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Storage Ring Beam Dynamics

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Fermilab

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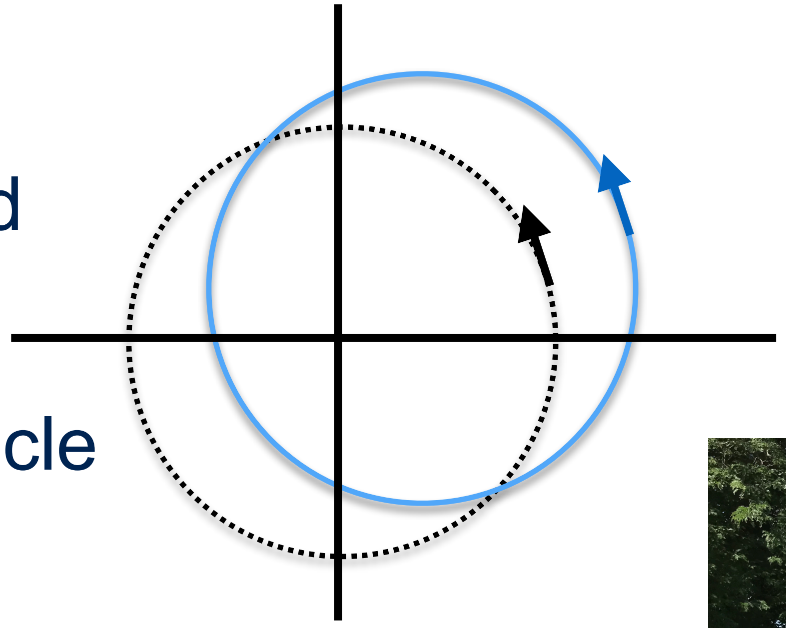
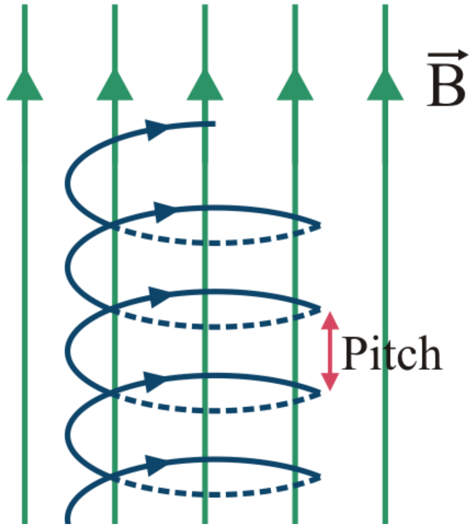
Outline

- The Storage Ring
- Reaching equilibrium
  - debunching
  - final momentum distribution
  - Equilibrium phase space
- Distortions due to realities
  - Closed orbit distortions
  - Tunes vs. quad plate alignment
  - Introduction to nonlinear effects
  - Choice of betatron tunes and resonances
  - Lost muon processes

The Storage Ring

- The problem with a "uniform field"
  - pure horizontal motion is stable and bounded
  - however, given any vertical component of momentum, the particle will spiral into the vertical aperture





1935 U. Chicago cyclotron magnet, Fermilab Village

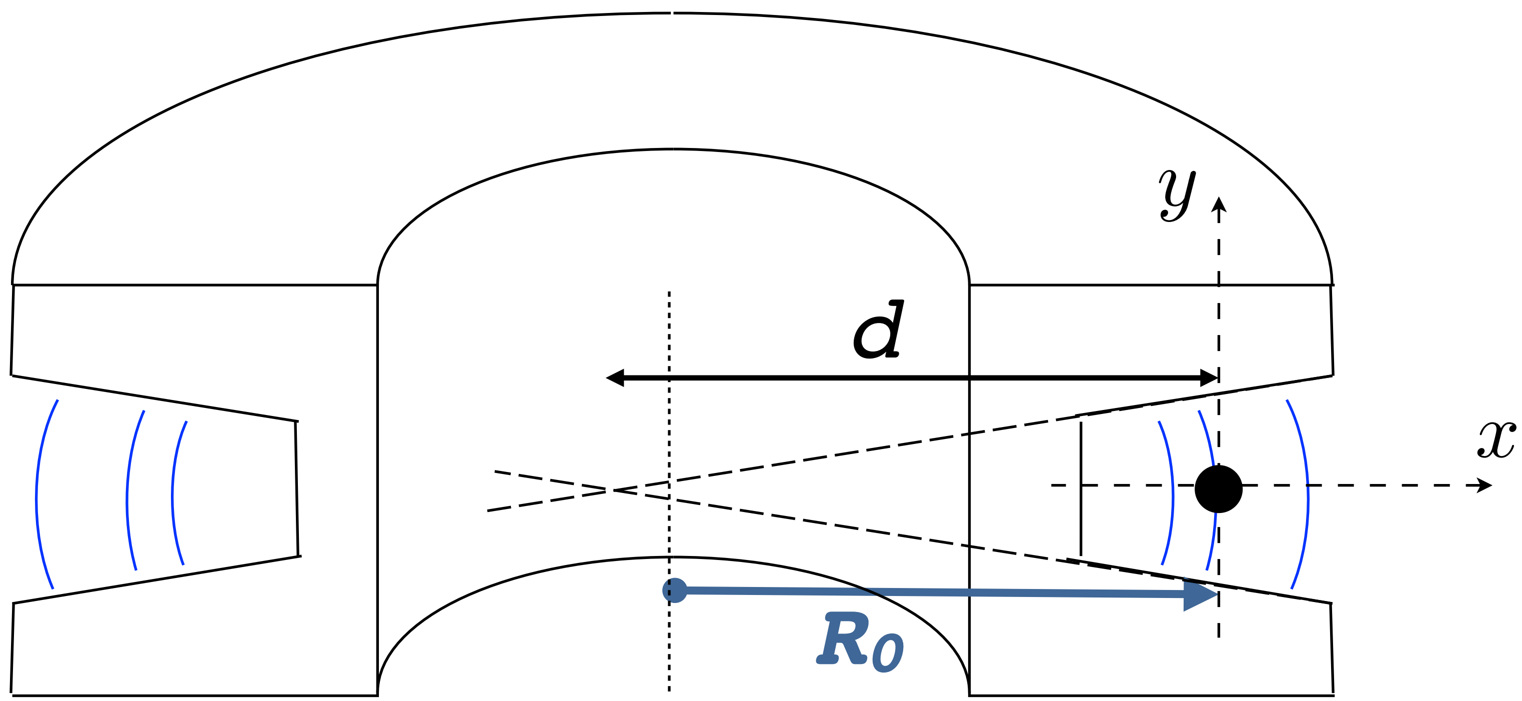
- Classical *Weak Focusing*: shape the field to yield a radial component for y ≠ 0, thus providing vertical steering for out-of-plane particles

The Storage Ring

- Classical ***Weak Focusing:***
  - (early synchrotrons)

wedge-shaped pole tips provide: $$B = B_0 \left(\frac{R_0}{r}\right)^n$$

n is determined by adjusting the opening angle between the poles $$n \approx \frac{R_0}{d}$$



n = "field index"

$$B = B_0 \left(\frac{1}{1 + x/R_0}\right)^n \approx B_0(1 - \frac{n}{R_0}x)$$

Yields equations of motion:

$$x'' + \frac{1-n}{R_0^2}\, x = 0$$

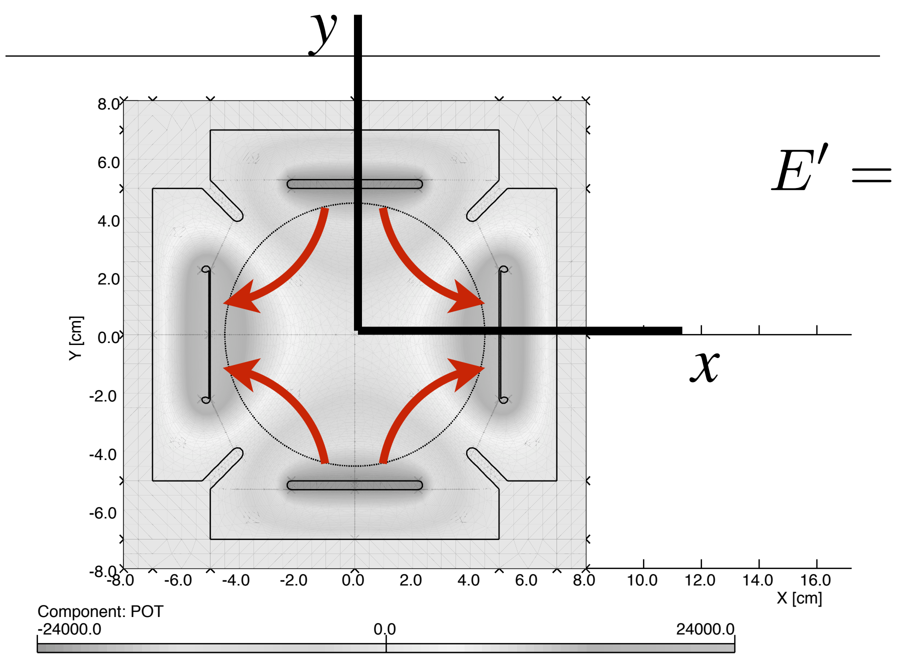
$$y'' + \frac{n}{R_0^2}\, y = 0$$

must have $0 \leq n \leq 1$ for stability

For E989 the magnetic field needs to be ***uniform***, but can create a similar effect using *electrostatic quadrupoles* for the vertical focusing

The Storage Ring

- Our implementation of weak focusing (discrete quads)

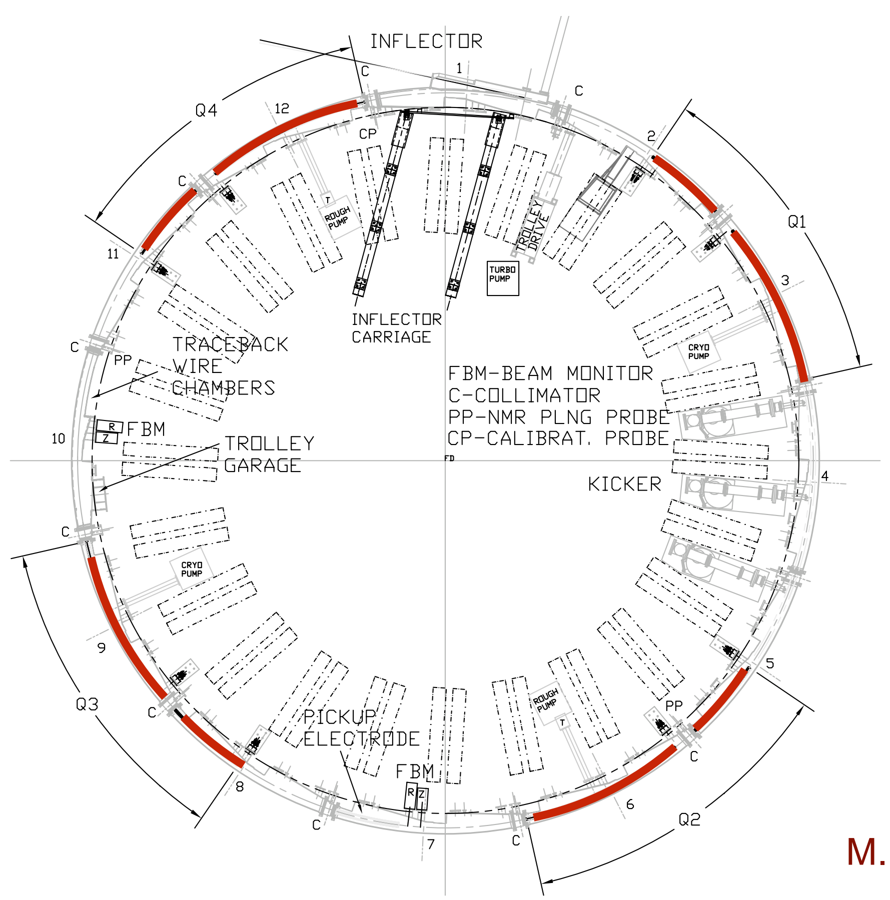


$$E' = |\partial E_y/\partial y| \approx 2V/a^2$$

$$\begin{aligned} x'' + K_x(s)x &= 0 \\ y'' + K_y(s)y &= 0 \end{aligned}$$

where

$$\begin{aligned} K_x(s) &= \frac{1}{R_0^2} - \frac{E'(s)}{vB_0R_0} \\ K_y(s) &= \quad + \frac{E'(s)}{vB_0R_0} \end{aligned}$$



Here, the weak focusing is not everywhere, but rather at discrete locations

Hence, the equilibrium beam size would be slightly larger in some spots than in others

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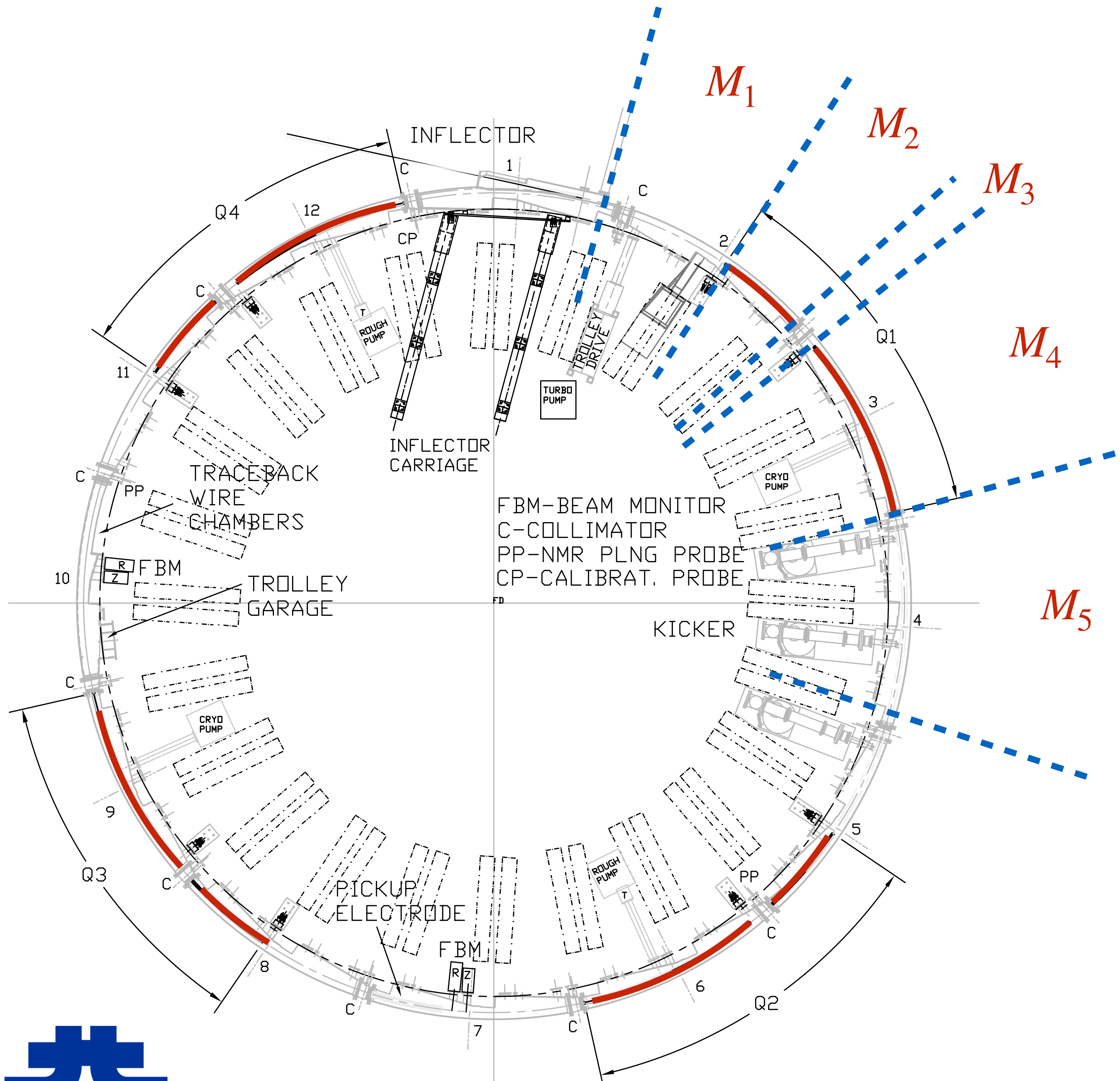
- Matrix description of discrete focusing regions



$$x'' + K_x(s)x = 0$$
$$y'' + K_y(s)y = 0$$

where

$$K_x(s) = \frac{1}{R_0^2} - \frac{E'(s)}{vB_0R_0}$$
$$K_y(s) = + \frac{E'(s)}{vB_0R_0}$$

$$M_{1/4} = M_5M_4M_3M_2M_1$$

$$M_{tot} = (M_{1/4})^4$$

The Storage Ring Optics

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- As seen earlier, Courant-Snyder parameters (α, β, γ) along with the emittance (ϵ) characterize the size, shape and orientation of the beam phase space ellipse; these functions vary along the trajectory of the beam
- In a synchrotron we can determine a set values for these parameters that that will behave periodically with the same periodicity as the optical layout of the ring. These are the *periodic* β functions of the ring:

