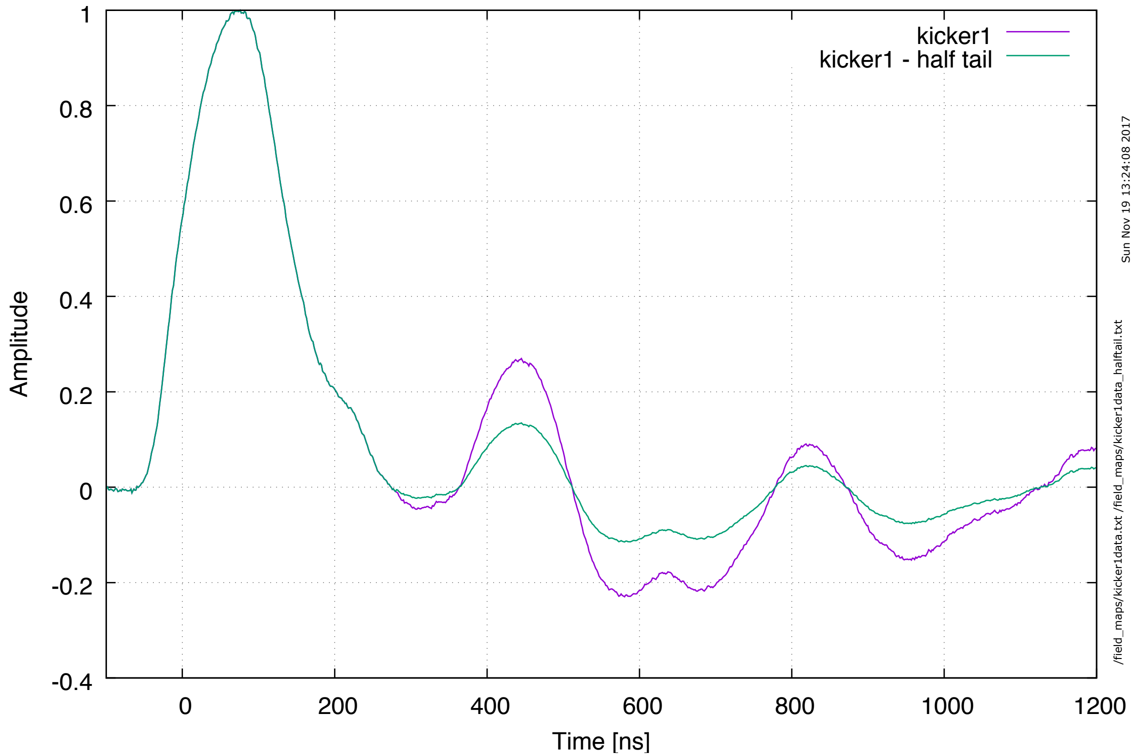


Measured/ideal = 70%

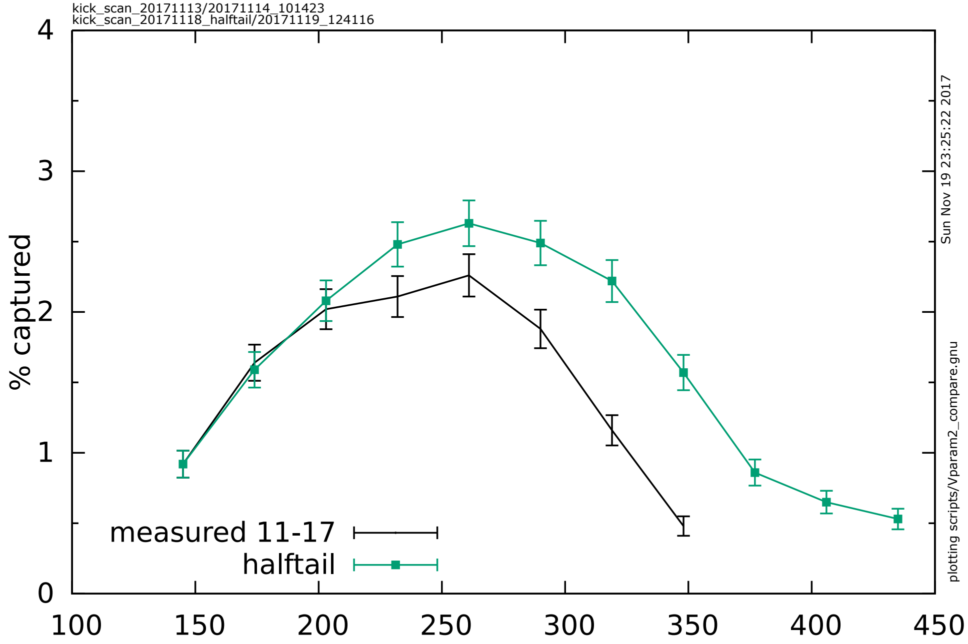
Capture vs pulse shape



Residual CBO vs pulse shape



Suppose we are able to reduce the secondary pulses by 1/2



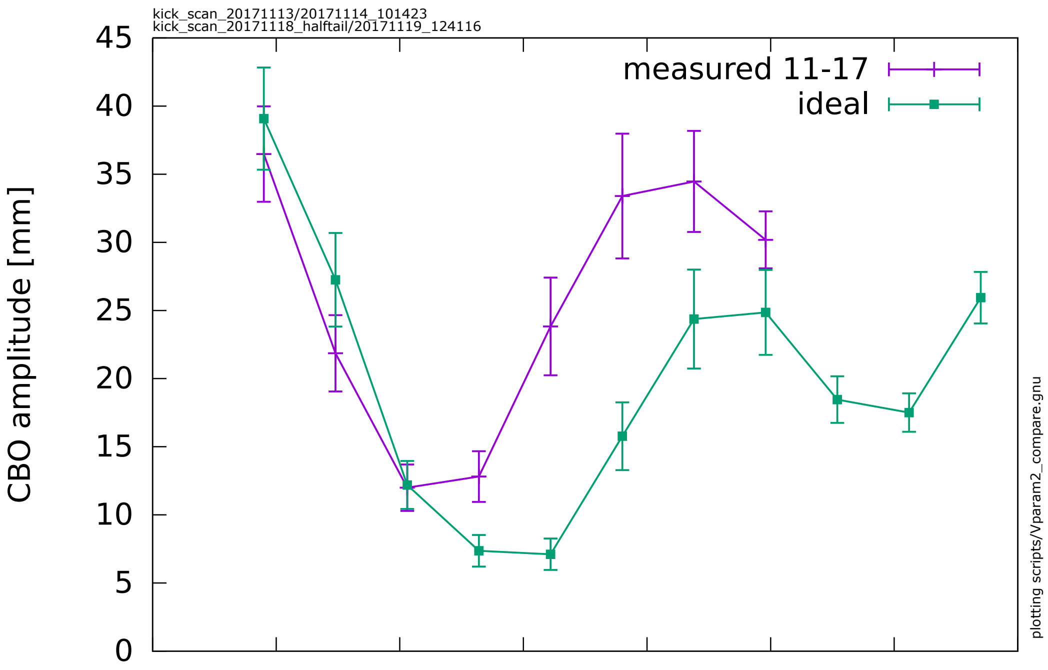
Compare

- Measured pulse shape
- Measured with ½ tail

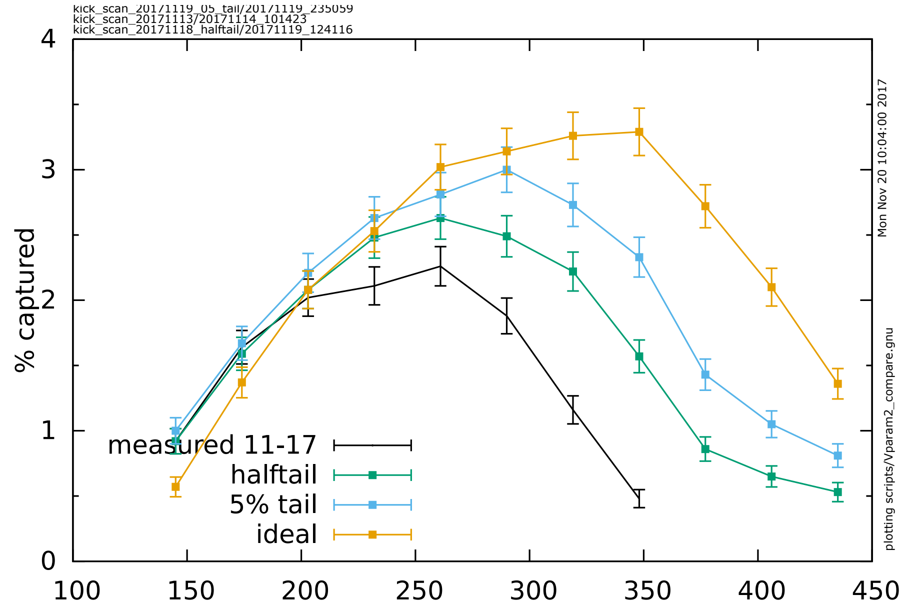
Capture efficiency

Reduced tail/ideal = 78%

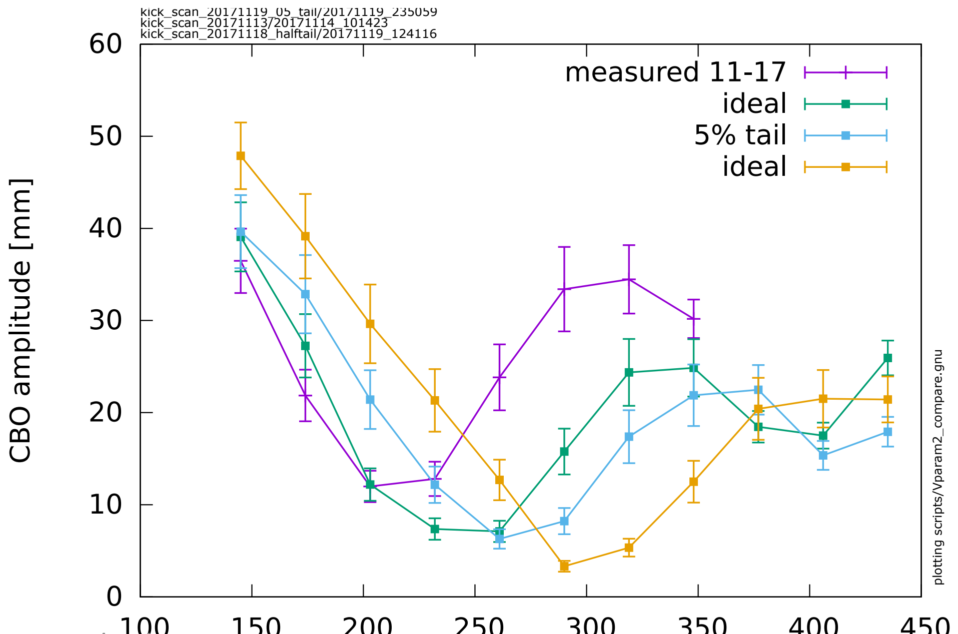
Measured/reduced tail = 86%



Residual CBO

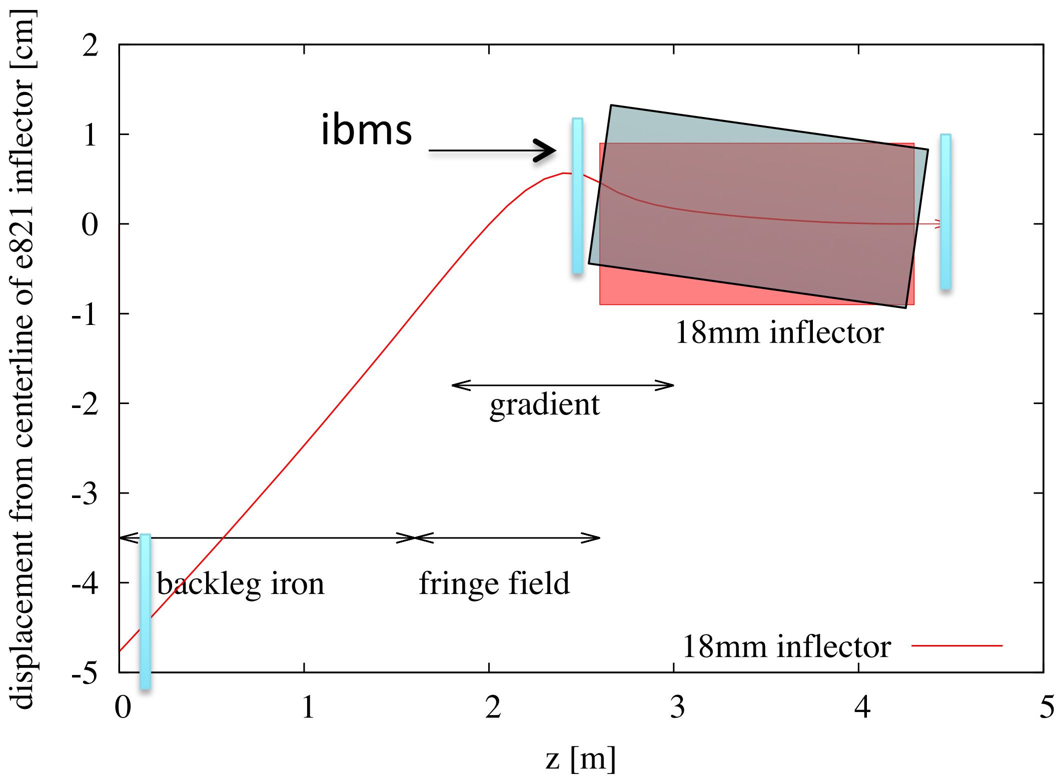