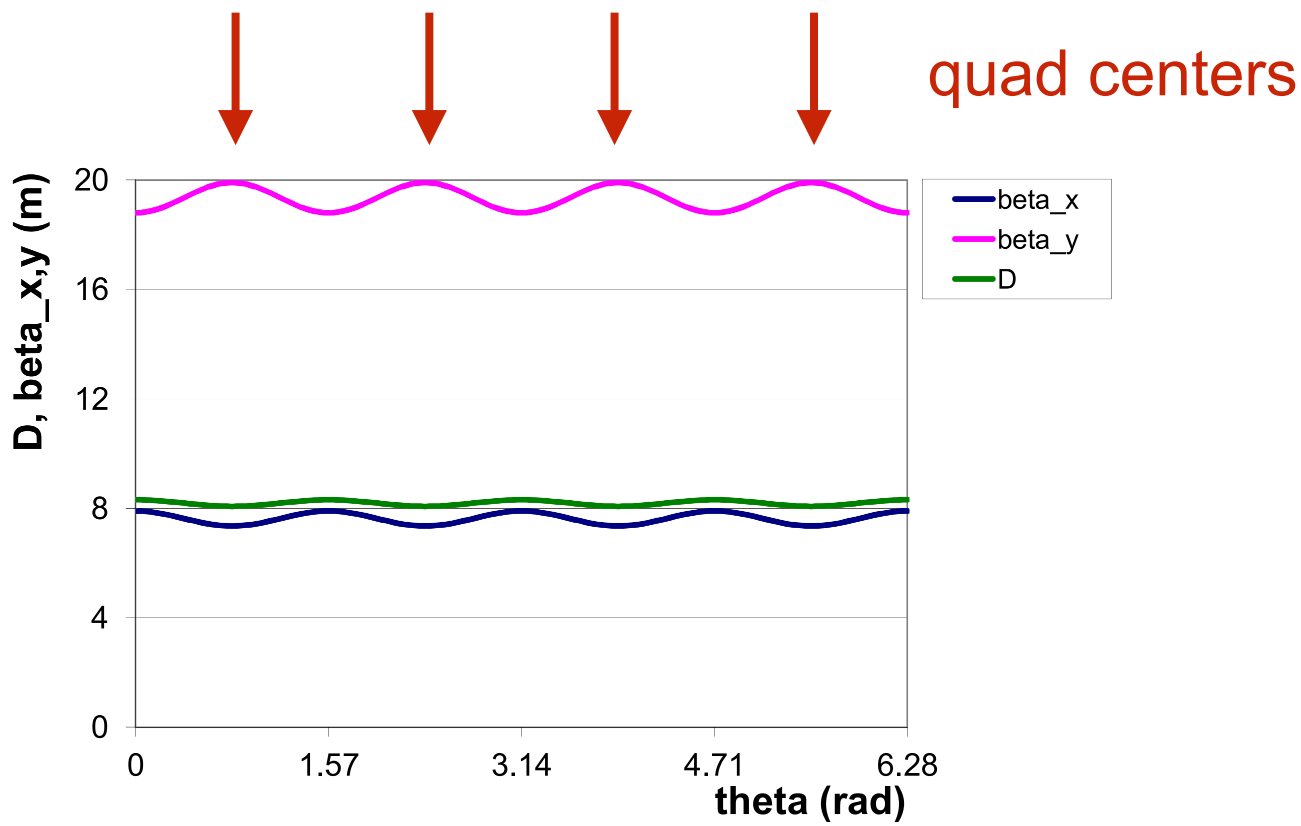
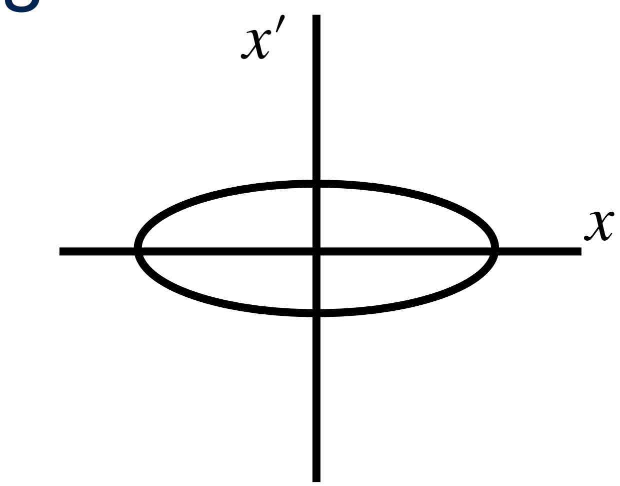
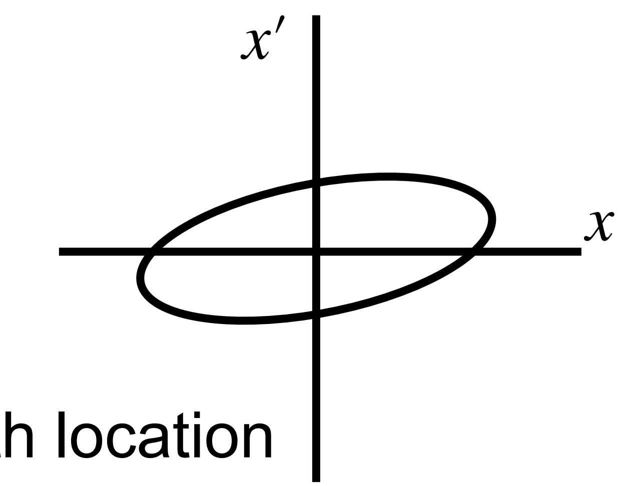


Figure 13.19: E821 horizontal and vertical beta functions.

(from the TDR)

SH Oscillator:



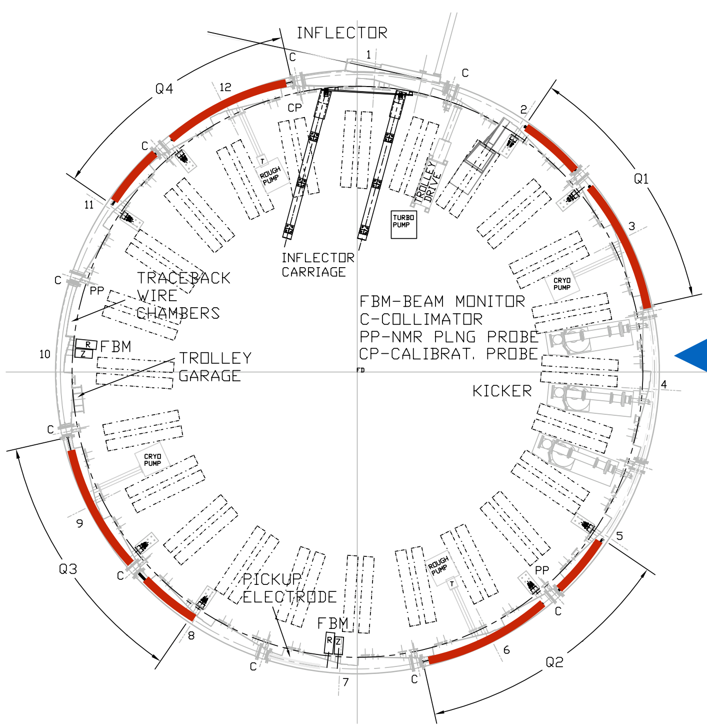


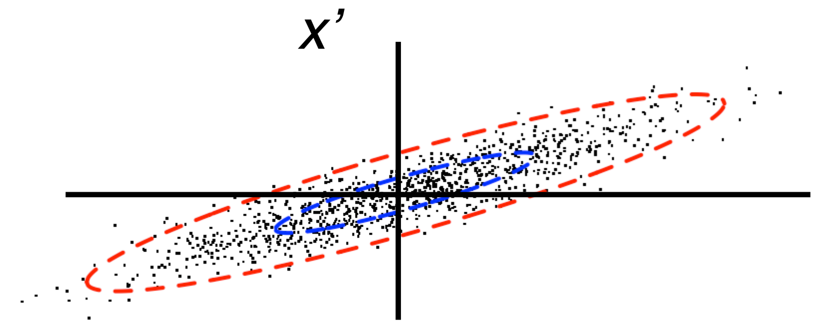
Synchrotron:
'tilt', etc. vary with location

"Normalized" Phase Space Description of Betatron Motion

- If pick a particular point in the storage ring as our "observation point" then we can describe the periodic motion with a phase space plot using the particular set of periodic Courant-Snyder variables for that location:

choose observation point:

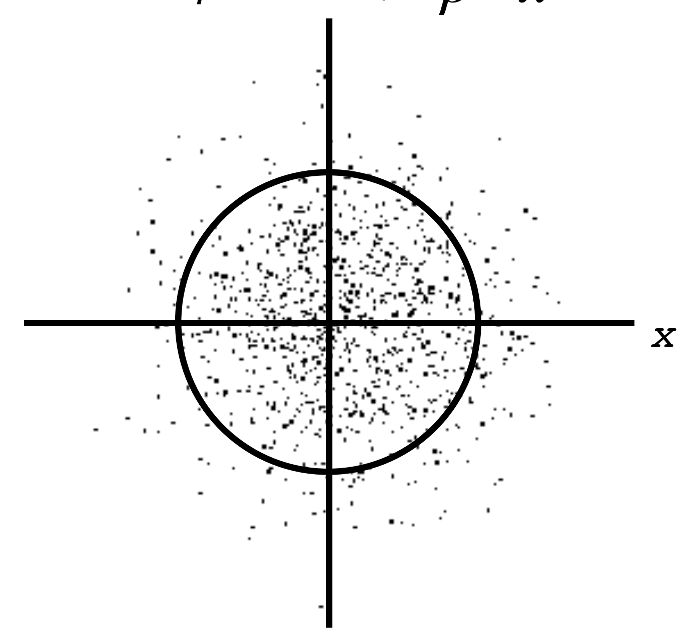




$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = \epsilon/\pi$$

or, equivalently:

$\beta x' + \alpha x \approx \beta \cdot x'$



$$x^2 + (\beta x' + \alpha x)^2 = \beta\epsilon/\pi$$

$\beta \approx$ 7.8 m *in our ring*

$\alpha \equiv -\frac{1}{2}(d\beta/ds)$ $\lesssim$*0.1-0.2 in our ring*

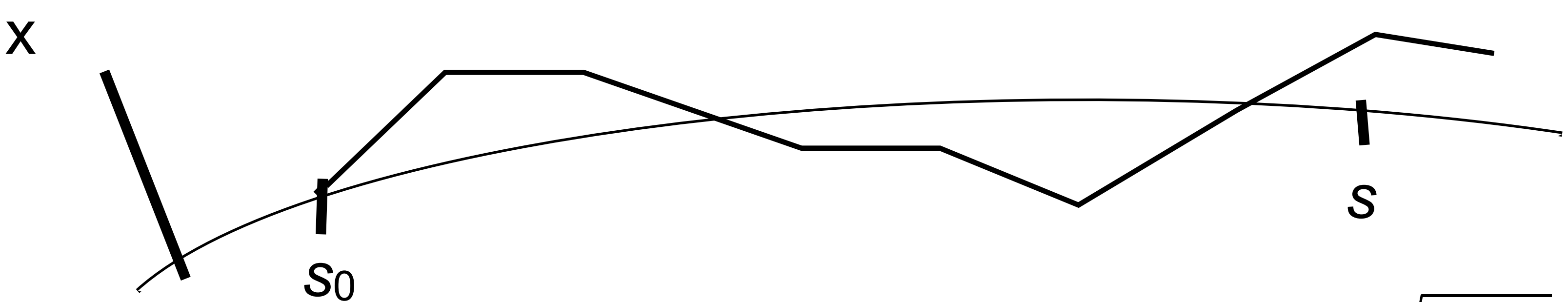
$\gamma \equiv \frac{1+\alpha^2}{\beta}$ $\approx$ *1/β in our ring*

Phase Advance and Betatron Tune

- The transverse oscillatory motion of an individual particle can now be written in terms of the periodic functions. These so-called "Betatron Oscillations" (first observed in the ***Betatron*** accelerator — at UIUC! — in the 1940s) can be written in terms of the periodic functions:

The general equation of motion: $x'' + K(s)x = 0$

Then solve (piecewise)… $\beta'' + 4K\beta = const.$



$$\Longrightarrow x(s) = \sqrt{\frac{\beta(s)}{\beta_0}}\left[x_0 \cos\Delta\psi + (\alpha_0 x_0 + \beta_0 x_0')\sin\Delta\psi\right]$$

The phase advances by amount:

$$\psi(s) = \int \frac{ds}{\beta(s)}$$

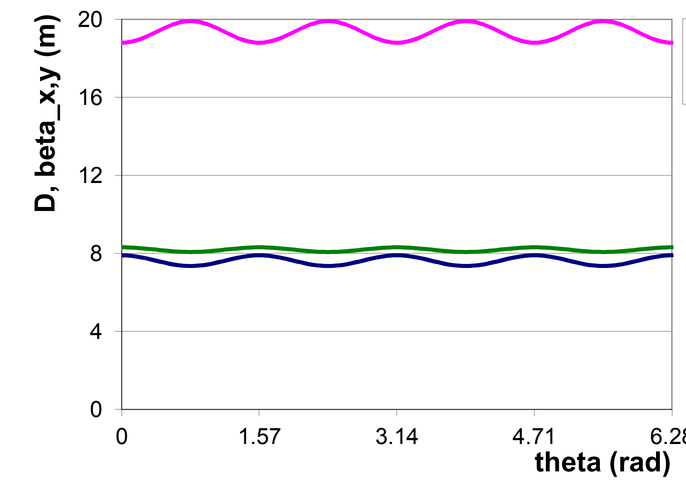
total phase advance in one revolution: $\Delta\psi_{tot} = 2\pi\nu$