Capture vs pulse shape

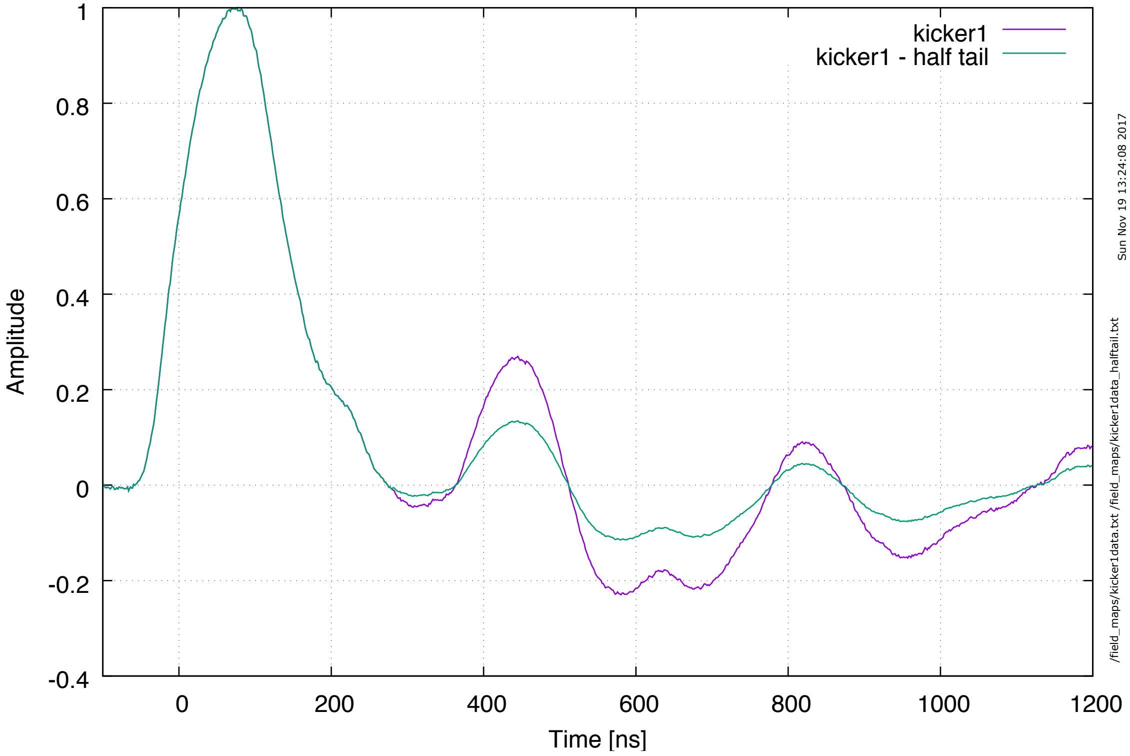


Residual CBO vs pulse shape

Trajectory through injection channel

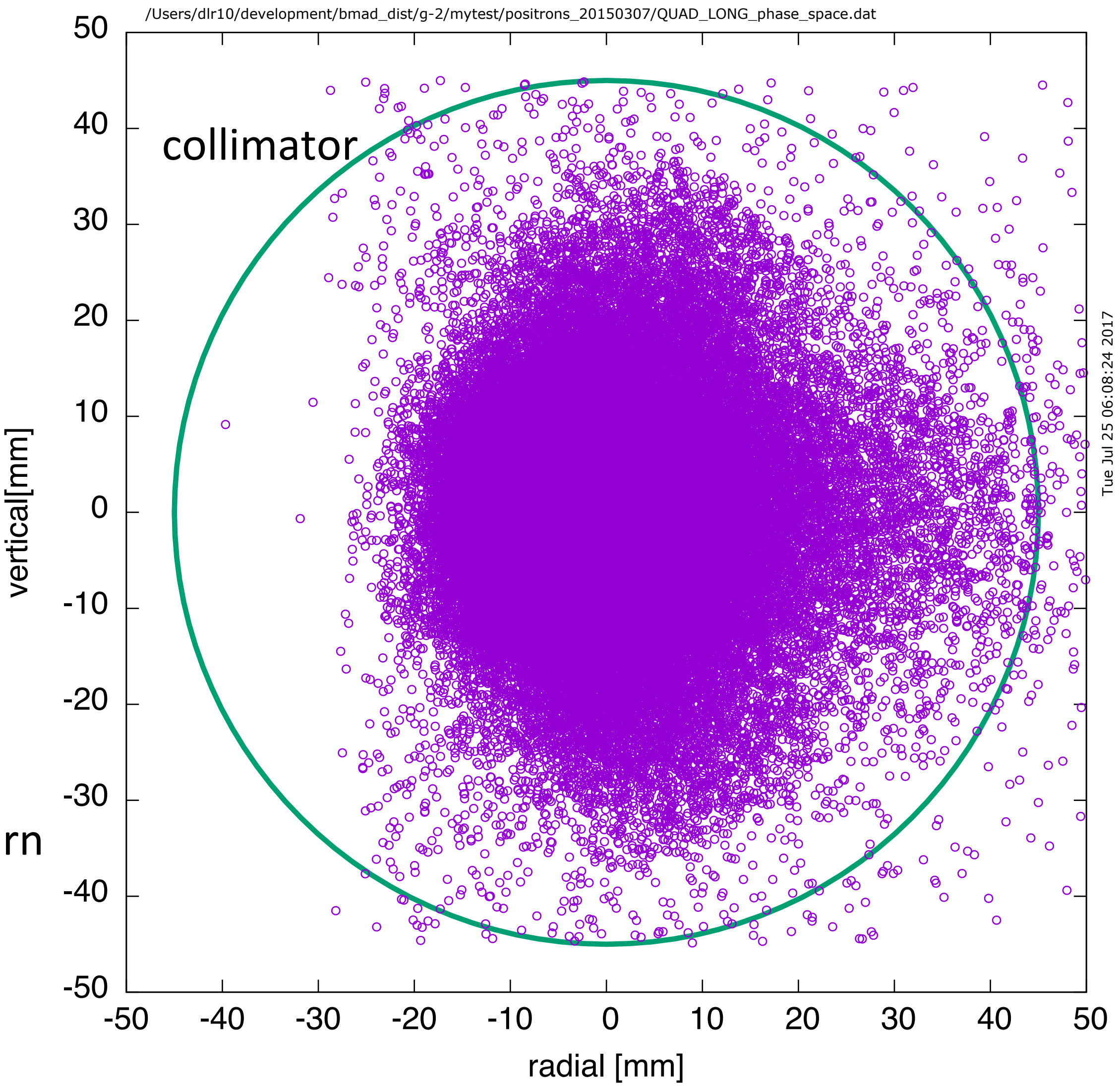


Beam exits centered in inflector tangent to and displaced 77mm from ring central orbit



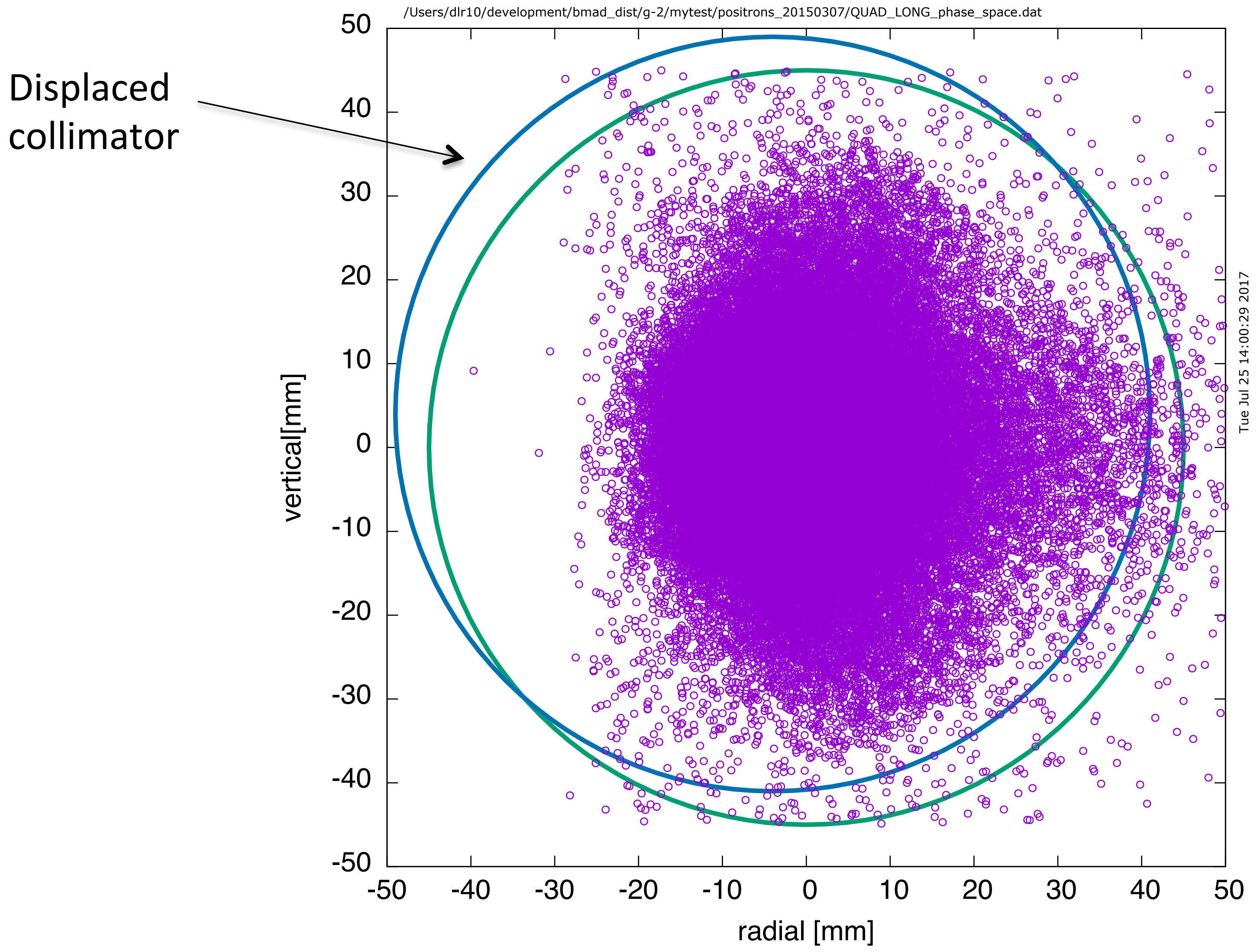
kicker1
kicker1 - half tail
Amplitude
Time [ns]
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
200
400
600
800
1000
1200
Sun Nov 19 13:24:08 2017
/field_maps/kicker1data.txt /field_maps/kicker1data_halftail.txt