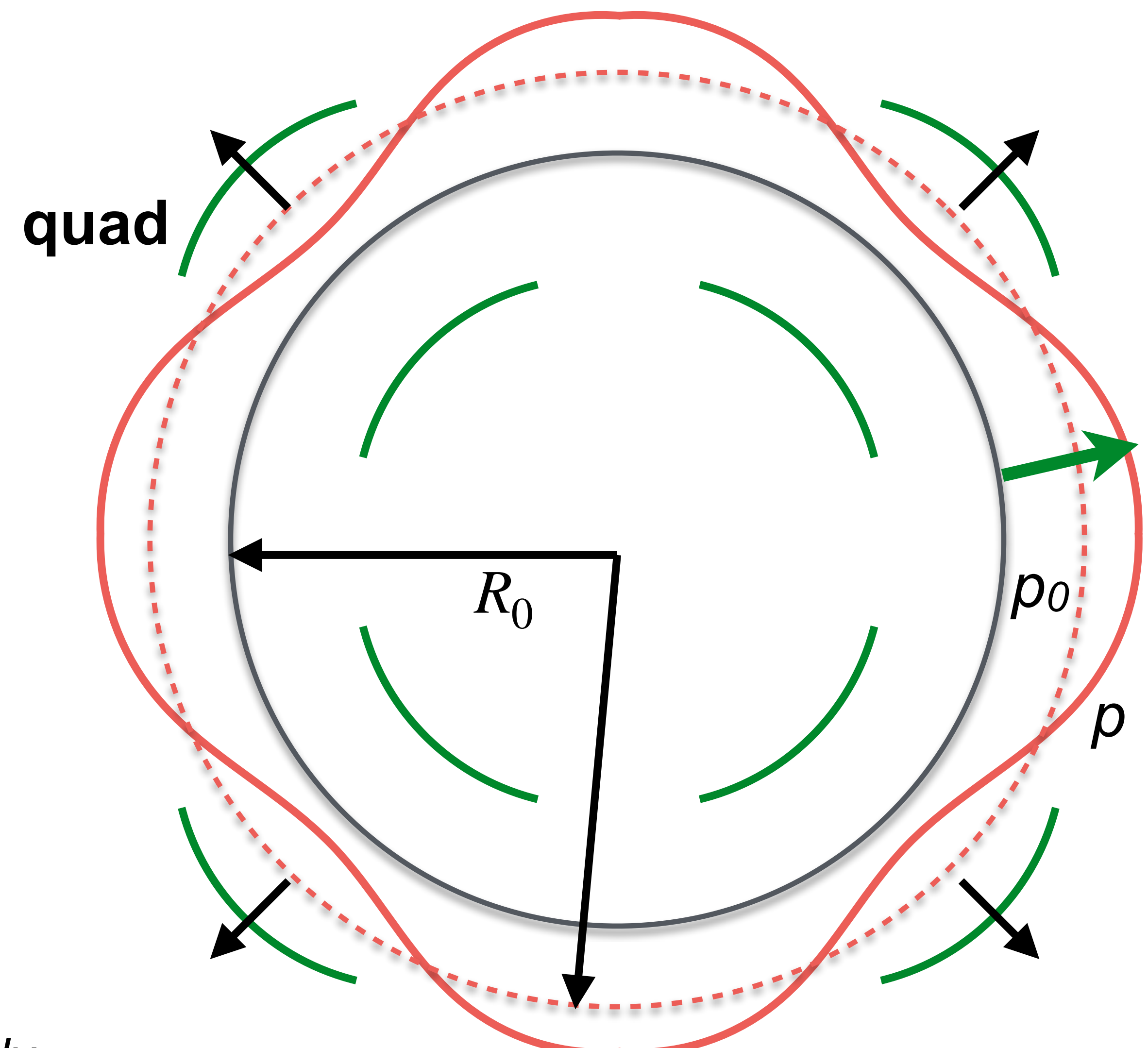
the ***tune***

"the tune" = no. of betatron oscillations in one revolution about the ring

Dispersion

- Periodic Orbit of an Off-Momentum Particle





$$\Delta x_e(s) = D(s) \cdot \frac{\Delta p}{p_0}$$

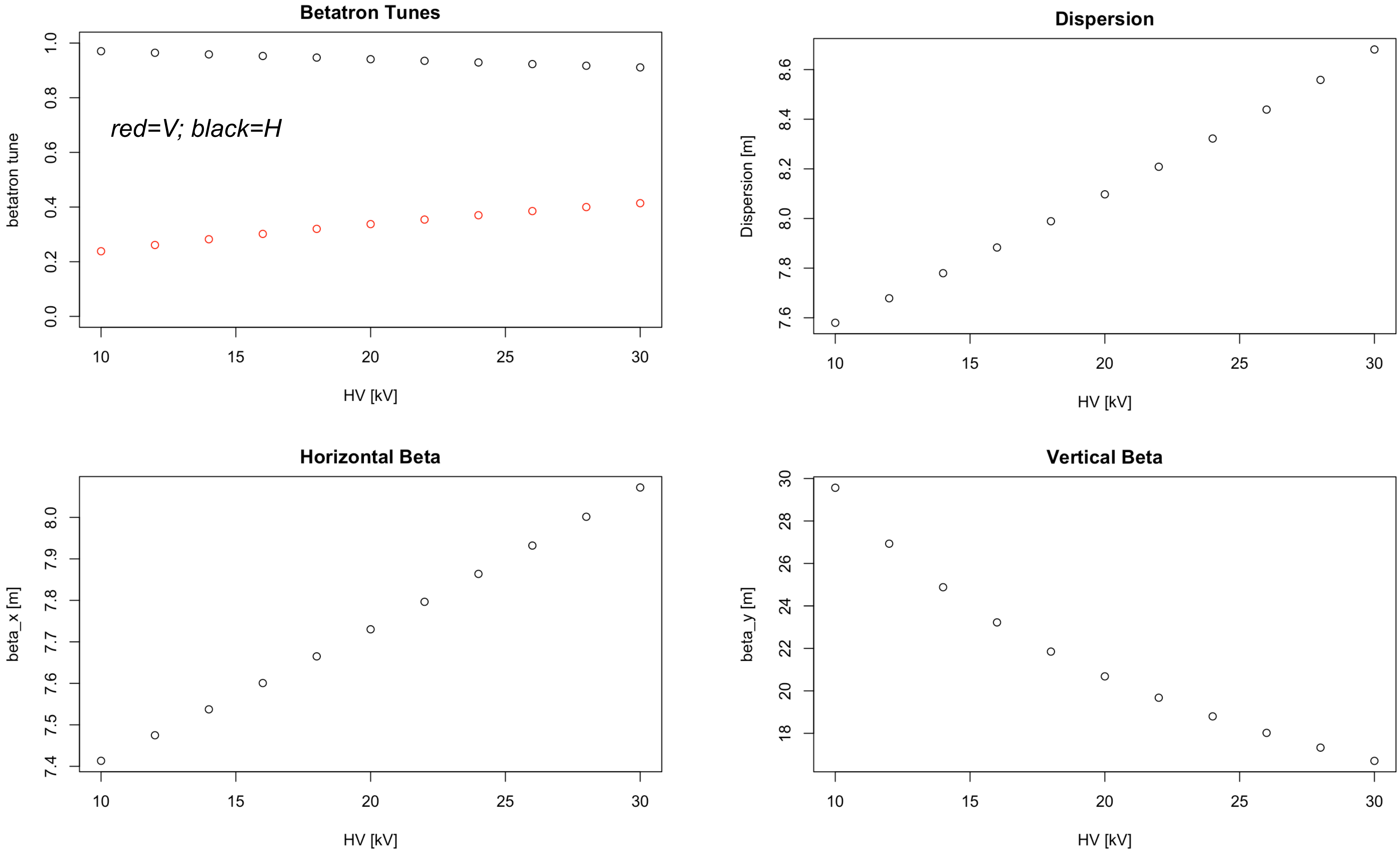
$R = R_0(1 + \Delta p/p)$

schematic only,
highly *exaggerated*

$D(s)$ will depend upon the quad voltage, hence upon the field index, n

R_0 = 7.112 m
D ≈ 8 m

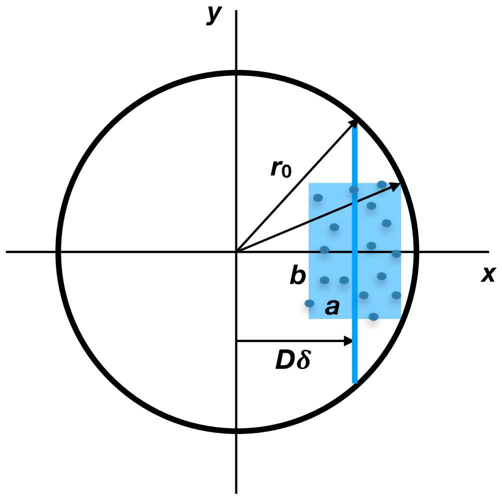
Variations with High Voltage



Reaching Equilibrium

- Debunching

$$\frac{dt}{t} = \frac{dL}{L} - \frac{dv}{v} = \frac{dL}{L} - \frac{1}{\gamma^2}\frac{dp}{p} = \left(\langle\frac{D}{\rho}\rangle - \frac{1}{\gamma^2}\right)\frac{dp}{p}$$

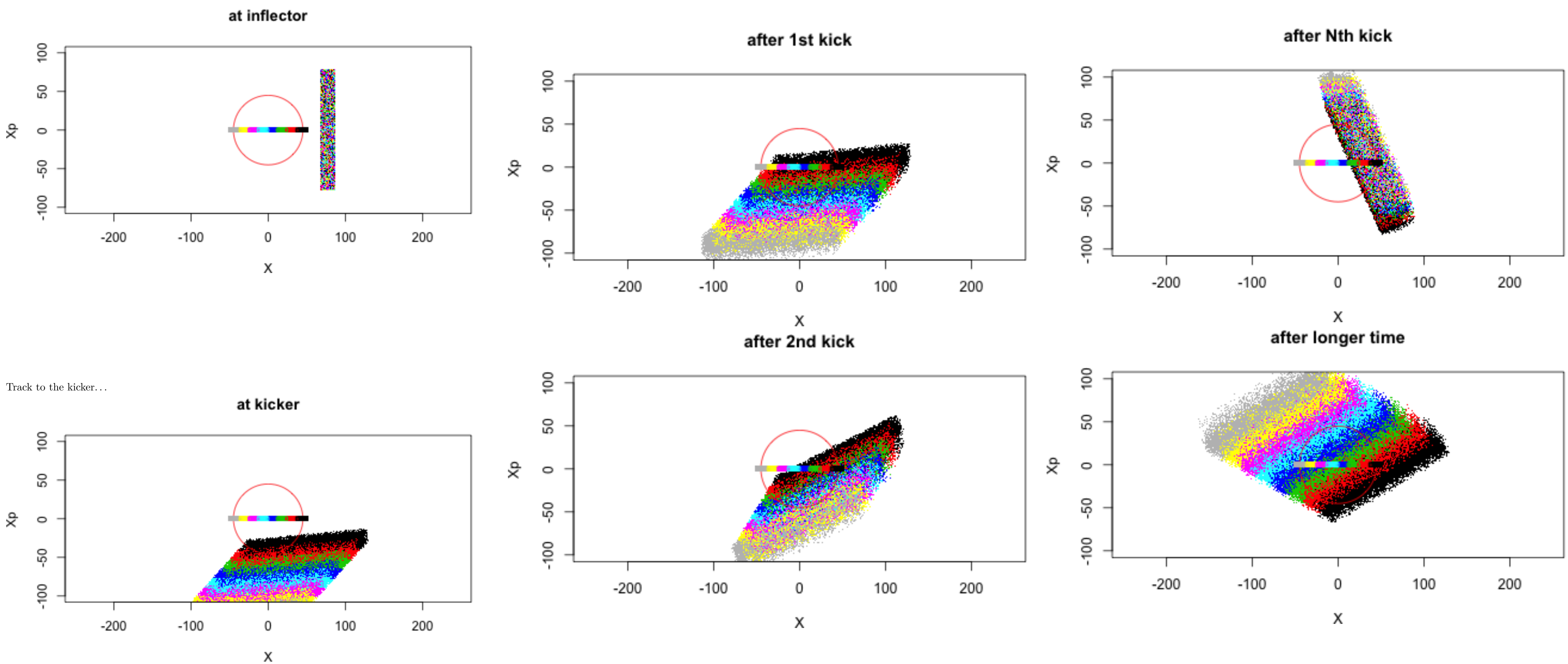


- Survivable particles
  - available aperture as a function of momentum
  - interplay of horizontal and vertical amplitudes due to circular aperture

- Initial beam losses

- The final momentum distribution

Final Distribution After Injection Kicking



Chromatic Decoherence

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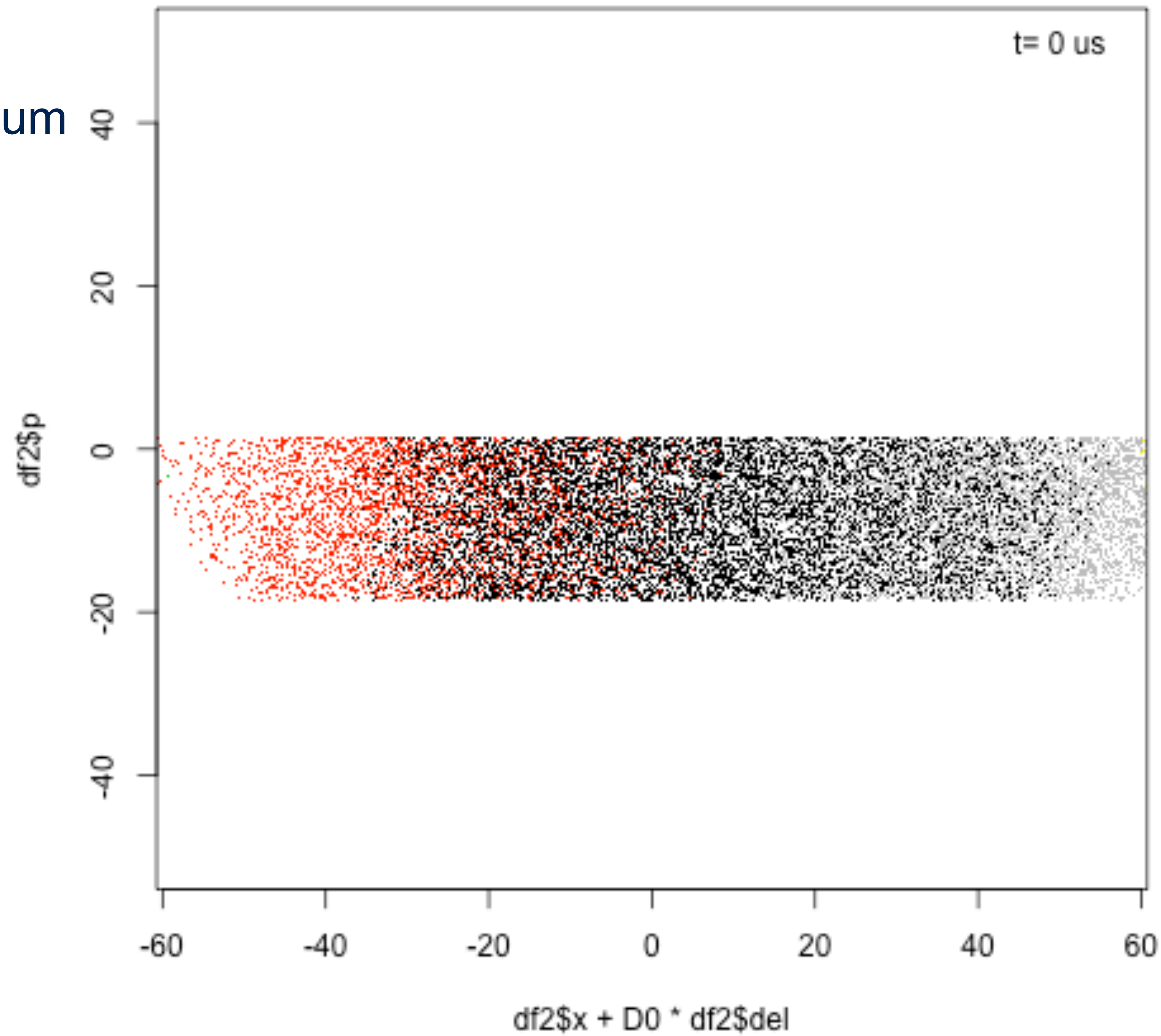
- Chromaticity
  - tune depends upon momentum