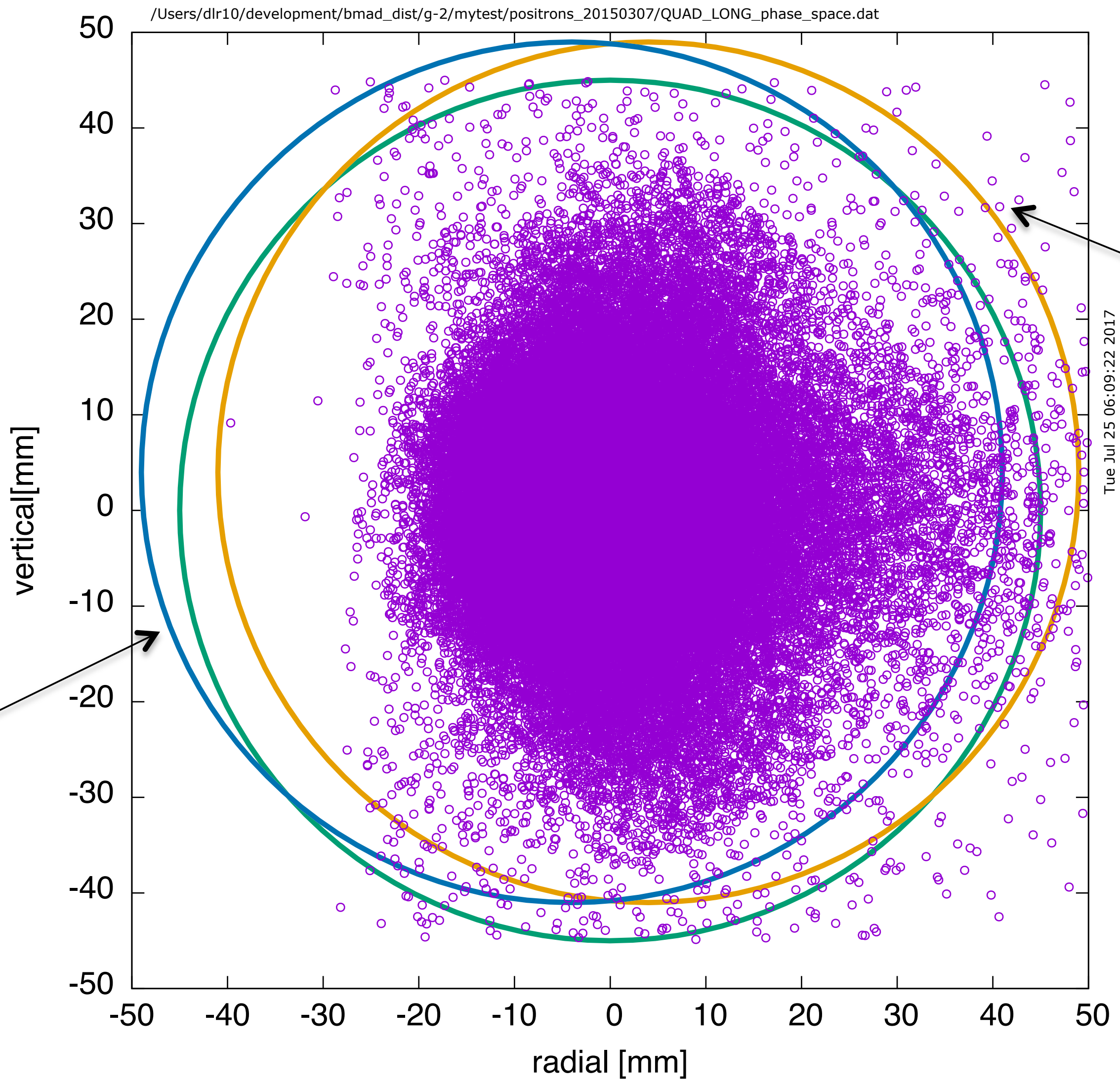
Scraping



Distribution on first turn

Displace collimator to scrape off tails





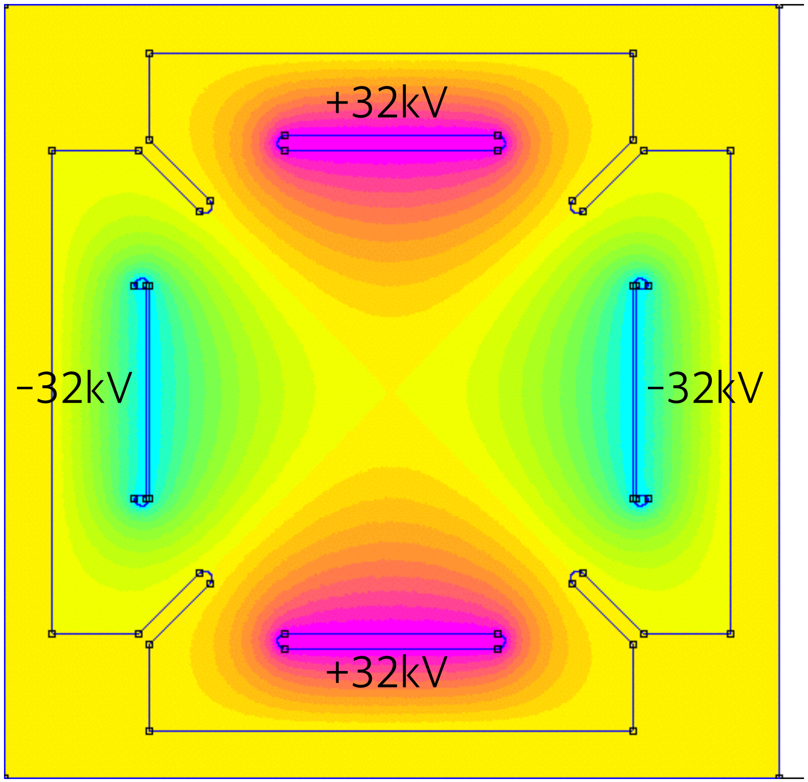
Displaced radially out and up

Displaced radially in and up

It is difficult to move the scrapers

Instead we displace the muon closed orbit

Nominal field configuration



Reduce voltage on bottom plate and orbit shifts downward

Reduce voltage on inner(outer) plate and orbit shifts out(in)

From the NIM paper

Table 10
Quadrupole plate voltage [kV] at injection time for the negative muon storage polarity

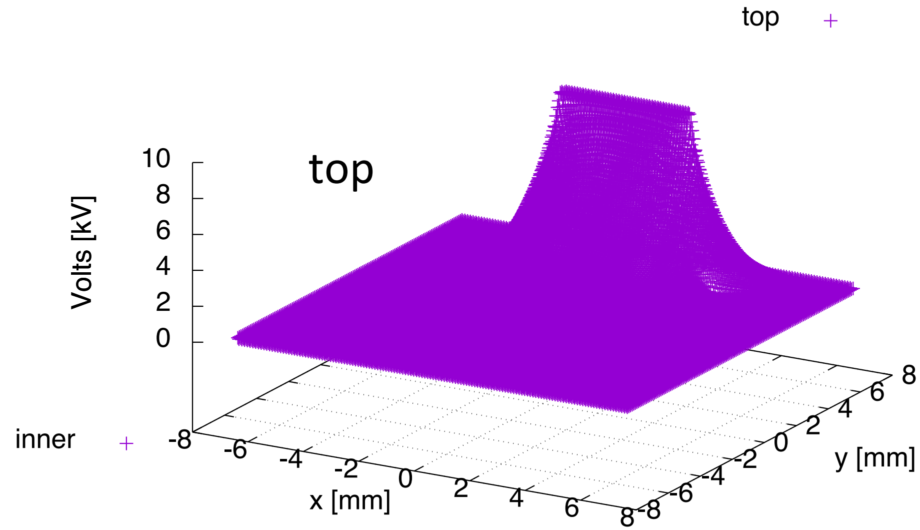
Quad section	Top plates	Bottom plates	Inner plates	Outer plates
Q_1	−24	−17	+24	+24
Q_2	−24	−17	+17	+24
Q_3	−24	−17	+24	+24
Q_4	−24	−17	+24	+17

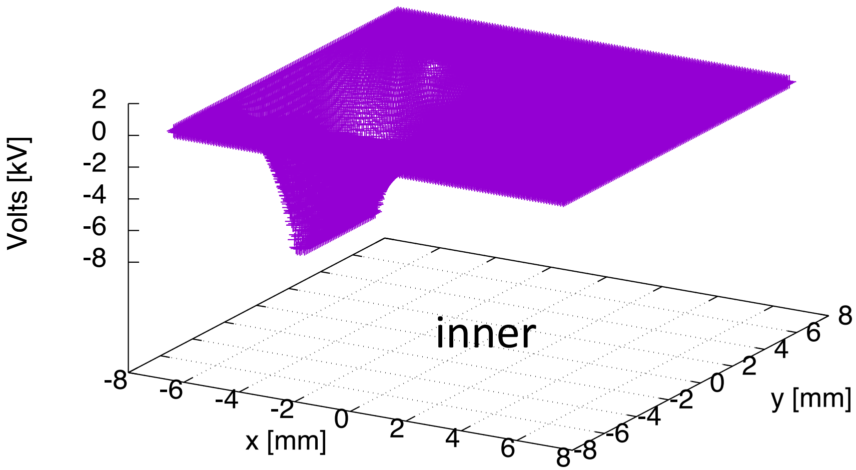
Temporary values (kV) for E989

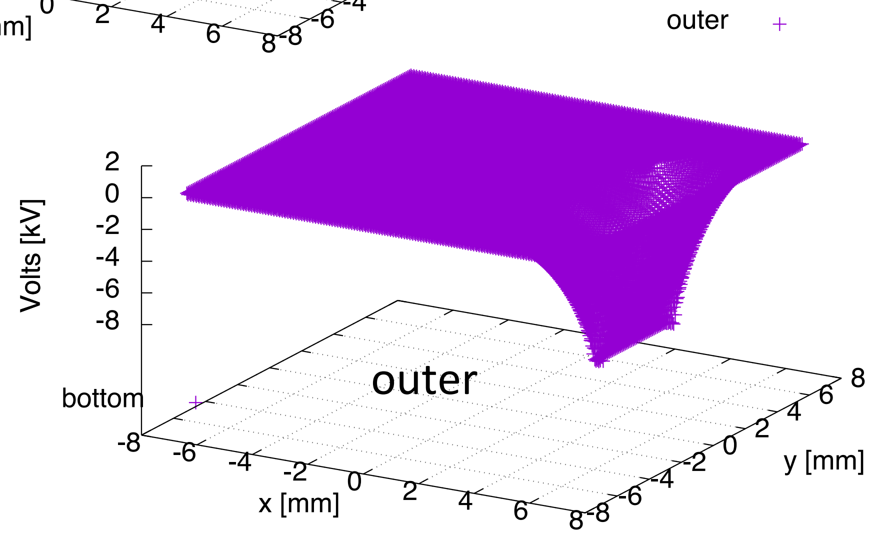
	top	bottom	inner	outer
Q1	+32	+22.7	-32	-32
Q2	+32	+22.7	-22.7	-32
Q3	+32	+22.7	-32	-32
Q4	+32	+22.7	-32	-22.7

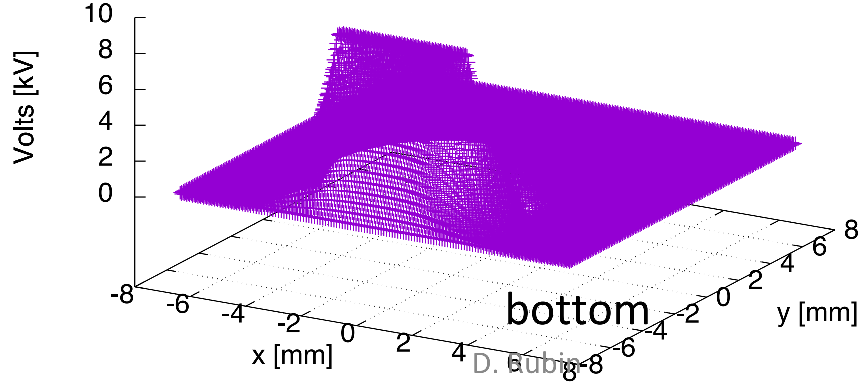
*scaling NIM paper value to 32kV, but some document says 75% for the interim voltage.

Potential map for each of four quad plates









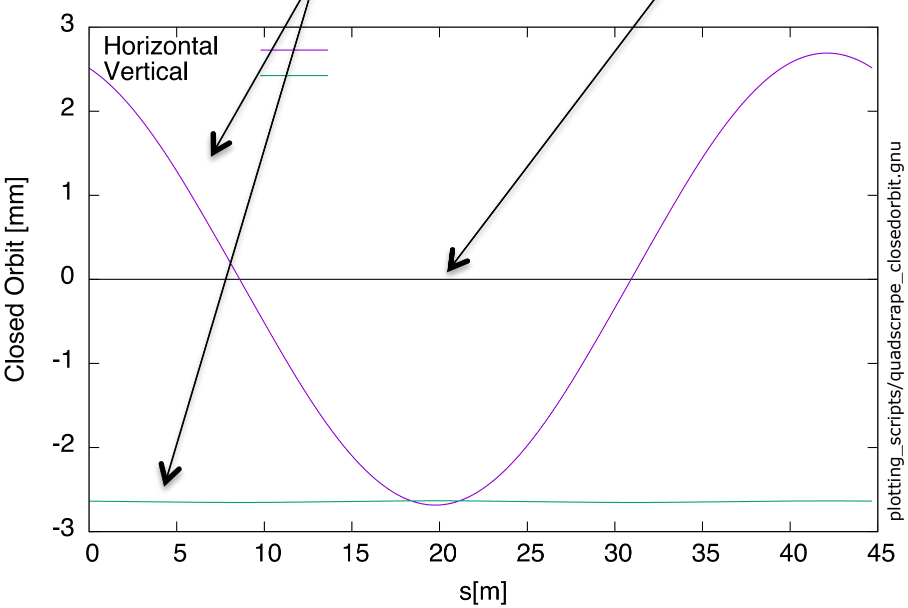
Closed orbit, tunes, and β at injection and after ramp

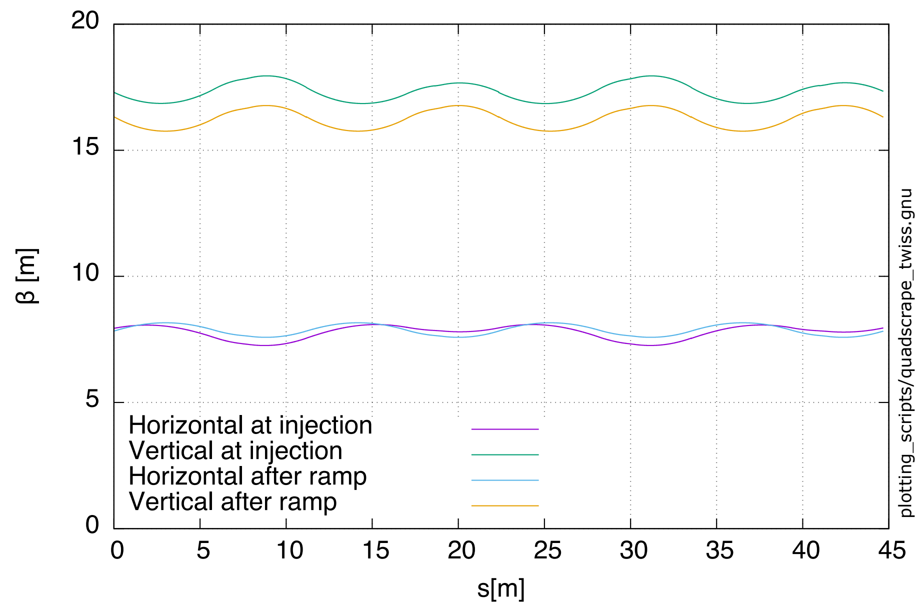
Quad voltage at injection

	Top	Bottom	Inner	Outer
Q1	32	22.7	-32	-32
Q2	32	22.7	-22.7	-32
Q3	32	22.7	-32	-32
Q4	32	22.7	-32	-22.7