The Momentum Distribution

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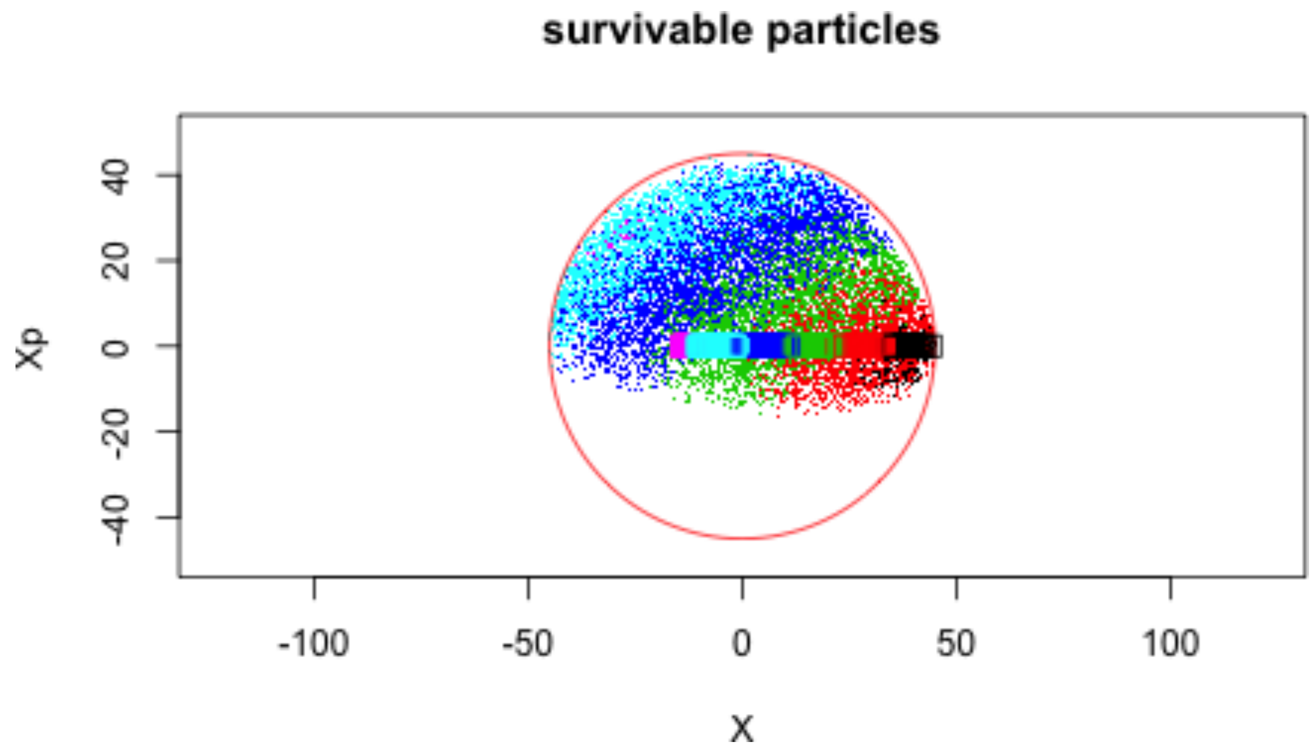
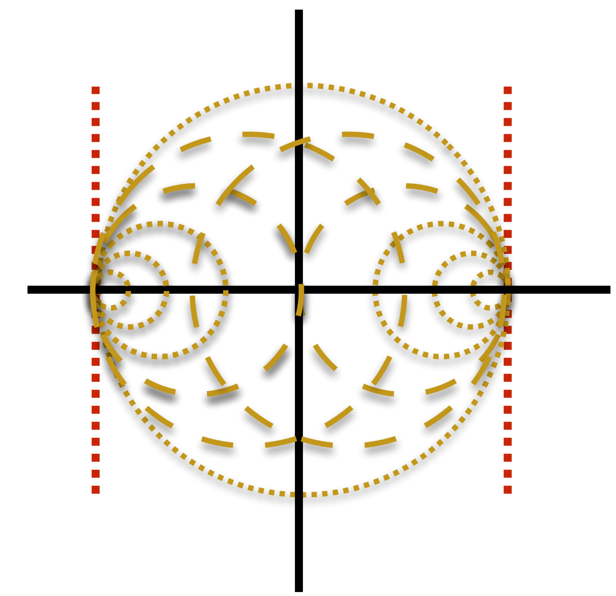
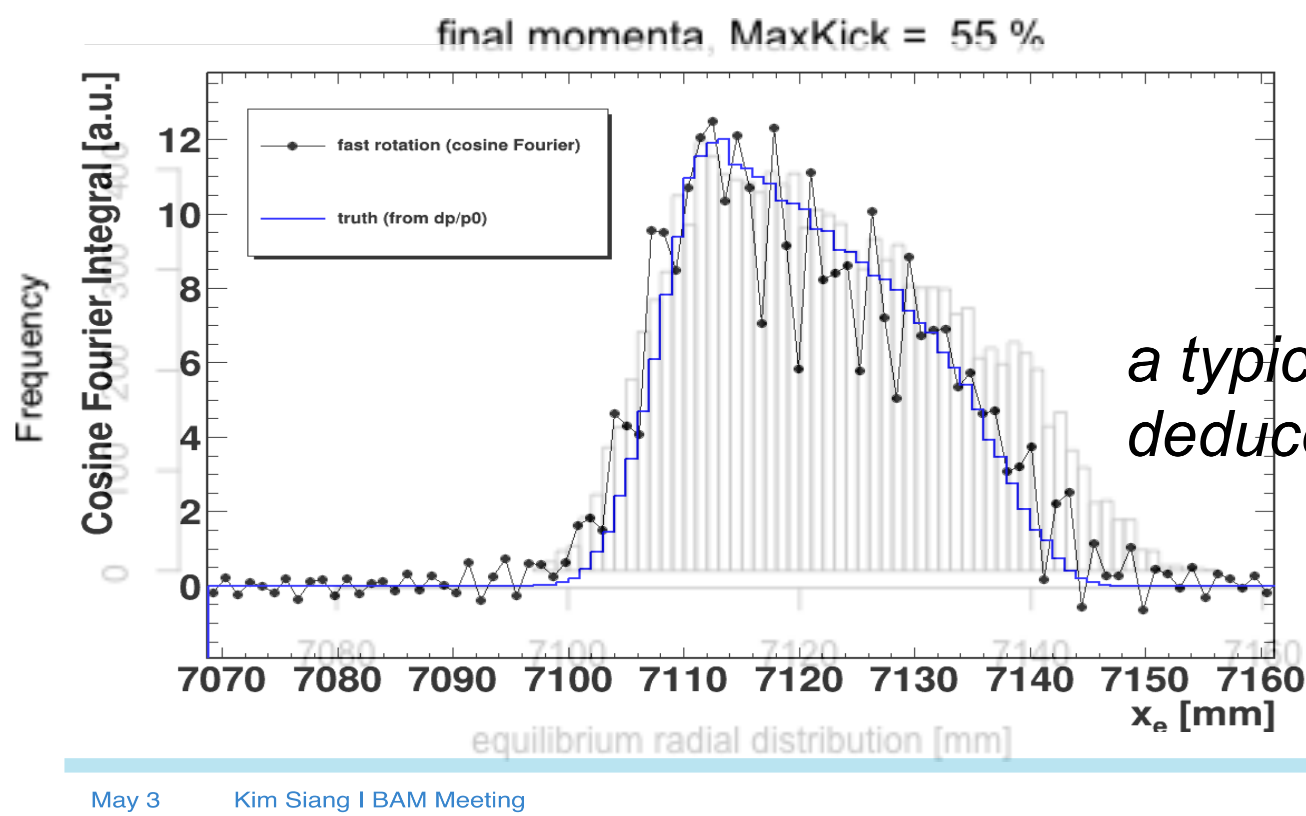


$a_\beta < r_{ap} - D\delta$



look at momentum distribution of the particles that can survive long-term:



a typical distribution, deduced from data analysis

Distortions Due to Reality

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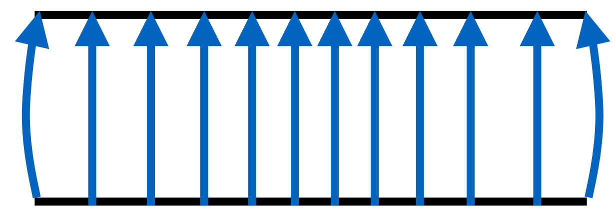
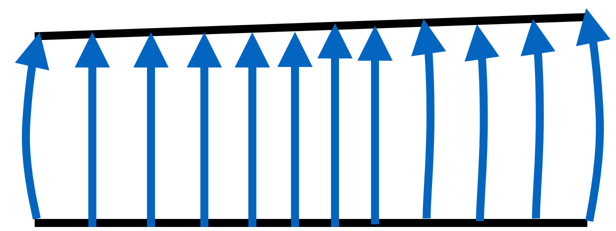
- Field imperfections
  - systematic errors
  - random errors

the trouble with a "finite plane"…



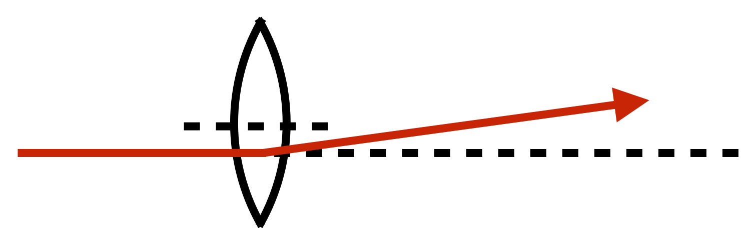
"b_2" from edge

- Alignment Imperfections
  - inherently random



"b_1" from imperfect pole alignment

- Feed-down Effects
  - misaligned quadrupole — steers beam (i.e., like a "dipole" magnet would)

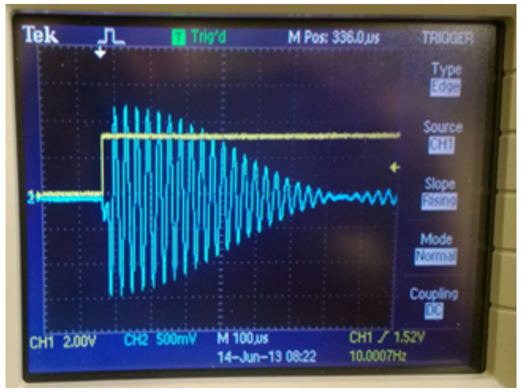


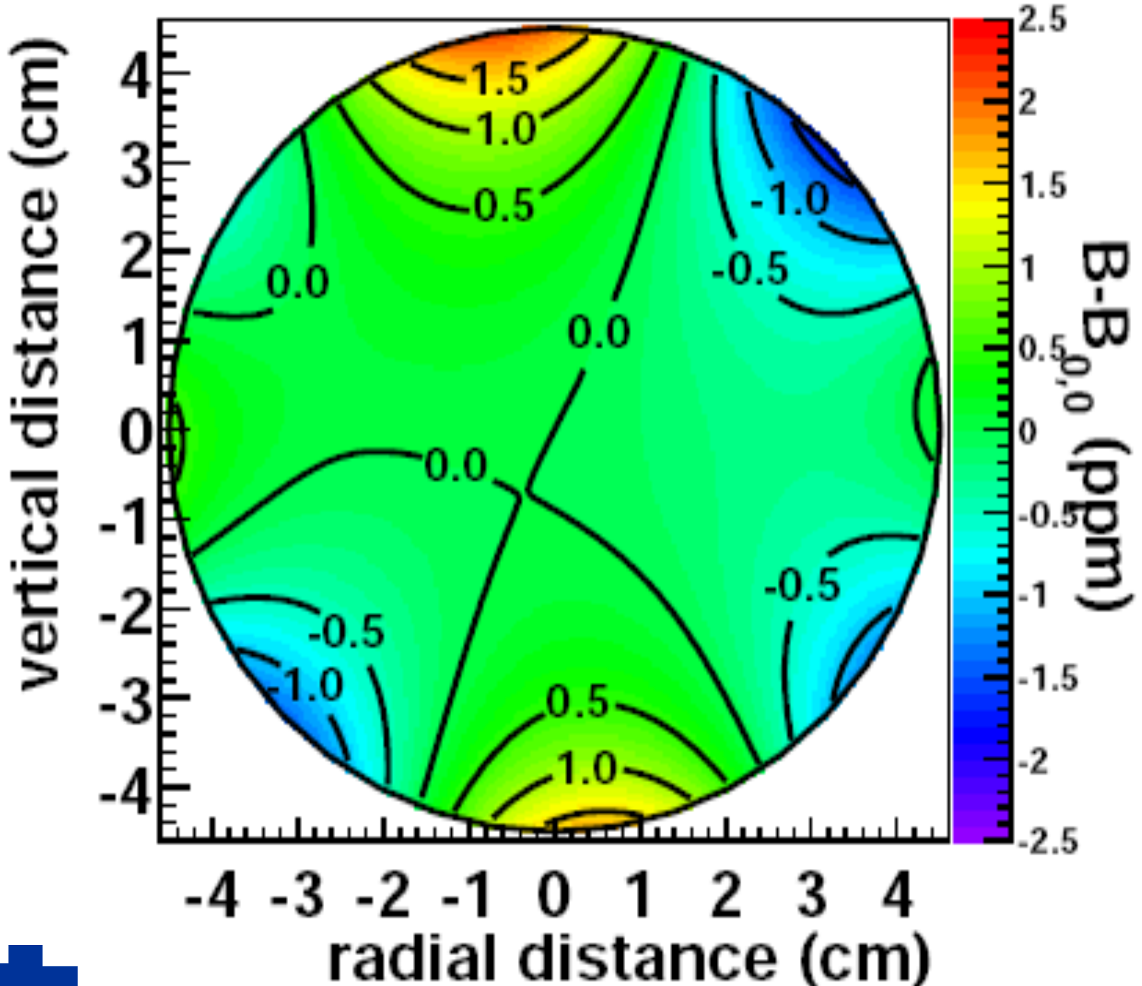
"b_0" from imperfect quad alignment

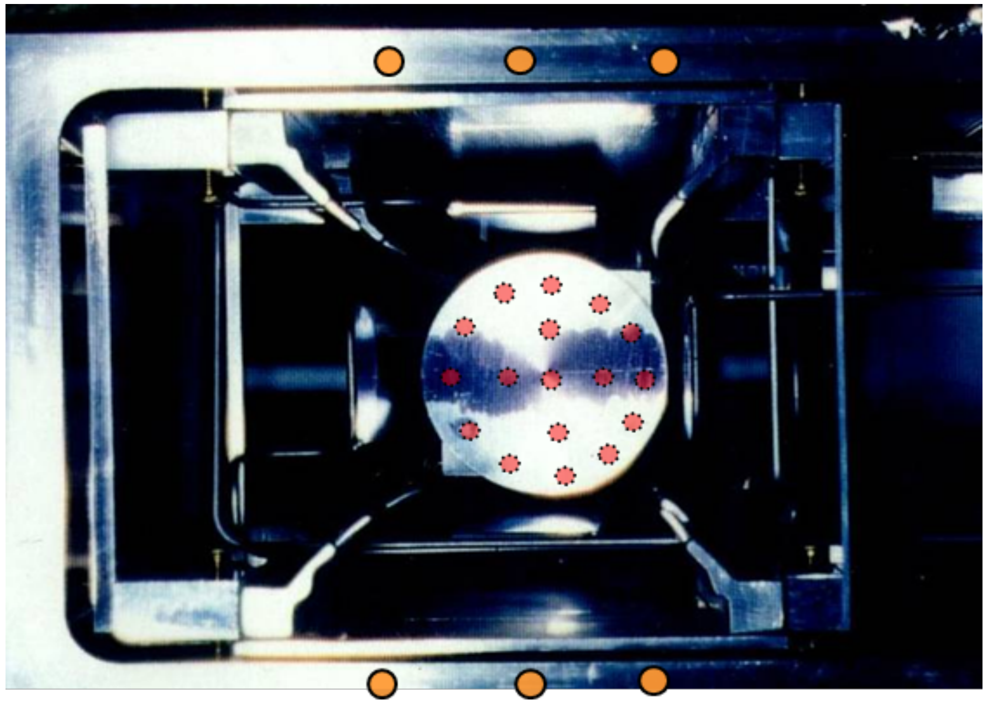
  - offset trajectory through a sextupole field — adds extra focusing (like a quad)