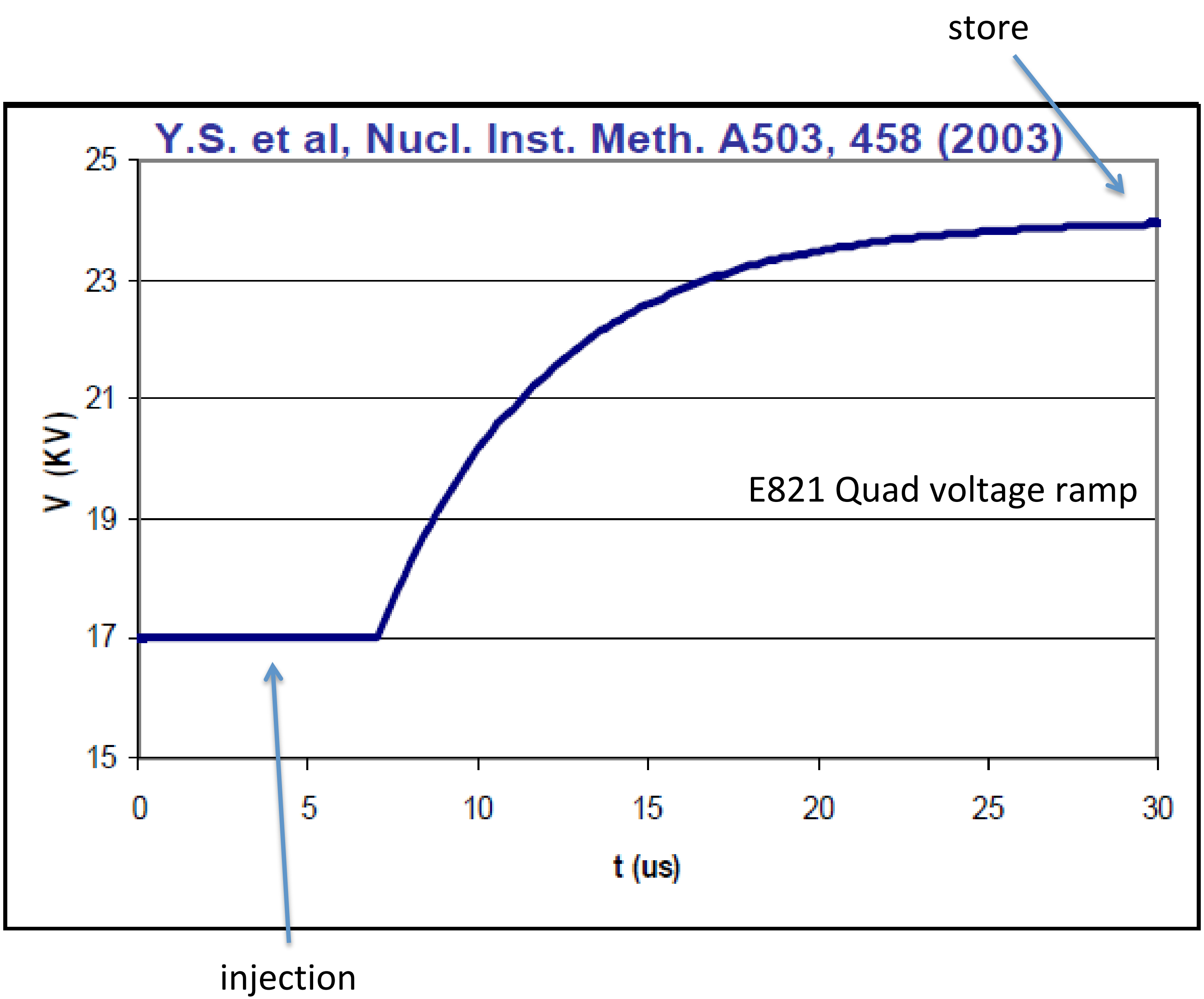
Closed orbit at injection

Closed orbit after ramp

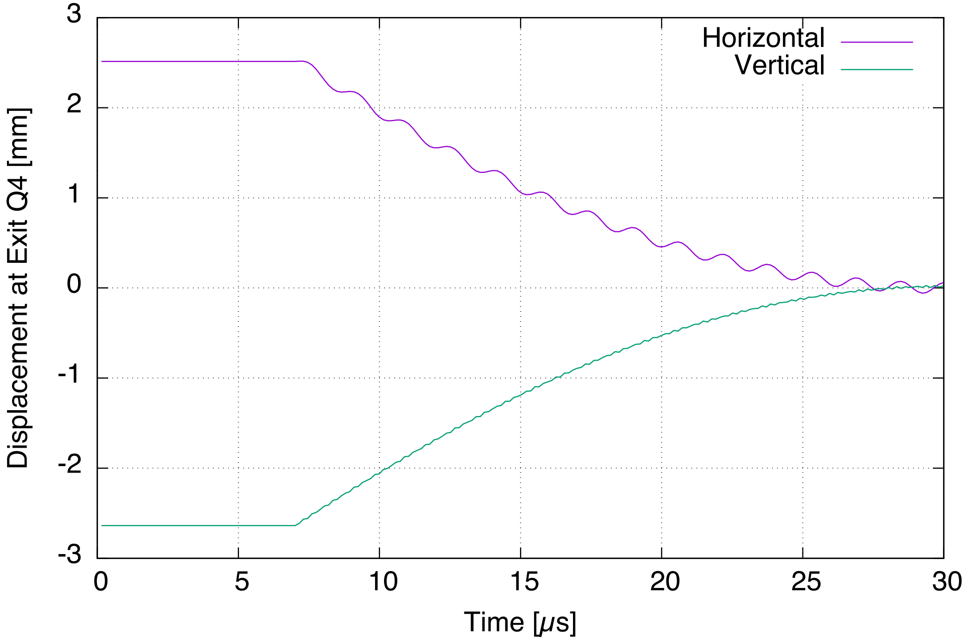




	Tunes at Injection	Tunes after ramp
Q_x	0.9137	0.9039
Q_y	0.4090	0.4403

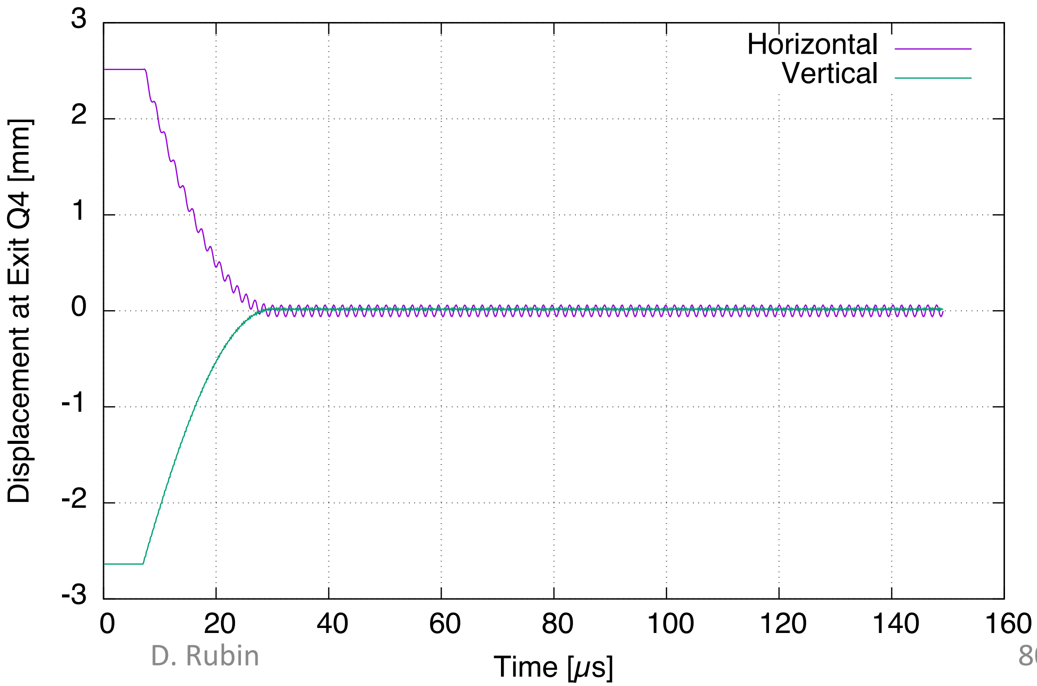

store
Y.S. et al, Nucl. Inst. Meth. A503, 458 (2003)
25
23
21
19
17
15
V (KV)
E821 Quad voltage ramp
0
5
10
15
20
25
30
t (us)
injection

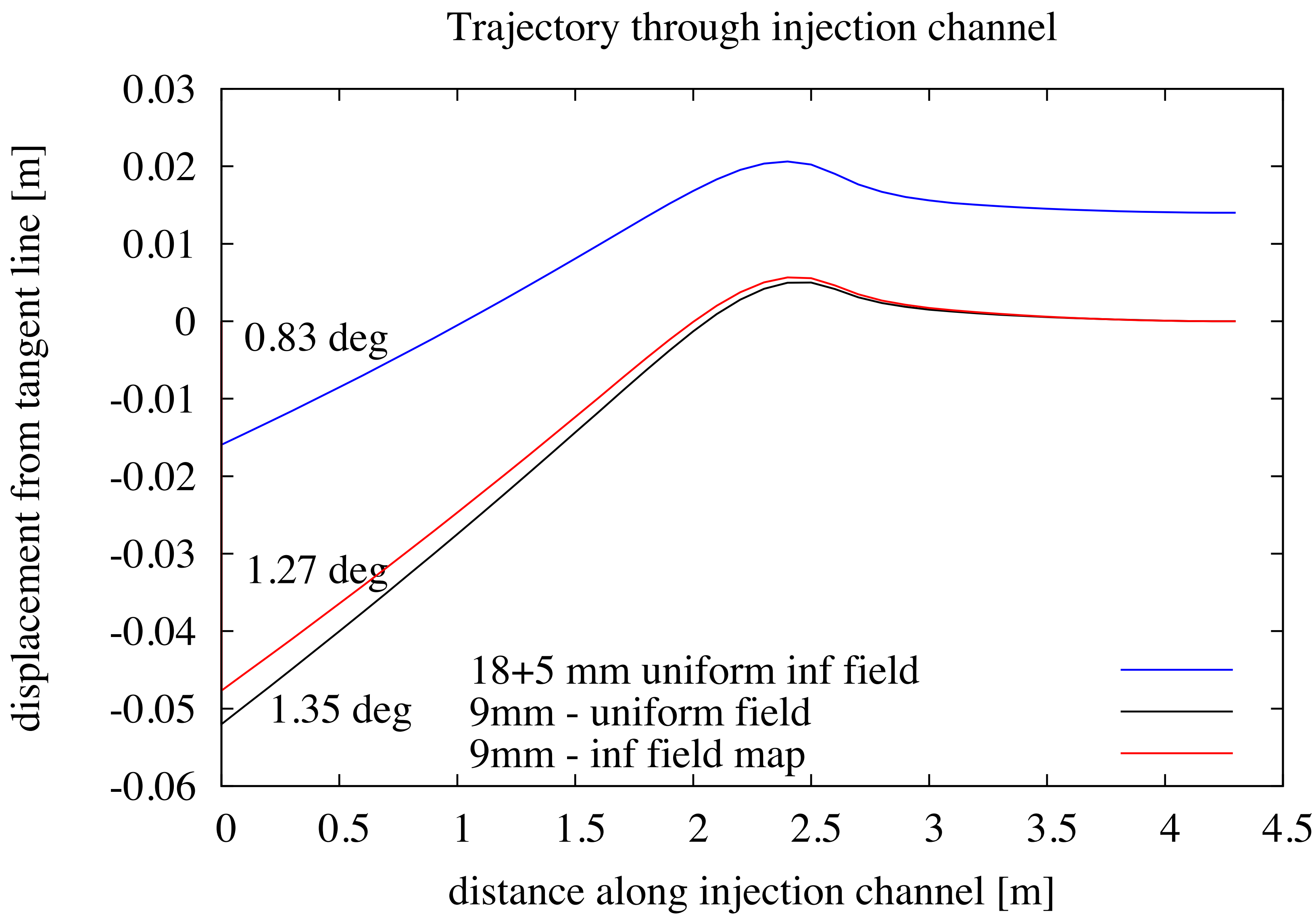
- At t=0, inject muons on to displaced closed orbit?
- Track 1000 turns
- Quad plate voltage ramps as in E821
- (Plot shows orbit at exit of Q4 on each turn)

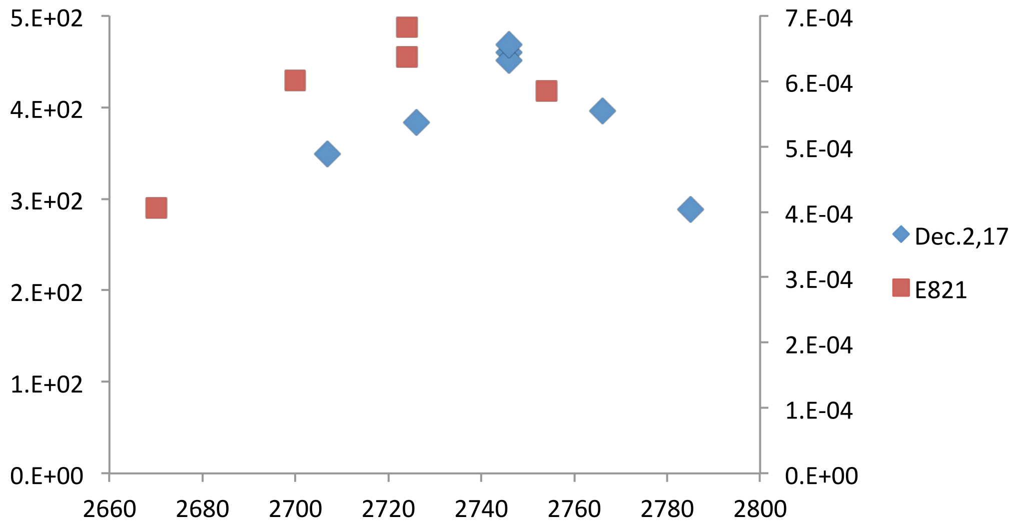


Orbit shifted to center of aperture as quads ramp to nominal (symmetric voltage)