The Storage Ring and Field Precision

- Collection of NMR probes on a railed system map out the field quality periodically over time
- Hall probe system, as well as vertical orbit distortion detection, provides information on local radial field distortions







Multipole Representations

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- Magnetic Multipole Coefficients:

$$B_y + iB_x = B_0 \left\{ 1 + \sum_{n=0}^{\infty} (b_n + ia_n)(x+iy)^n \right\}$$

for ideal dipole magnet, error terms:

$$\frac{\Delta B_y}{B_0} + i\frac{\Delta B_x}{B_0} = \sum_{n=0}^{\infty} (b_n + ia_n)(x+iy)^n$$

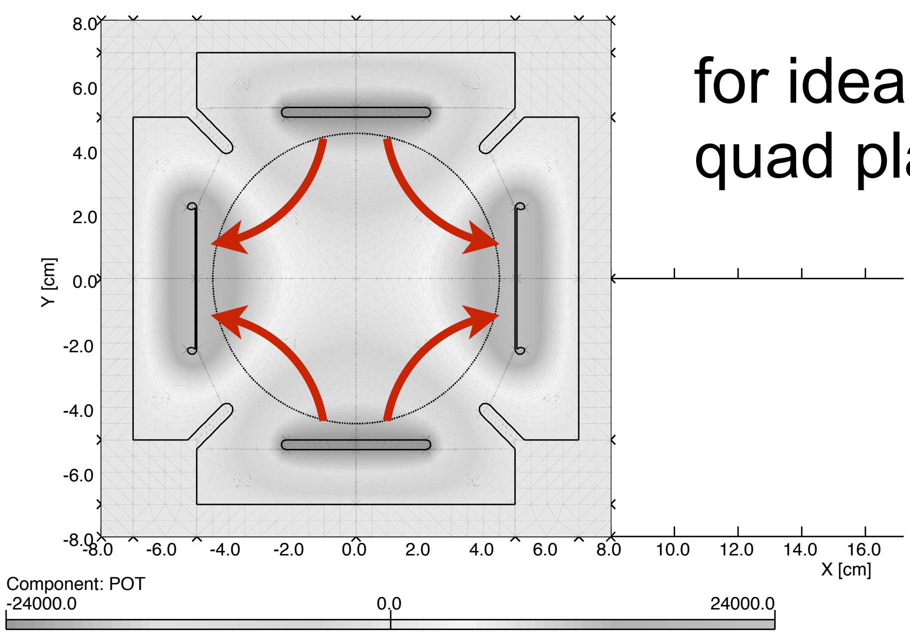
b's are "normal" terms
a's are "skew" terms

- Note: above assumes no curvature of the coordinates.
  - OK for large bending radius, but really not quite good enough for R = 7.1 m
  - small corrections come into play

Multipole Representations

- Electric Multipole Coefficients:

$$V = V_0 \left\{ 1 + \Re\left[\sum_{n=0}^{\infty} (v_n + i w_n) \left(\frac{x+iy}{a_0} \right)^n \right] \right\}$$



for ideal finite electrostatic quad plates, along $y = 0$:

v's are "normal" terms
w's are "skew" terms

$$\frac{\partial E_x}{\partial x} = -\frac{\partial E_y}{\partial y} = -\frac{\partial^2 V}{\partial x^2} = -\frac{V_0}{a_0^2} \sum_{n=2}^{\infty} n(n-1) v_n \left(\frac{x}{a_0} \right)^{n-2}$$

- Again: assumes no curvature of the plates. Exact representation is just a little more complicated.

Distortions Created from Error Fields

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$$x'' + K_x(s)x = 0 \longrightarrow = -\frac{e\Delta B_y(s,x)}{p_0} \longrightarrow = -\frac{B_0 b_n x^n}{B\rho} = -\frac{b_n x^n}{R_0}$$

- Closed Orbit Distortions

$$x'' + K_x x = -\frac{b_0}{R_0}$$

similarly for quad field errors

- Tune, Amplitude Function Distortions

$$x'' + K_x x = -\frac{b_1}{R_0}x \longrightarrow x'' + \left(K_x + \frac{b_1}{R_0}\right)x = 0$$

- Nonlinear Distortions

$$x'' + K_x x = -\frac{b_2}{R_0}x^2$$

Effect of Dipole Steering Error(s)

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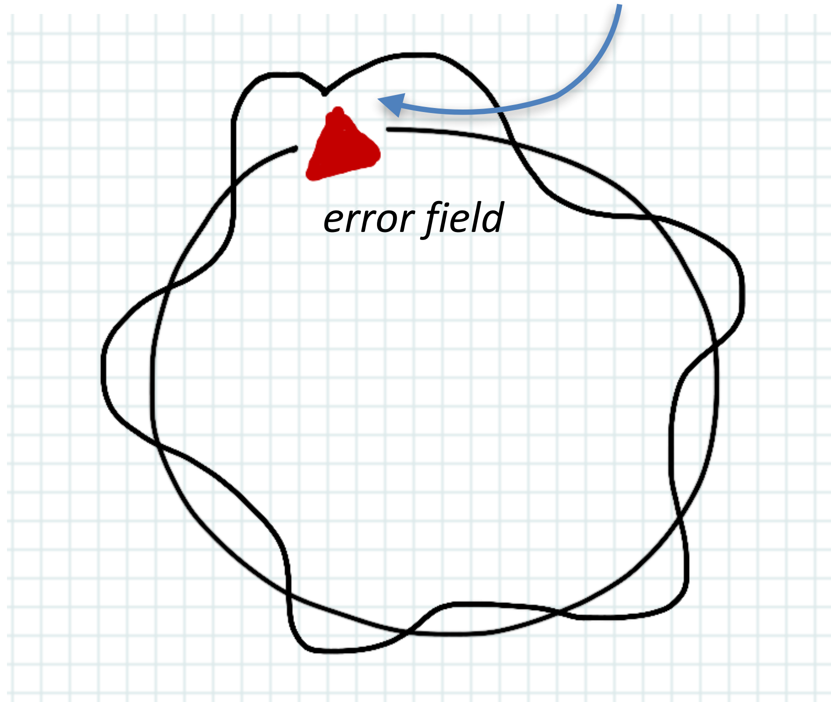
- Want to find the one trajectory which, upon passing through a localized error field, will come back upon itself
  - this is the "closed" trajectory, or ***closed orbit***

$$M_0 \begin{pmatrix} x_0 \\ x_0' \end{pmatrix} + \begin{pmatrix} 0 \\ \Delta\theta \end{pmatrix} = \begin{pmatrix} x_0 \\ x_0' \end{pmatrix}$$



- When find x_0, x'_0, can find x,x' downstream:

$$\begin{pmatrix} x_0 \\ x_0' \end{pmatrix} = (I - M_0)^{-1} \begin{pmatrix} 0 \\ \Delta\theta \end{pmatrix}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} \left(\frac{\beta}{\beta_0}\right)^{1/2} (\cos\Delta\psi + \alpha_0 \sin\Delta\psi) & \sqrt{\beta_0\beta}\sin\Delta\psi \\ -\frac{1+\alpha_0\alpha}{\sqrt{\beta_0\beta}}\sin\Delta\psi - \frac{\alpha-\alpha_0}{\sqrt{\beta_0\beta}}\cos\Delta\psi & \left(\frac{\beta_0}{\beta}\right)^{1/2}(\cos\Delta\psi - \alpha\sin\Delta\psi) \end{pmatrix} \begin{pmatrix} x_0 \\ x_0' \end{pmatrix}$$

Closed Orbit Distortion from Single Error

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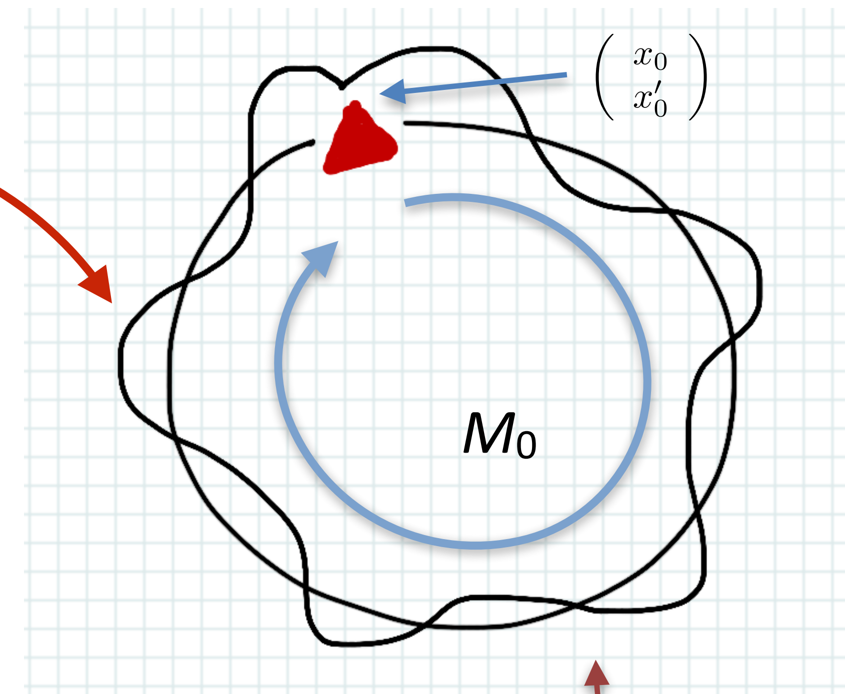
$$\Delta x(s) = \frac{\Delta\theta\sqrt{\beta_0\beta(s)}}{2\sin\pi\nu}\cos\left[|\psi(s)-\psi_0| - \pi\nu\right]$$

as ν —> integer, huge distortions
a resonance!



ν oscillations

If have a collection of errors about the accelerator, then at any one point:

$$\Delta x(s) = \sum_i \frac{\Delta\theta_i\sqrt{\beta_i\beta(s)}}{2\sin\pi\nu}\cos\left[|\psi(s)-\psi_i| - \pi\nu\right]$$

With such error sources in the ring, particles will oscillate about this new closed orbit; hence this will be the mean of the observed betatron oscillations

Note: exact same analysis applies for vertical steering errors

Ex: Vertical Distortion due to Radial Fields

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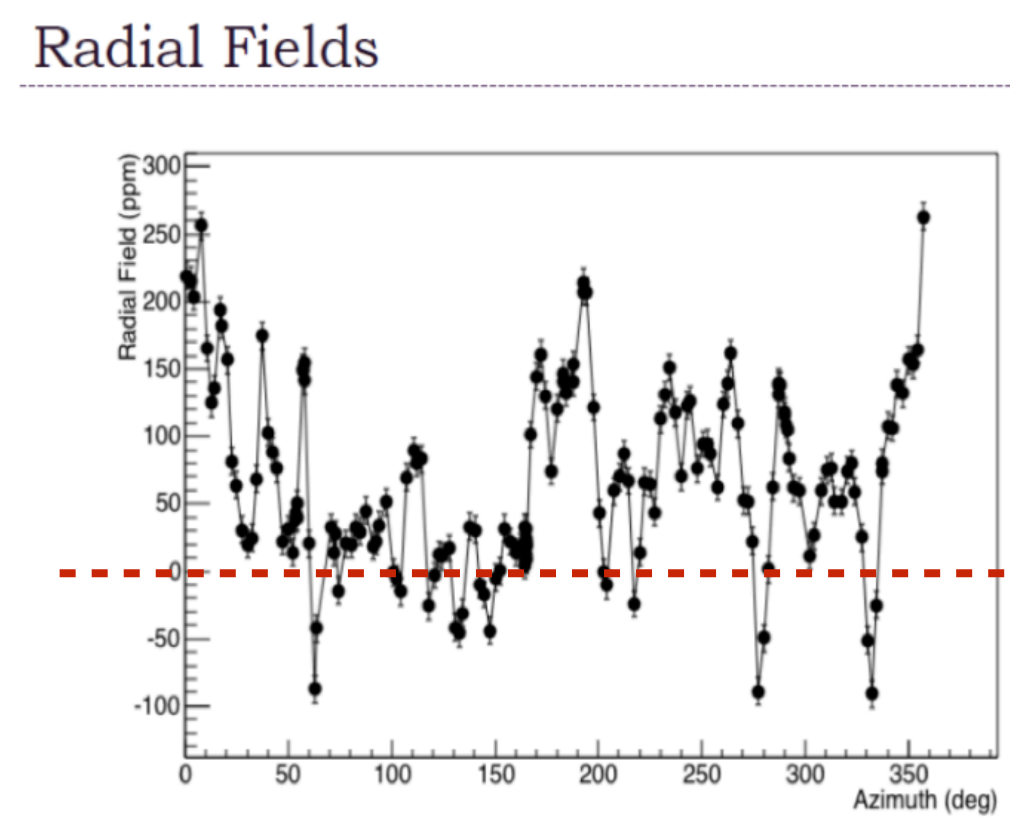
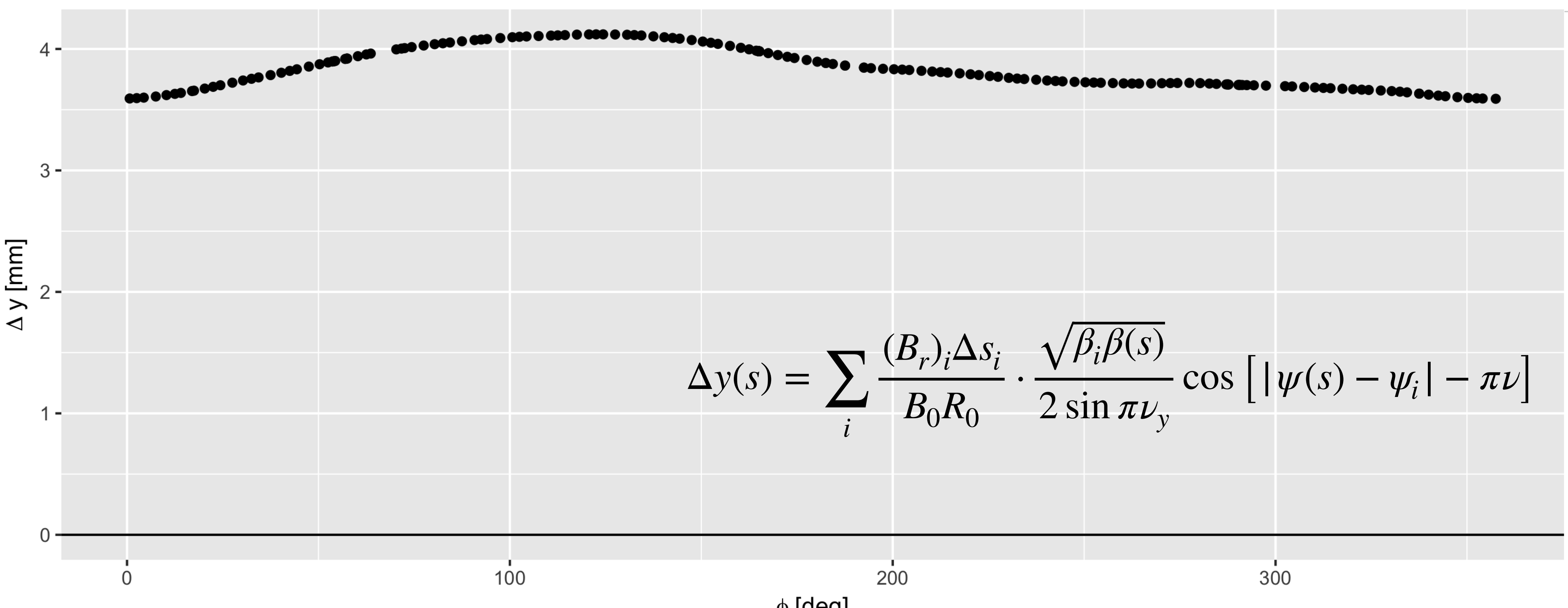
- Using data from Rachel Osofsky, can predict the vertical offsets around the ring due to residual radial fields
- When these data were taken, the offsets observed were on the scale of 4 mm; agrees well with the calculation
- Since then, further trimming has reduced this to ~2 mm or less





$$\Delta y(s) = \sum_i \frac{(B_r)_i \Delta s_i}{B_0 R_0} \cdot \frac{\sqrt{\beta_i \beta(s)}}{2 \sin \pi \nu_y} \cos\left[|\psi(s) - \psi_i| - \pi\nu \right]$$

Tunes and Lost Muons

- Tune Shift from Gradient Error
  - misaligned quad plate pair
  - orbit offset through nonlinear field imperfection

$$\Delta\nu = \frac{1}{4\pi}\beta_0\frac{\Delta B'\ell}{B\rho} \quad\longrightarrow\quad \frac{1}{4\pi}\beta_0\frac{e\Delta E'\ell}{p_0 v}$$

- Amplitude Function Distortion from Gradient Error
  - misaligned quad plate pair
  - orbit offset through nonlinear field imperfection

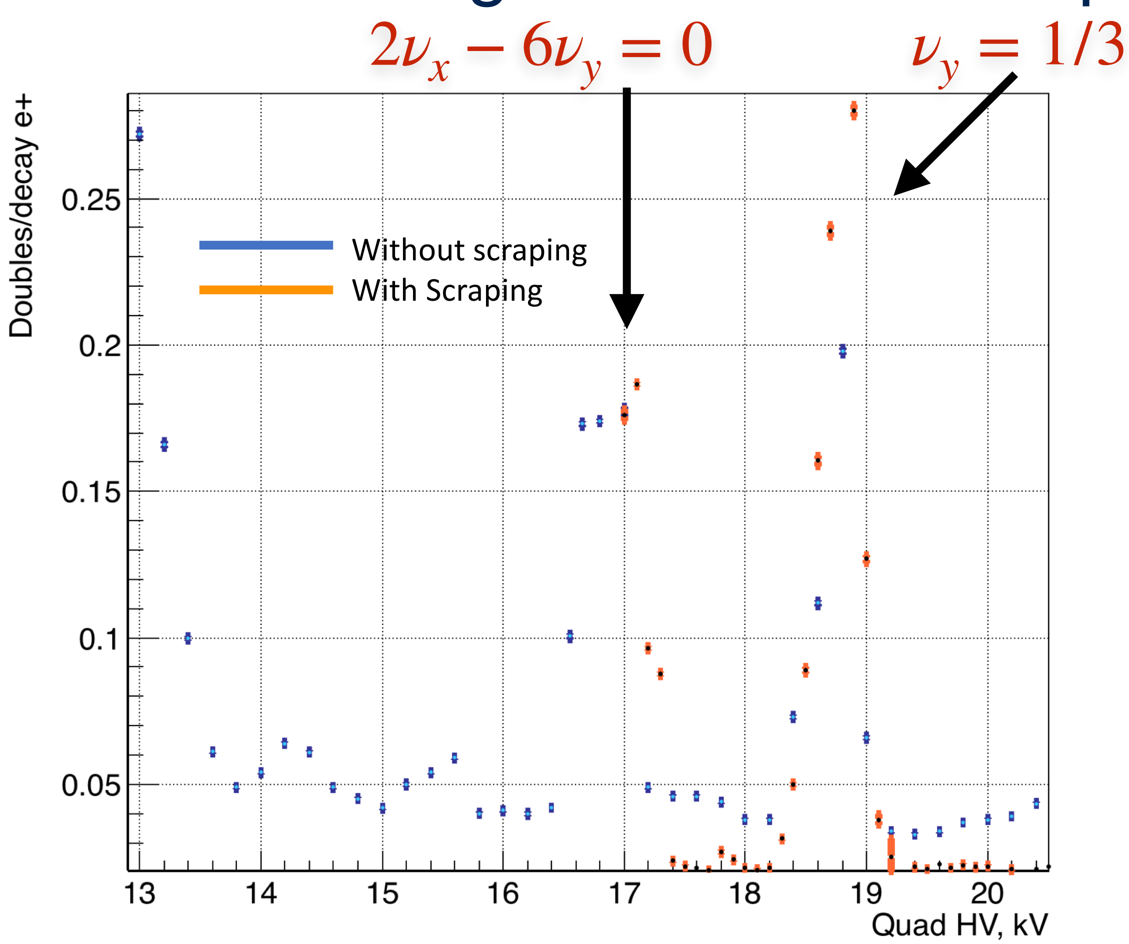
$$\frac{\Delta\beta}{\beta_0} \approx -\beta_0\frac{\Delta B'\ell}{B\rho}\sin 2\psi(s)$$

Muon Losses Prior to Decay

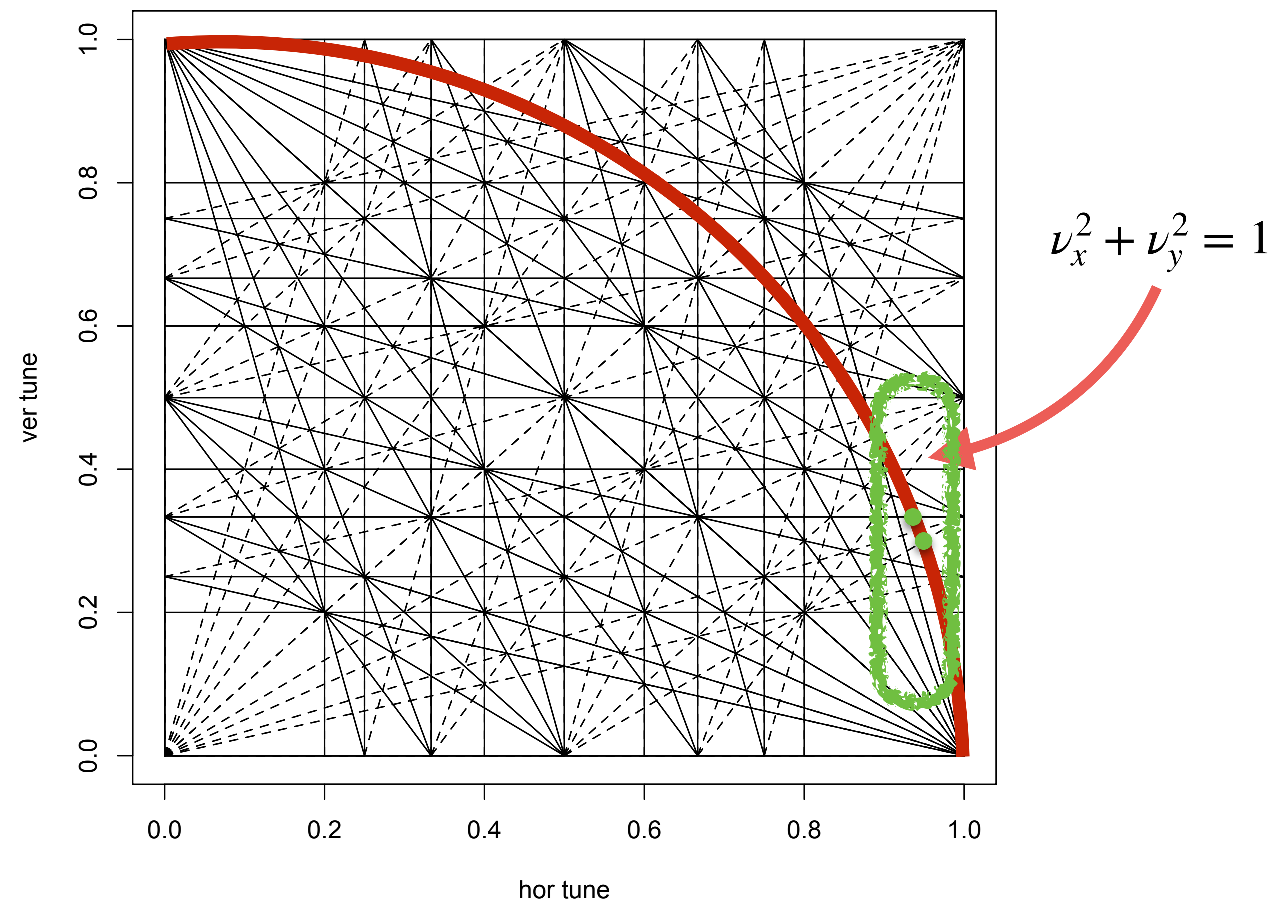
- So-called "lost muons" are identified as particular "hits" in the detector system that occur simultaneously on 2 or 3 consecutive detectors, assumed to be a single muon as opposed to 2 or 3 coincidental positrons
  - beam-gas scattering, field fluctuations, resonance conditions, …
  - Muon loss rate not due to decays must be taken into account in the analysis — will run at high and low vertical tunes, away from resonances

- Scan the quadrupole high voltage:
  - note: weak focusing — tunes are coupled: $\nu_x^2 + \nu_y^2 = 1$

$$m\,\nu_x \pm n\,\nu_y = k$$



Lost muon doubles vs. quad HVPS set-points for t > 30μs after injection, with scraping and without scraping.



Losing Muons Prematurely

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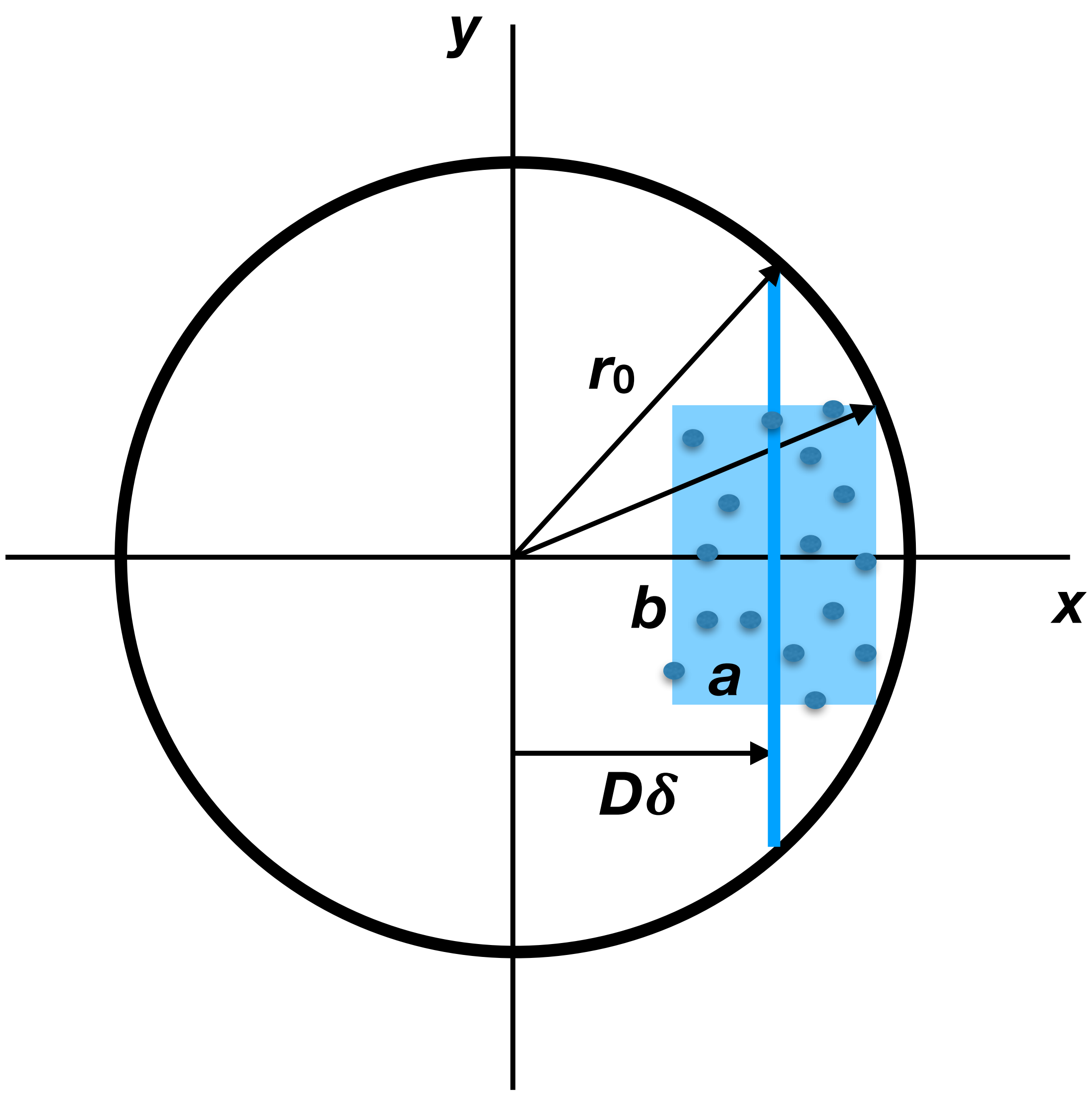
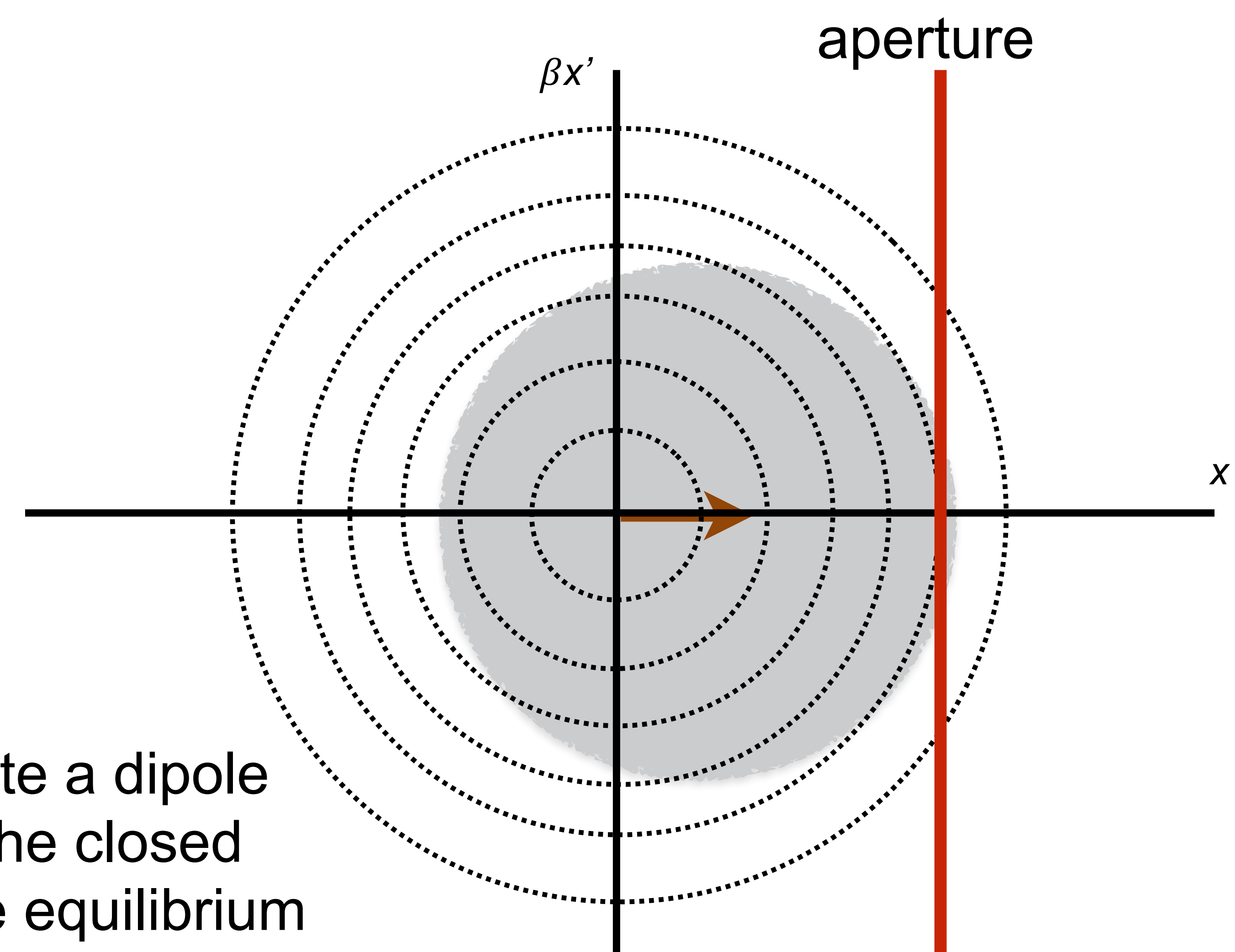
- When discussing *Lost Muons*, consider what the mechanism(s) might be:
  - Simply the time for a particle to eventually "find the aperture”?
    - » the "scraping" process (if the mountain won't come ...)
- Is it muons scattering off of the residual gas molecules in the vacuum chamber?
- We know that the rates go up when near “resonances” in the tune space; how do we interpret this in terms of phase space dynamics?