A small horizontal betatron oscillation is introduced by the ramp

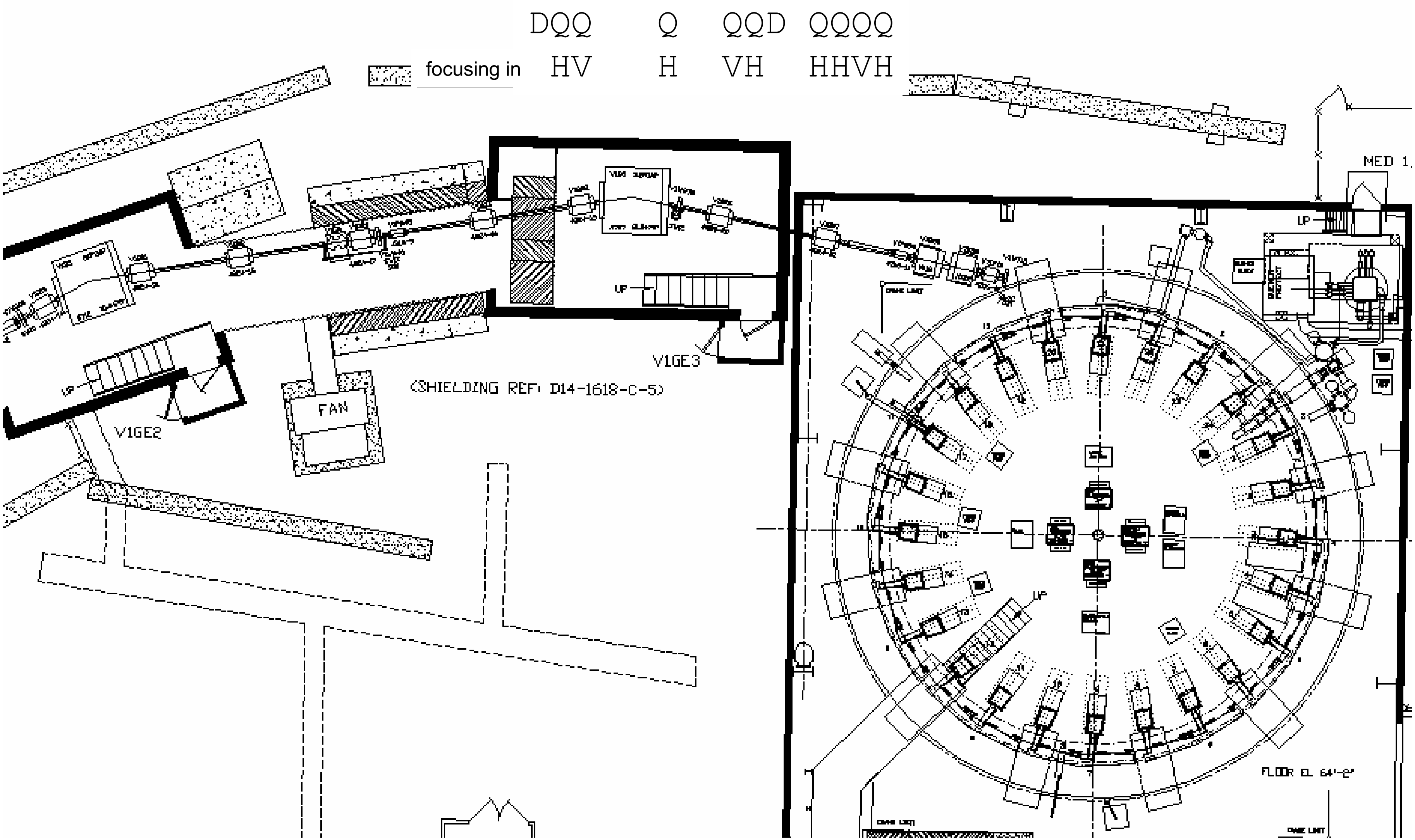



Trajectory through injection channel
displacement from tangent line [m]
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
distance along injection channel [m]
0.83 deg
1.27 deg
1.35 deg
18+5 mm uniform inf field
9mm - uniform field
9mm - inf field map



Higher inflector field compensates positive inflector exit angle

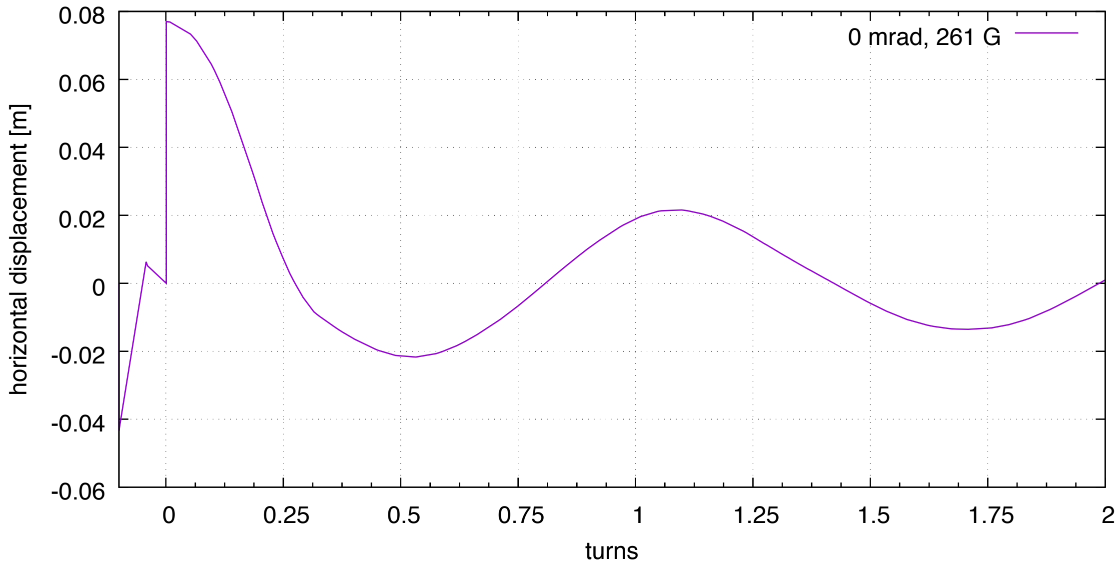
E 821 V line D5 to g-2 ring

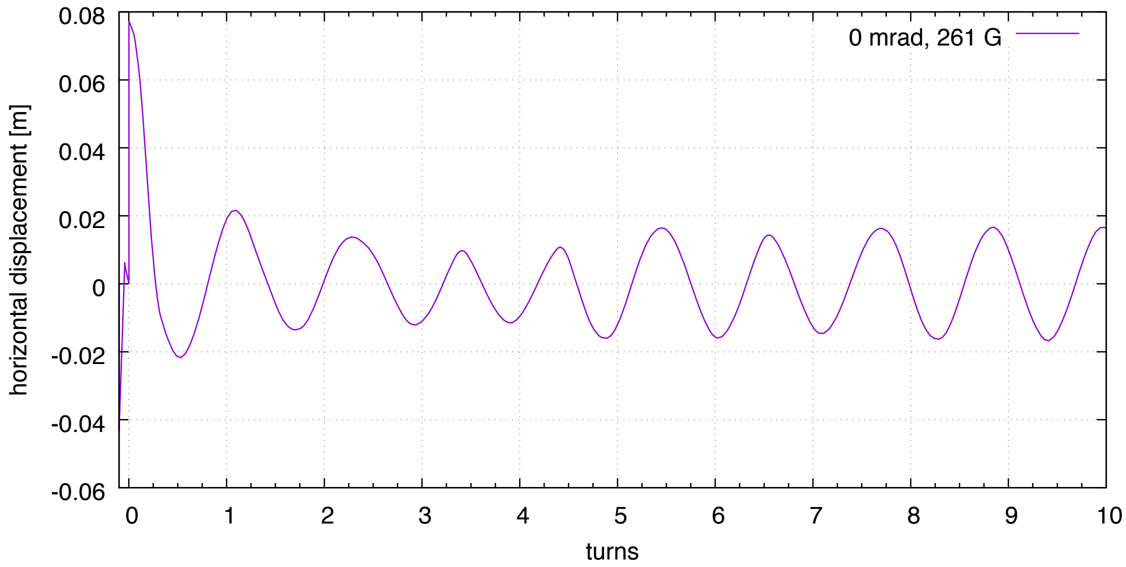


Optimum kick (minimize CBO) vs inflector exit angle − on momentum muon

$\theta=0$

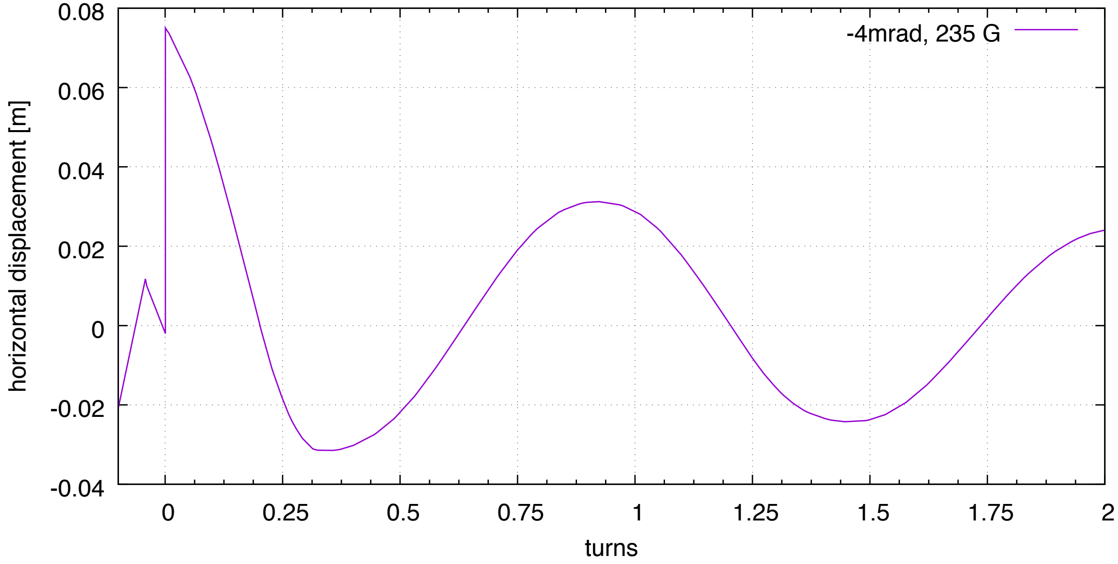
Muon trajectory is along tangent line (inflector center line)

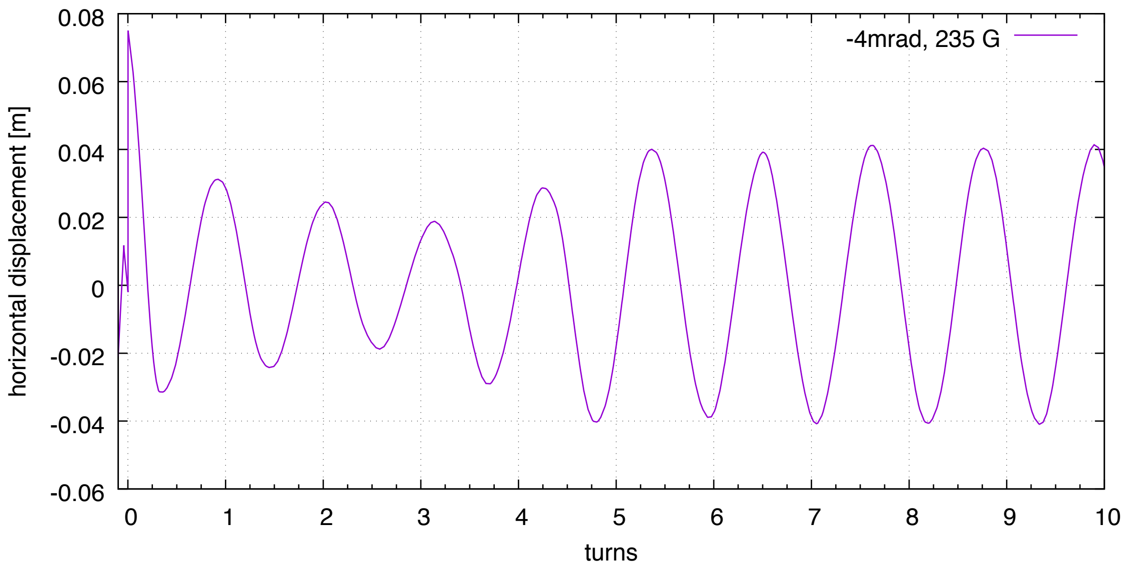




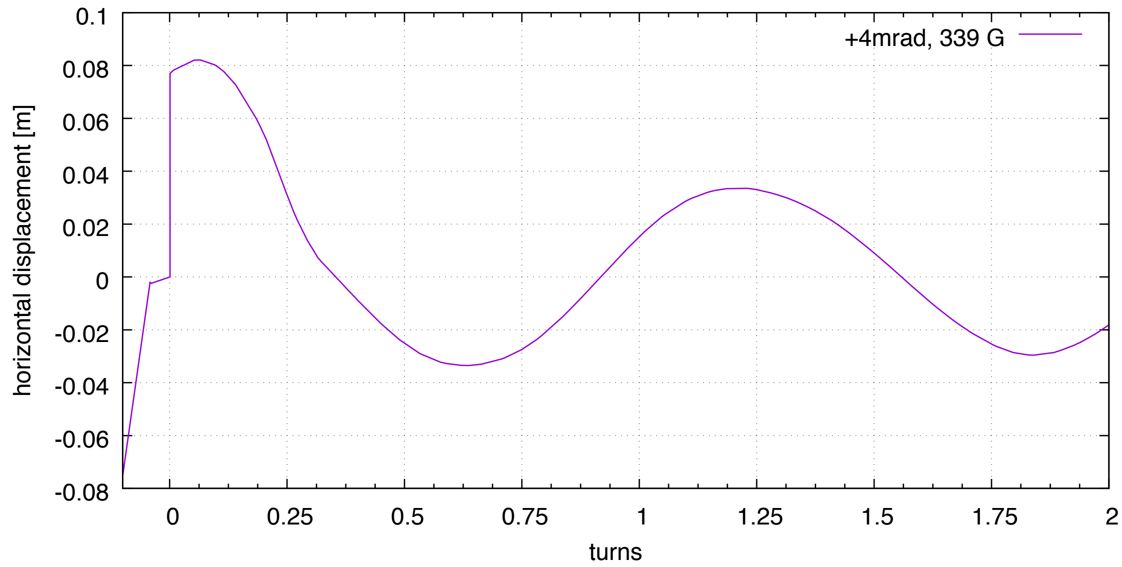
Optimum kick (minimize CBO) vs inflector exit angle − on momentum muon

Muon trajectory is tilted -4mrad with respect to tangent line (inflector center line)

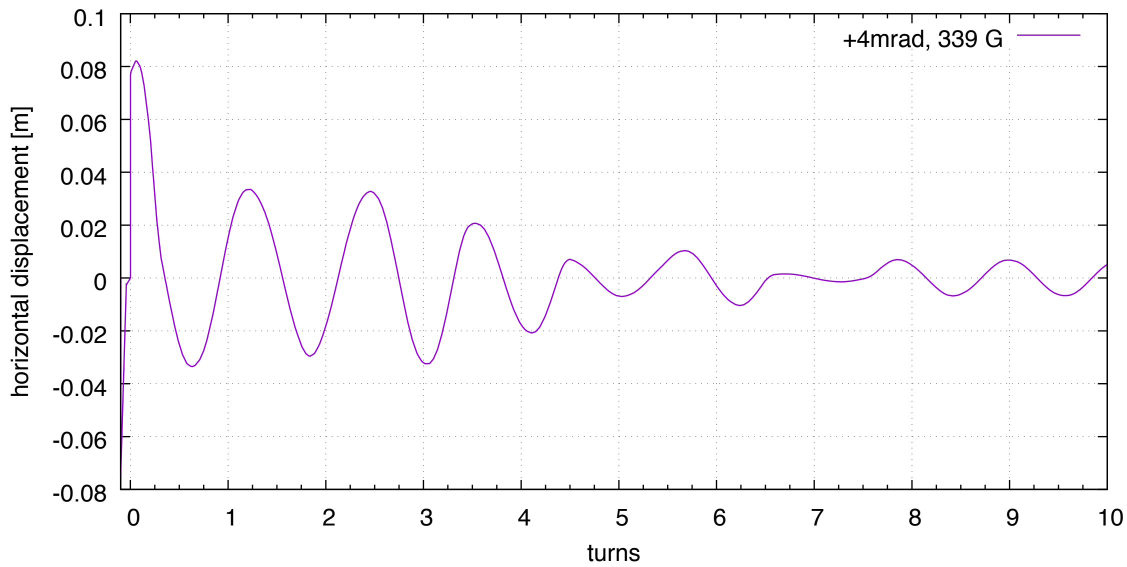




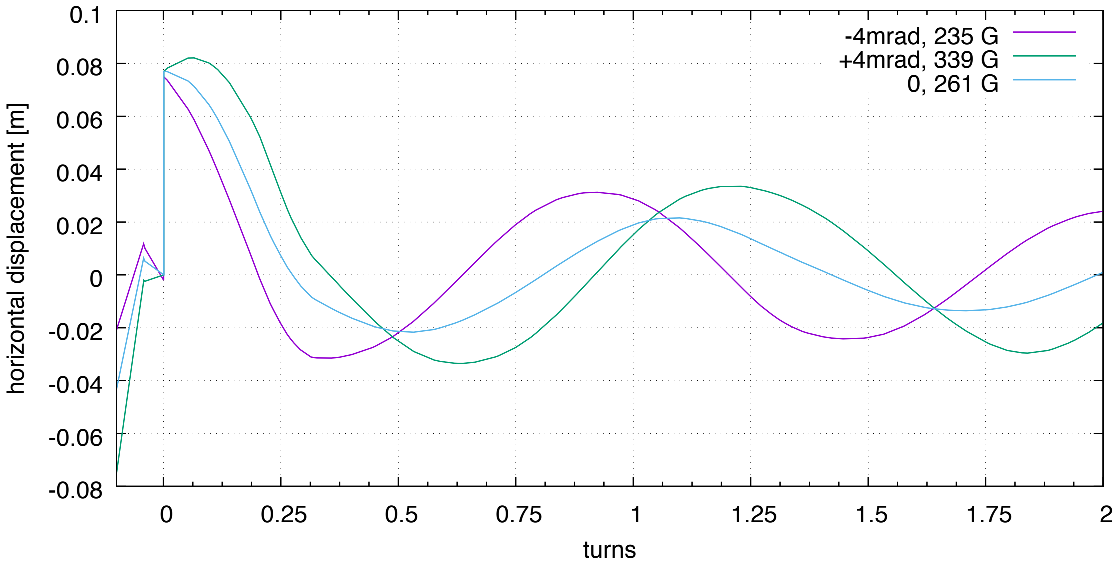
Optimum kick (minimize CBO) vs inflector exit angle − on momentum muon

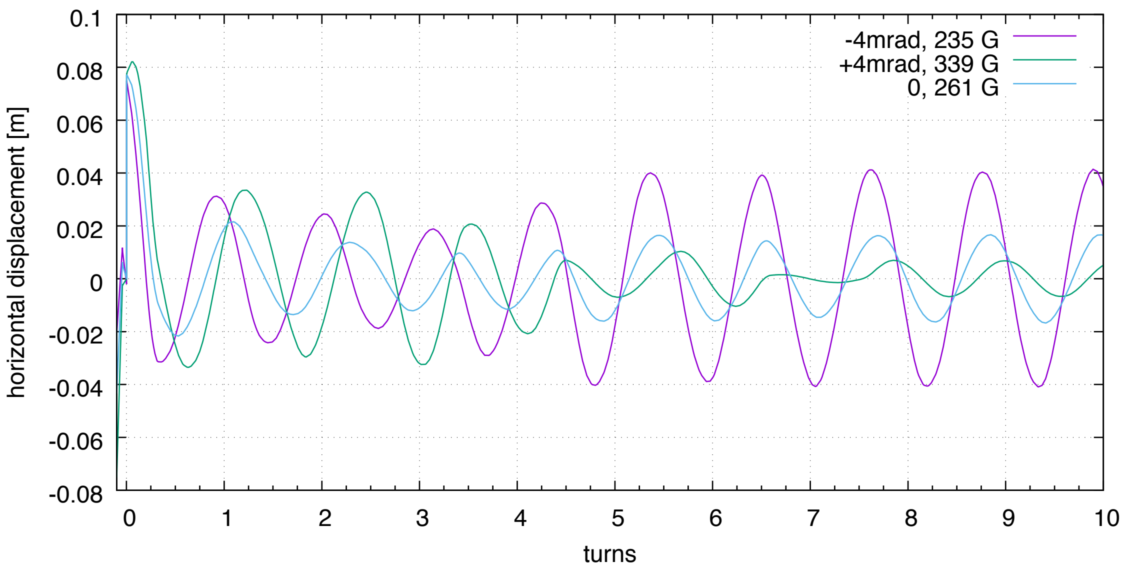


Muon trajectory is t +4mrad with respect to tangent line (inflector center line)



Optimum kick (minimize CBO) vs inflector exit angle − on momentum muon





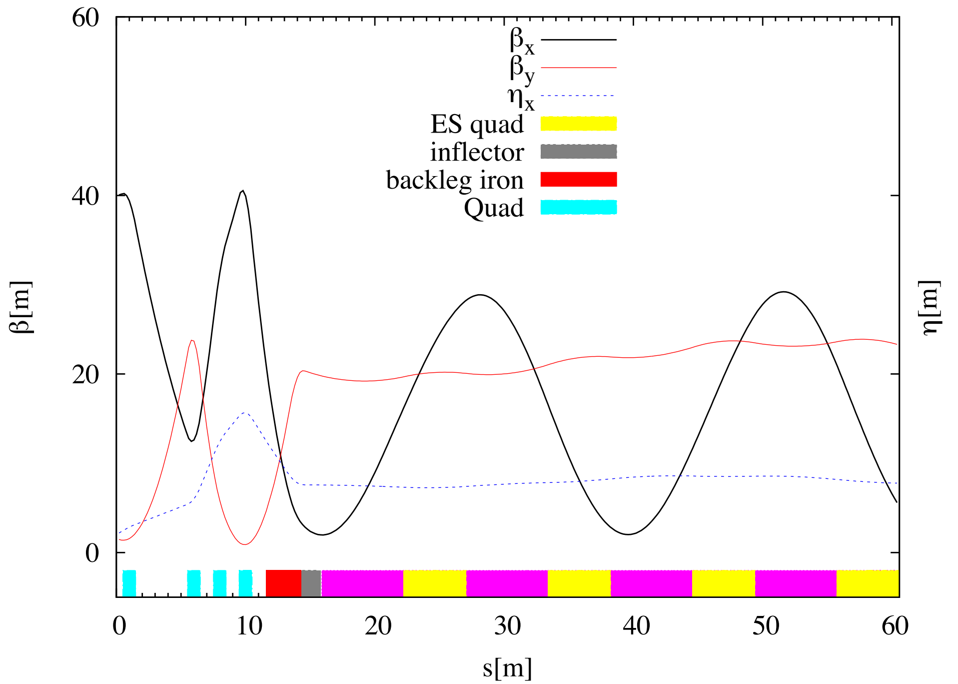
1.Dispersion at inflector exit matches dispersion into ring

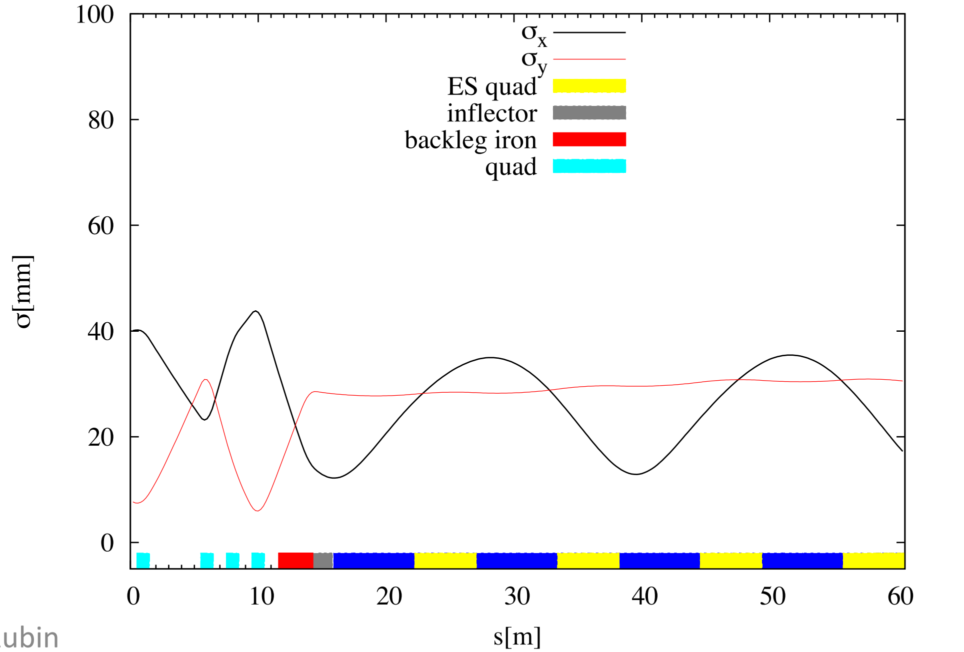
- Ring aperture = ±4cm
- Inflector aperture = ±9mm
- Ring dispersion η_0=8m
- => Energy acceptance in ring ΔE/E = 0.5 %
- => Energy acceptance through inflector ΔE/E = 0.11 %

Real acceptance = 0.11 % and no losses in ring

$$\sigma_x = [\beta\varepsilon + (\eta\delta)^2]^{1/2}, \qquad \delta=0.11\%$$

Beam fits comfortably into ring aperture
- 0.11% energy acceptance
- No energy modulation of beam size
- "breathing" at twice the betatron tune





2.Dispersion at inflector exit is zero

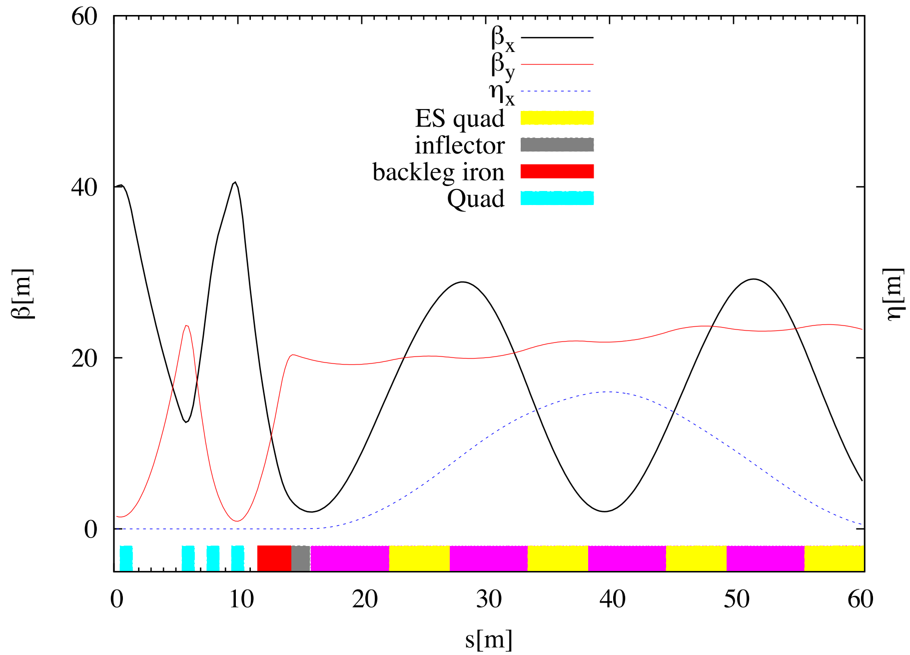
- Ring aperture = ±4cm
- Inflector aperture = ±9mm
- Ring dispersion η_{min}=0 => η_{max} = $2\eta_0$= 16m
- => Energy acceptance in ring ΔE/E = 0.25 %
- => Energy acceptance through inflector ΔE/E = ∞