Scraping

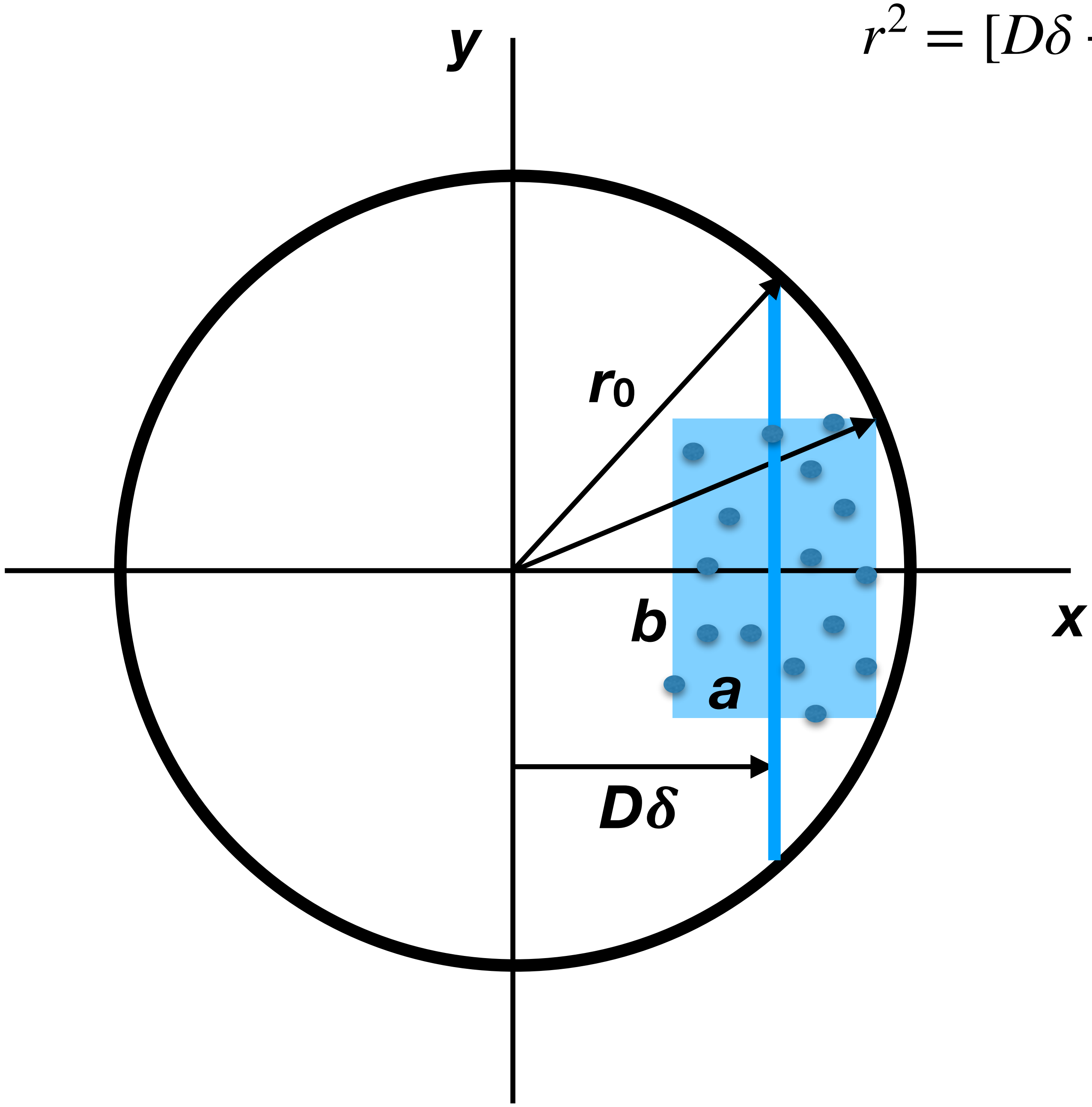
- Some small fraction of particles will live "on the edge"; will eventually encounter the collimator aperture





Use quad plates to create a dipole electric field, distorting the closed orbit (hor + ver) to move equilibrium orbits toward the aperture, then "back off"

Time to Reach the Aperture



$$r^2 = [D\delta + a\cos(\psi_{x0} + 2\pi\nu_x n)]^2 + [b\cos(\psi_{y0} + 2\pi\nu_y n)]^2$$

if *a* and *b* and the initial phase space coordinates conspire such that the particle can reach $r > r_0$, then particle can get lost eventually; the questions are,

How long does this take?

What would the average rate be?

can be several hundred turns…

Beam-Gas Scattering

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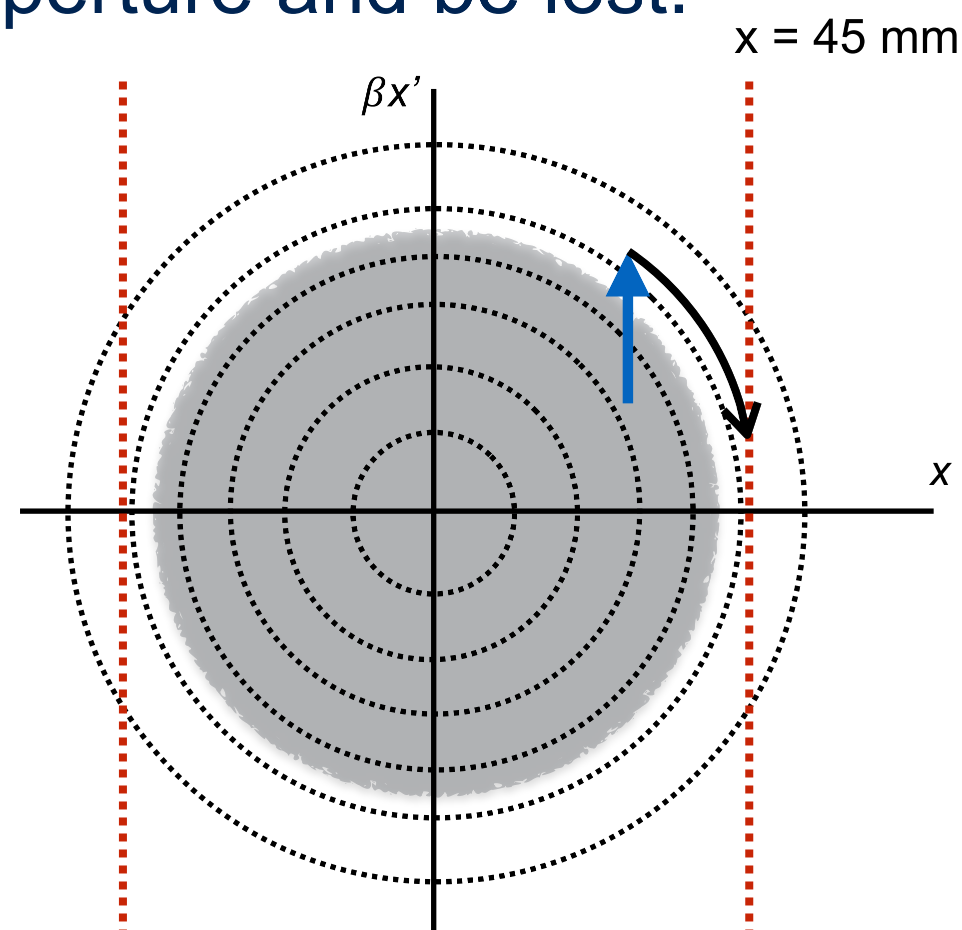
- As muons Coulomb scatter off of residual gas molecules, their betatron amplitudes will grow (on average) and particles can eventually reach the aperture and be lost.



$$R \equiv \frac{d}{dt}\langle W\rangle = \frac{d}{dt}\langle \pi a^2/\beta\rangle = \pi\beta\frac{d}{dt}\langle x'^2\rangle = \pi\beta\cdot\langle\dot{\theta}^2\rangle$$

assuming scattering events only alter x' and not x, and where

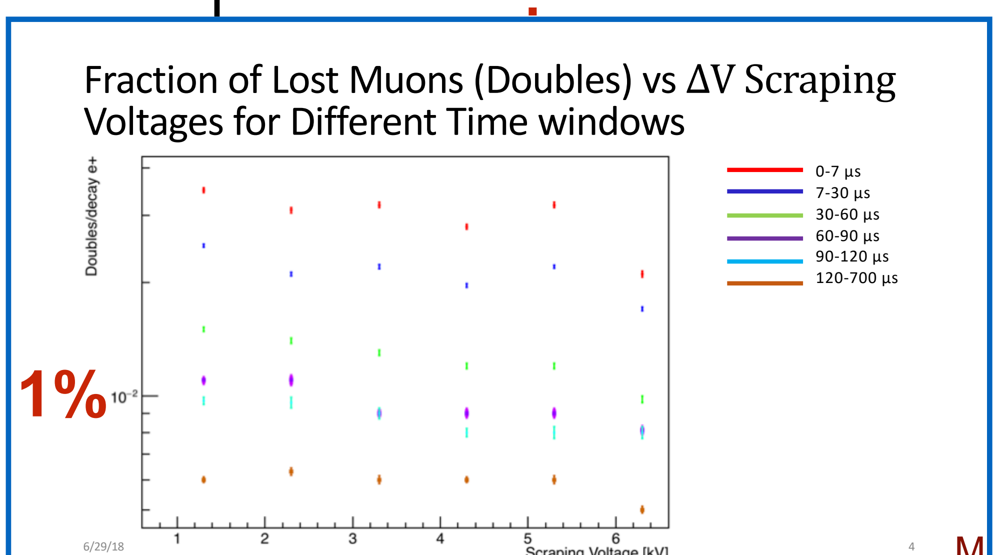
$$\langle\dot{\theta}^2\rangle \approx \left(\frac{13.6\ \mathrm{MeV}}{pv}\right)^2 \frac{v}{L_{rad}}$$

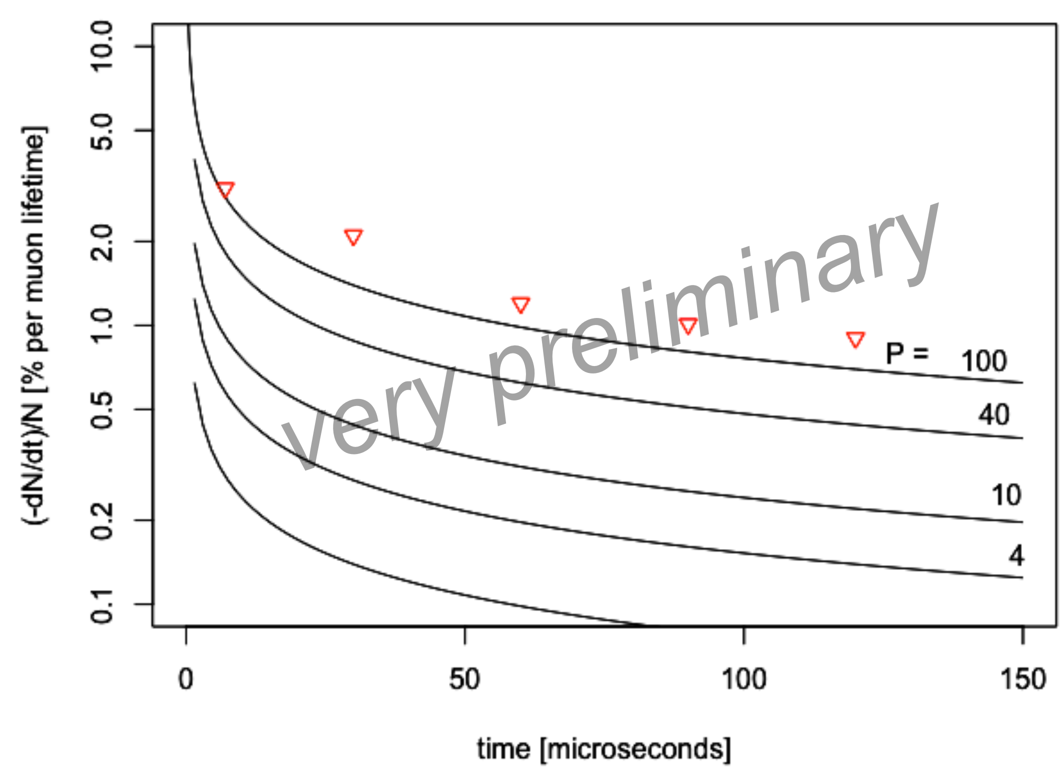
is the rate of increase of the variance of the scattering angle, θ, due to multiple Coulomb interactions. For the Muon g-2 Storage Ring and using "air" as our scattering material, we find that

$$\langle\dot{\theta}^2\rangle \approx \left(\frac{13.6}{3094}\right)^2 \frac{3\cdot 10^8\ \mathrm{m/s}}{\frac{36.6\ \mathrm{g/cm^2}}{1.205\ \mathrm{g}/\ell}\frac{10^3\mathrm{cm}^3}{\ell}}\cdot\frac{100\ \mathrm{cm}}{\mathrm{m}}\cdot\frac{P_{\mu torr}\cdot 10^{-6}}{760}$$

$$= (0.16\ \mathrm{mr})^2/\mathrm{s}\cdot P_{\mu torr}$$

where $P_{\mu torr}$ is the residual gas pressure in units of microtorr.