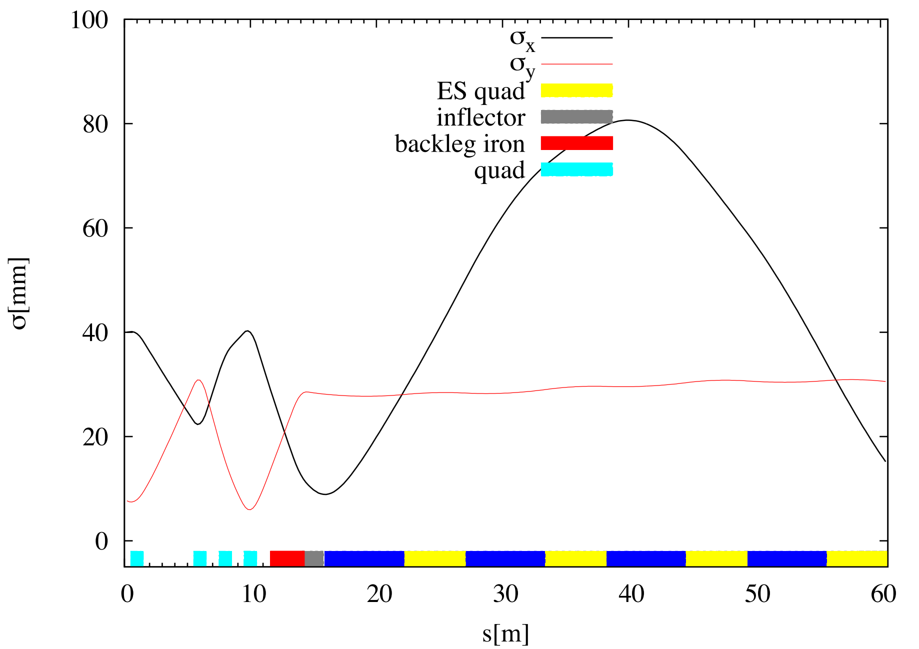
Real acceptance = 0.25 %
and all losses are in ring

$$\sigma_x = [\beta\varepsilon + (\eta\delta)^2]^{1/2}, \qquad \delta=0.05\%$$

All energies fit through inflector.

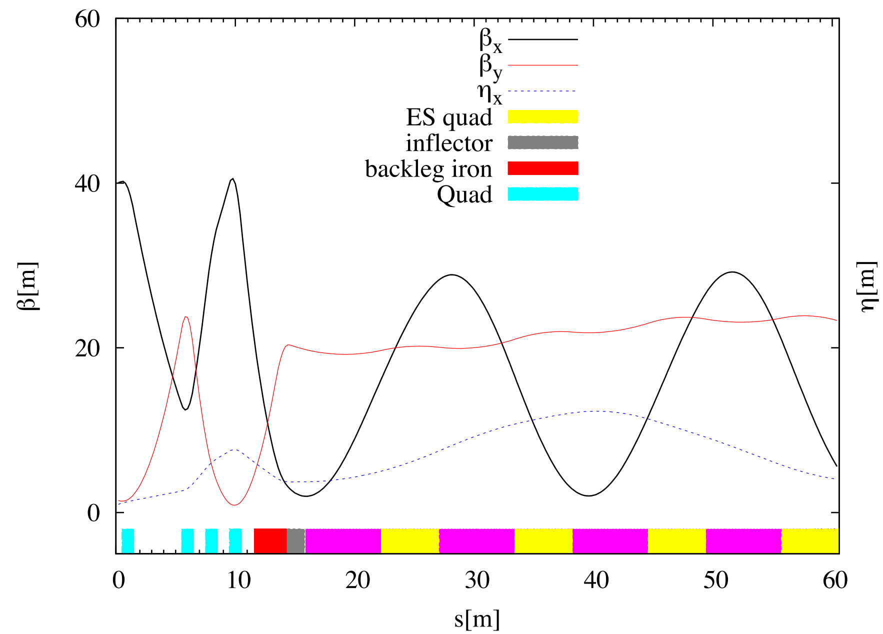
- All muons with ΔE/E > 0.25 % are scraped off in the ring
- Deep modulation of beam size (η_{max}/η_{min} = ∞) at the betatron tune
- "breathing" at twice the betatron tune





3.Dispersion at inflector exit is ½ match value

- Ring aperture = ±4cm
- Inflector aperture = ±9mm
- Ring dispersion η_{min}= ½η_0 => η_{max} = 1.5η_0= 12m
- => Energy acceptance in ring ΔE/E = 0.25 %
- => Energy acceptance through inflector ΔE/E = 0.009/4 = 0.23 %

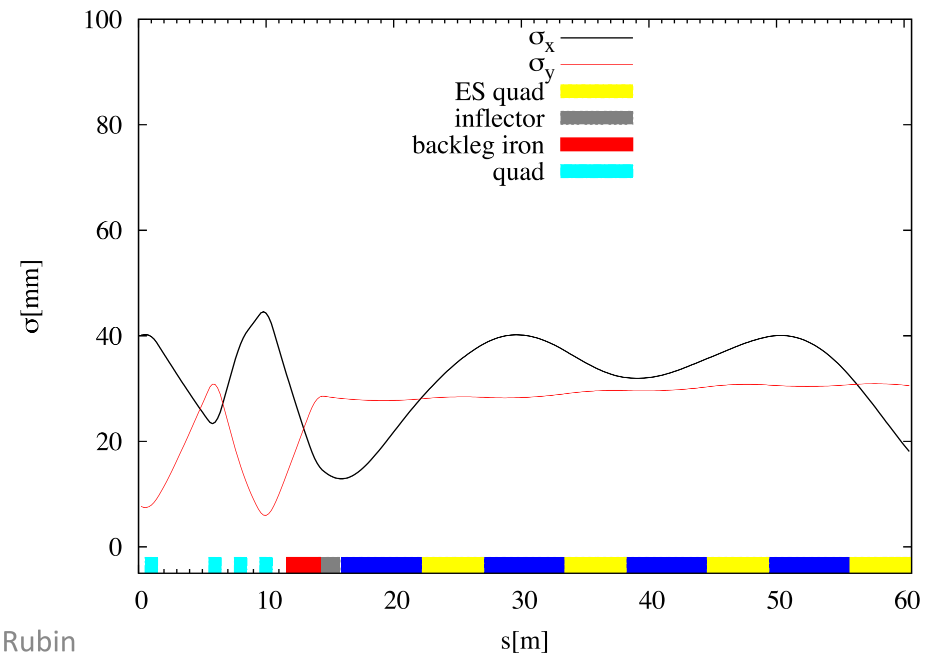


Real acceptance = 0.23 %
with no losses in ring

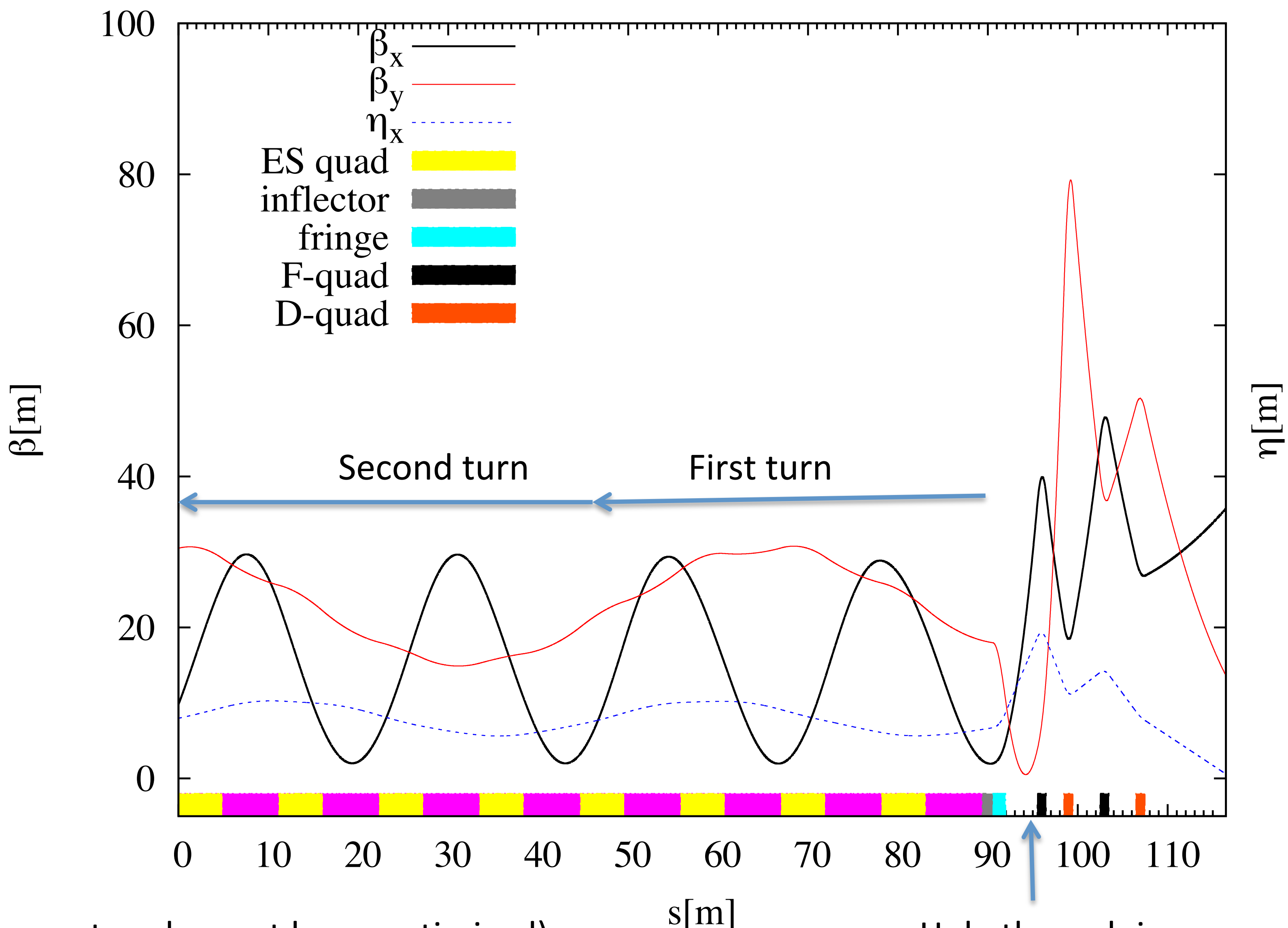
$$\sigma_x = [\beta\varepsilon + (\eta\delta)^2]^{1/2}, \qquad \delta=0.023\%$$

Acceptance of inflector ≤ acceptance of ring.

- All muons with ΔE/E > 0.23 % are lost in inflector
- Modest modulation of beam size (η_{max} /η_{min} = 3) at the betatron tune
- "breathing" at twice the betatron tune



β,α at s=0 are chosen so that 40mm-mrad beam fits through inflector aperture.
Quads upstream of iron (F-quads and D-quads) optimized to achieve reasonable values at 120m

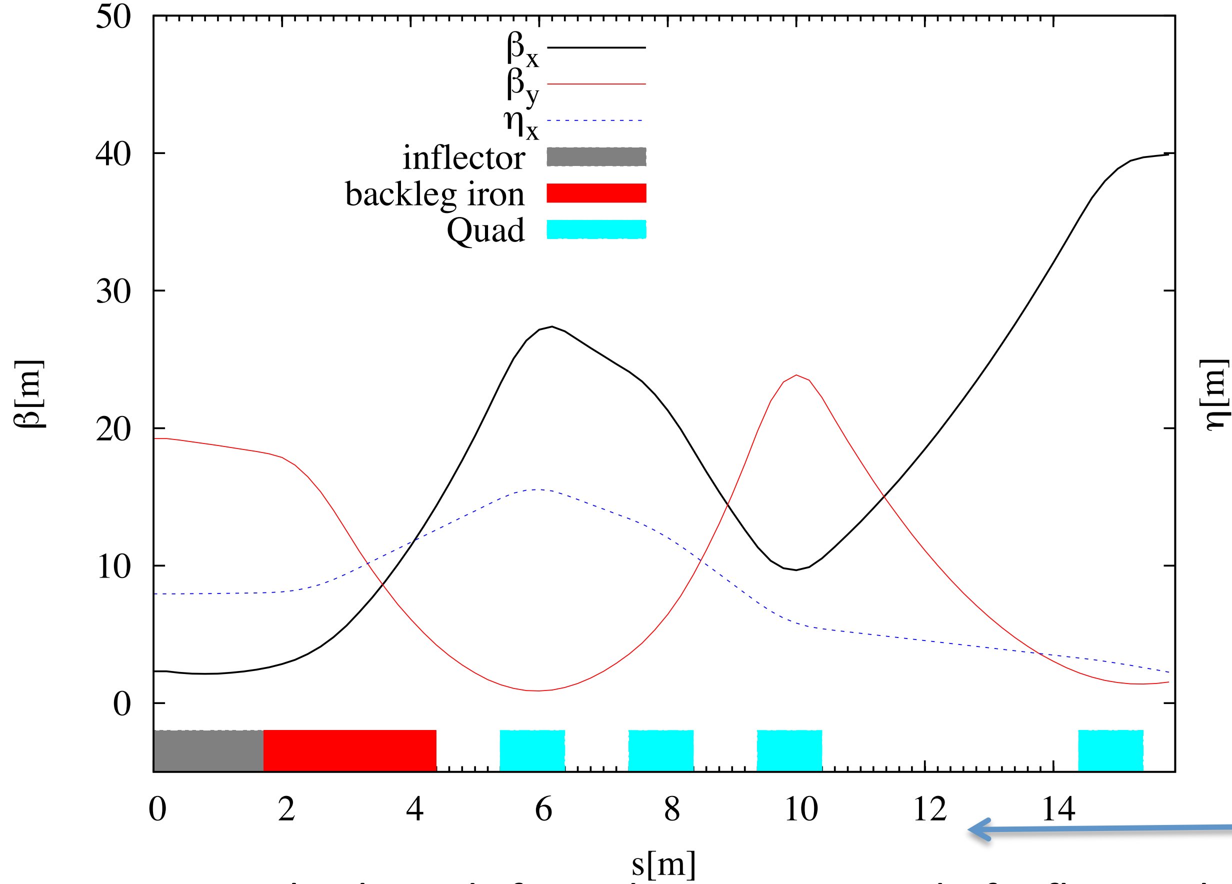


(Energy aperture has not been optimized)

Hole through iron

40 mm-mrad beam clears inflector if at exit
β_x=2.45, α_x= -0.41, β_y=19.1, α_y= 0.045

σ_E/E = 0.15% clears inflector if at exit
η= 7.96, η'=0.057

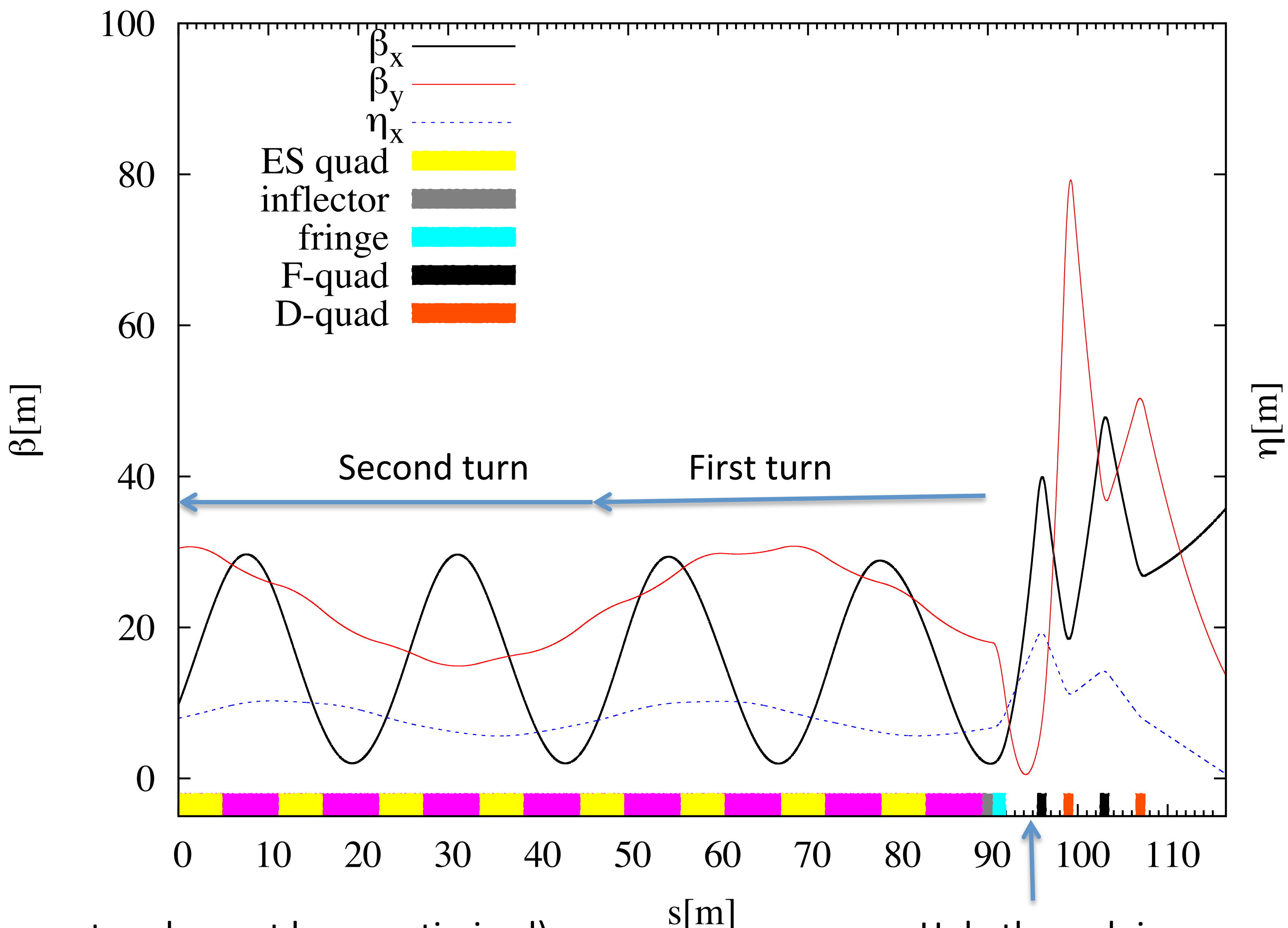


Quadrupoles optimized to minimize α, η, η' at entrance to beam line and with "reasonable" β

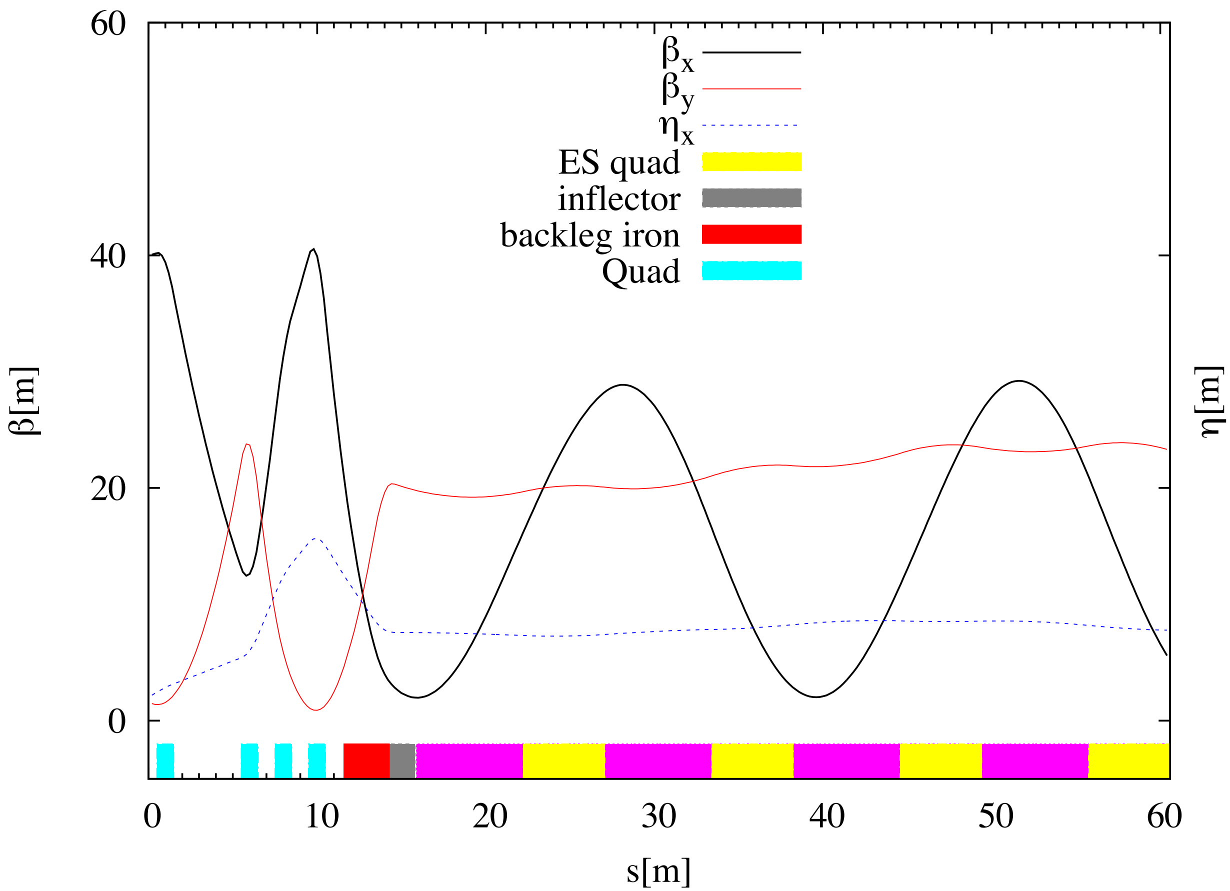
Beam travels right to left

Propagate backwards from downstream end of inflector through backleg iron and through beam line quadrupoles

β,α at s=0 are chosen so that 40mm-mrad beam fits through inflector aperture.
Quads upstream of iron (F-quads and D-quads) optimized to achieve reasonable values at 120m



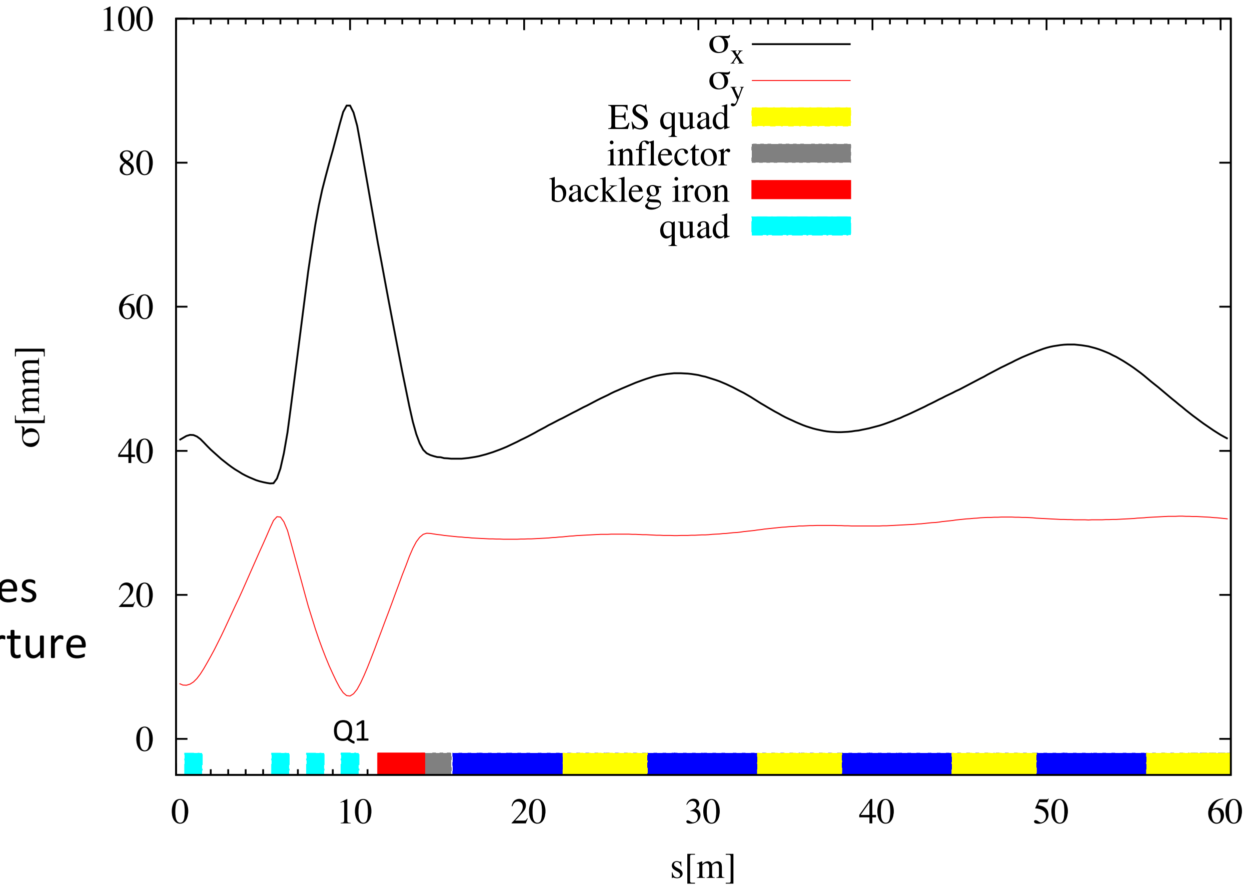
(Energy aperture has not been optimized)



Beam travels left to right

Propagate forward through beam line quadrupoles, backleg iron, inflector and into ring

$\sigma = [\beta\varepsilon + (\eta\delta)^2]^{1/2}$
ε = 40 mm-mrad, δ = 0.5%



Q1 requires large aperture

Propagate forward through beam line quadrupoles, backleg iron, inflector and into ring