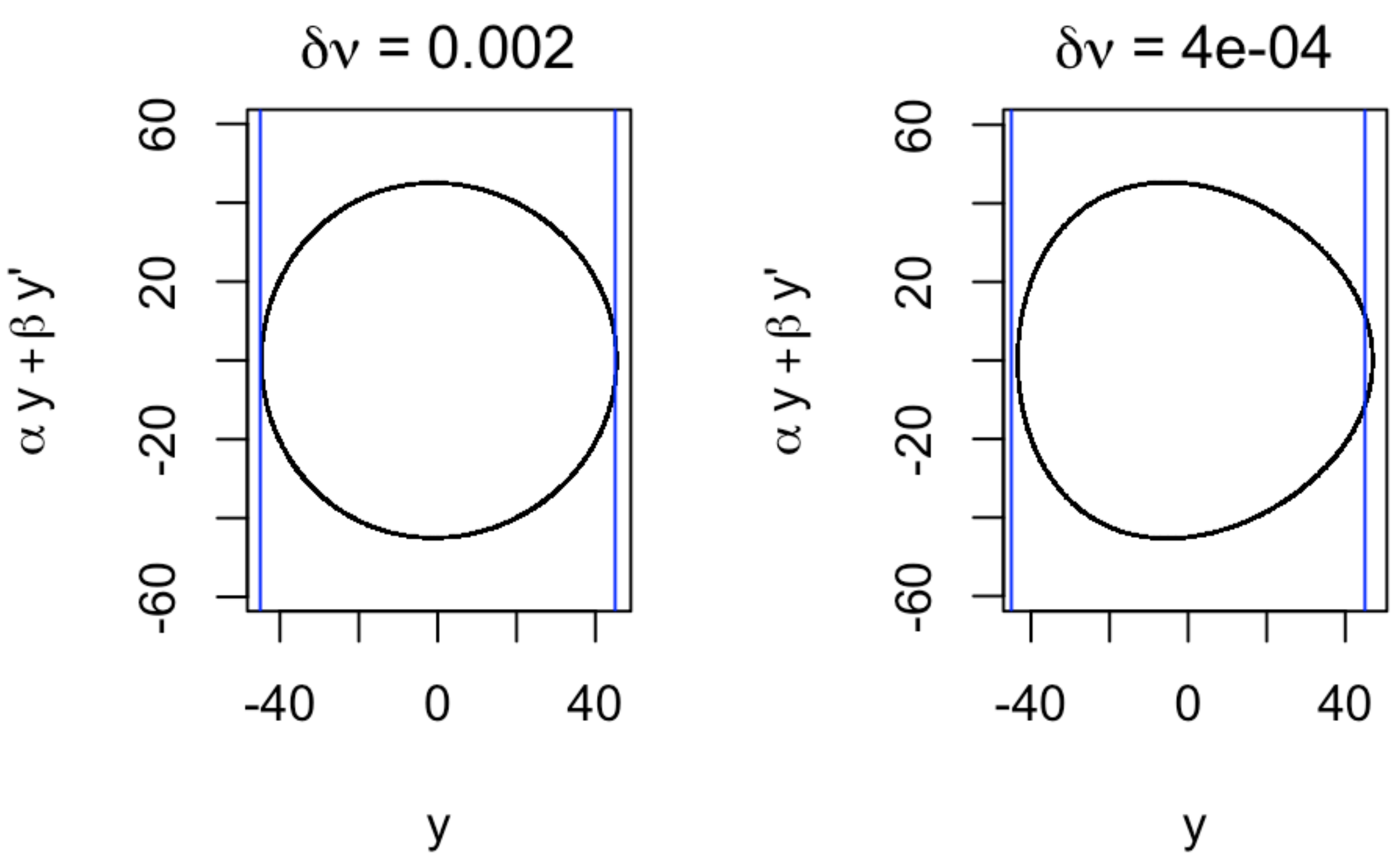




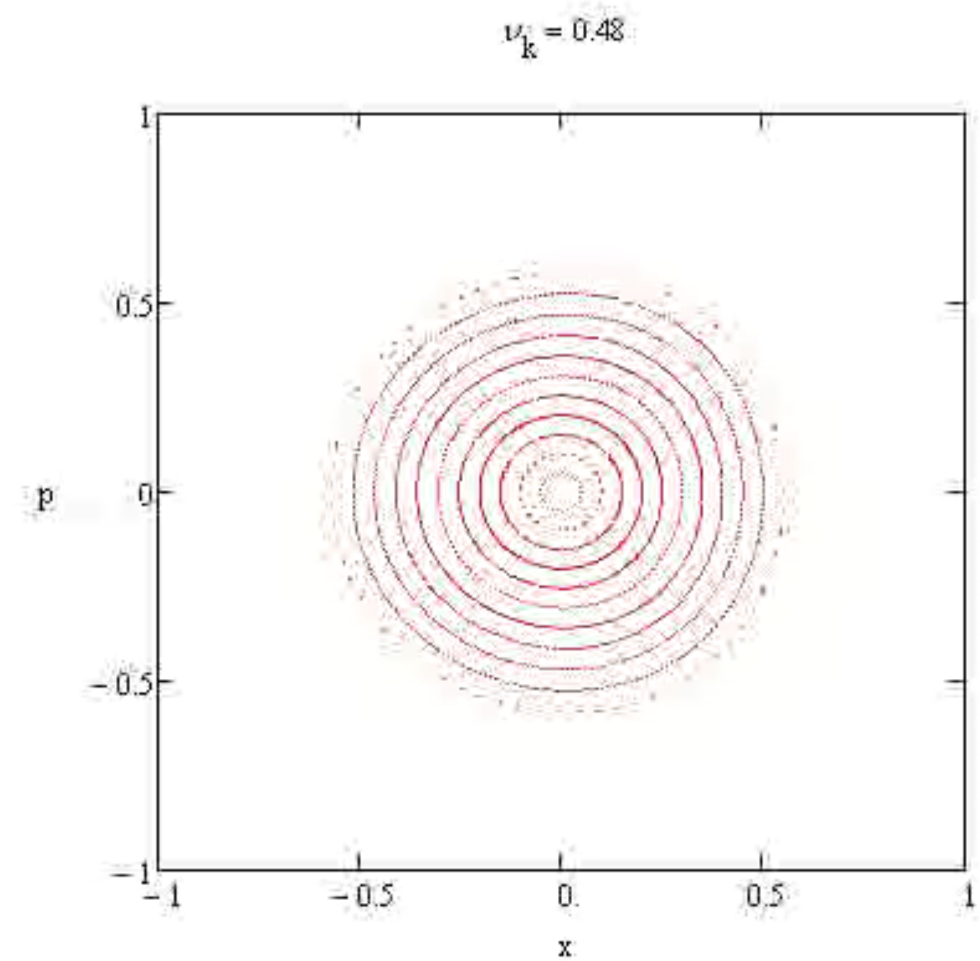
Resonances and Nonlinear Distortions

- Strong resonances can kick the particles out of the aperture for sure; but for E989 it is distortions of the phase space trajectories caused by the nonlinearities that guide particles into apertures

ex: weak, residual sextupole field,
tune near third-integer: $\delta\nu = \nu - \frac{1}{3}$



ex: phase space from *single sextupole* as function of tune:



Decoherence of Average Displacement

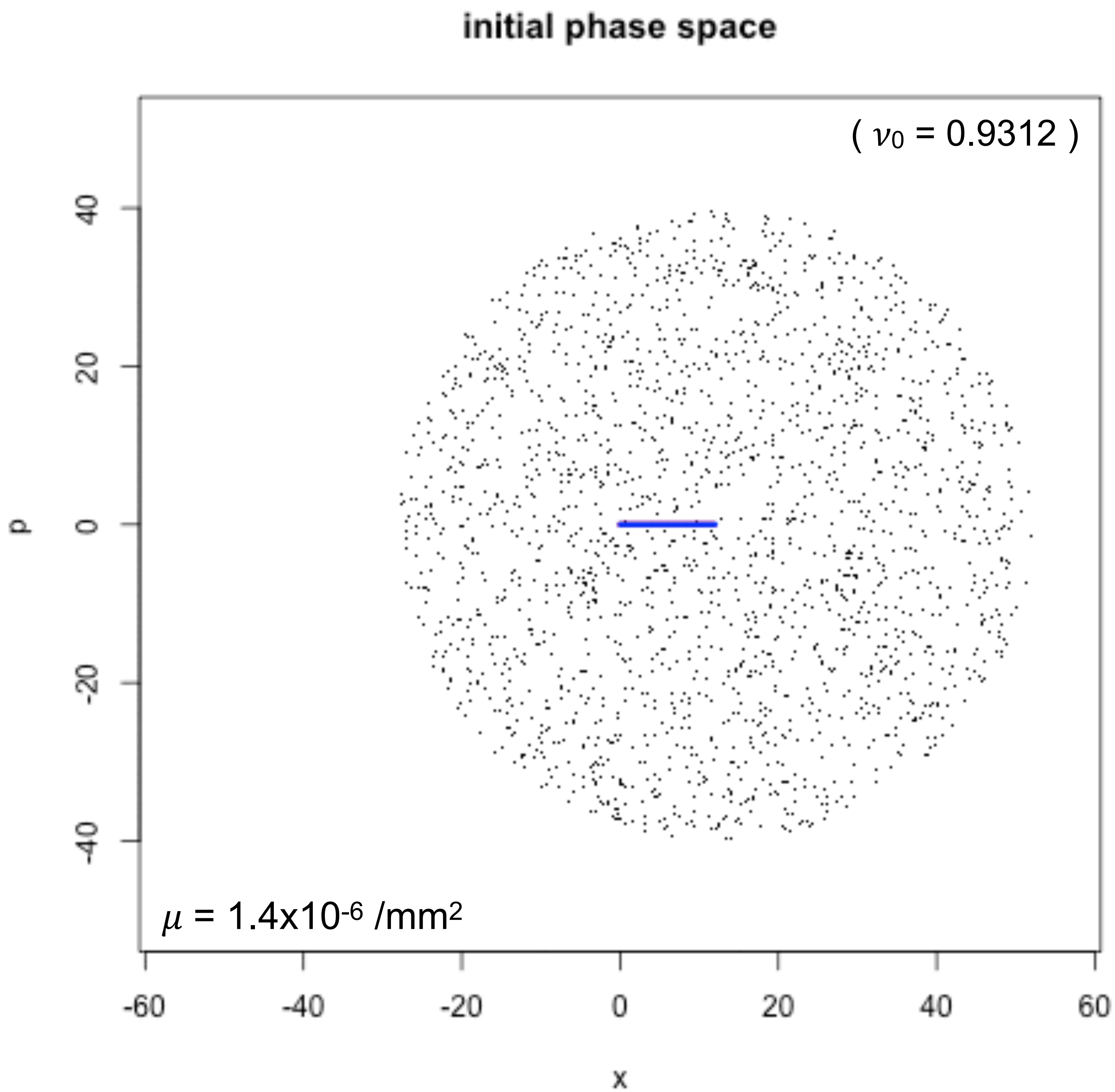
- The tunes will depend upon the particle momentum, as the particles' rigidities will be different — this is referred to as *chromaticity*

$$\text{chromaticity} \equiv \frac{\Delta\nu}{(\Delta p/p)}$$

- For our ring, the chromaticity is about 0.2 - 1 (abs. value) and the momentum spread is about 0.2% (rms). So, the tune spread due to chromaticity will be on the scale of ±0.002 or less. Thus the motion will be fairly coherent for a few hundred turns, but *will decohere* eventually.

- Also, as seen in previous slide, the betatron motion frequency will depend upon the amplitude of the motion (akin to a non-simple pendulum!) in the presence of nonlinear fields. This *nonlinear detuning* will also lead to oscillation decoherence and thus a diminishing signal amplitude as well as a *phase shift*

Nonlinear Deocherence Example



$$\nu = \nu_0 - \mu(x^2 + p^2)$$

blue:

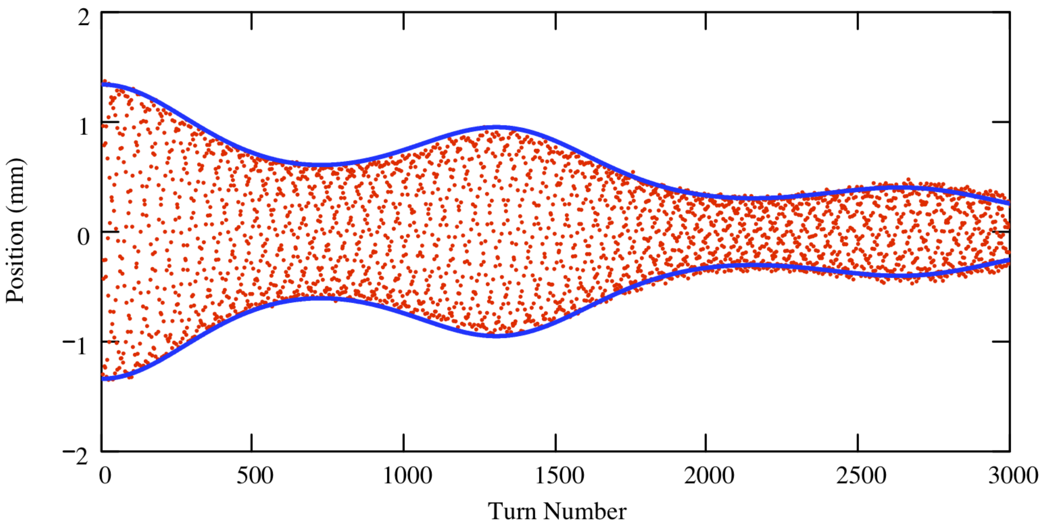
[$\cos(2\pi\nu_0)$, $\sin(2\pi\nu_0)$]

red:

[mean(x), mean(p)]

the “clumping” that occurs will create a phase shift of the centroid motion that evolves with time

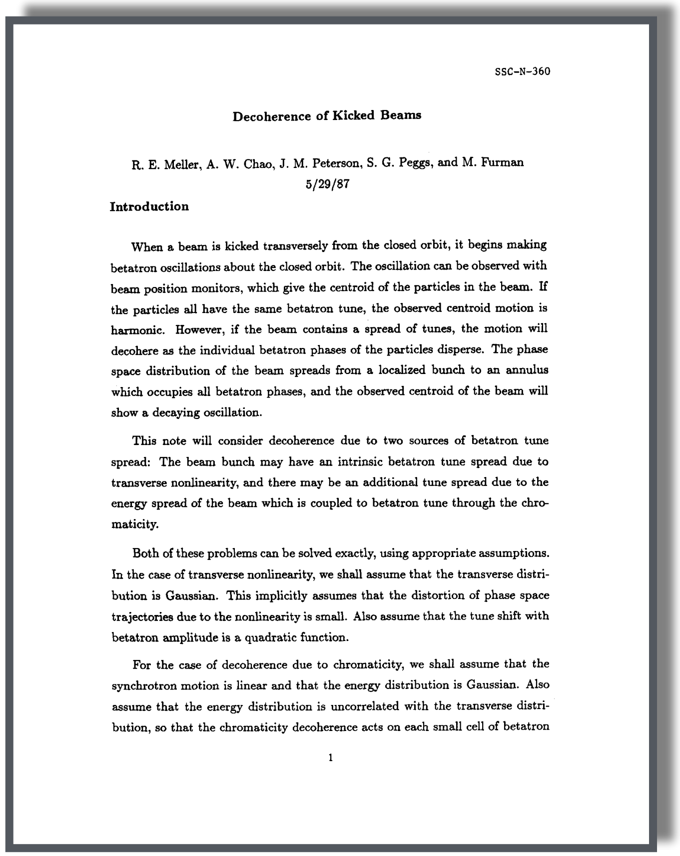
Have seen this in the past…



Tevatron Data

synchrotron oscillations — non-existent for us

$$\overline{x}(n) = a \cdot e^{-[2\sigma_s \xi \nu_s^{-1} \sin(\pi \nu_s n)]^2/2} \cdot \left(\frac{1}{1+(\nu_p n)^2} \; e^{-\frac{a^2}{2\sigma^2} \frac{(\nu_p n)^2}{1+(\nu_p n)^2}} \right) \cdot \cos[2\pi\nu_0 n + \Delta\overline{\phi}(n)]$$



SSC-N-360

Decoherence of Kicked Beams

R. E. Meller, A. W. Chao, J. M. Peterson, S. G. Peggs, and M. Furman
5/29/87

Introduction

When a beam is kicked transversely from the closed orbit, it begins making betatron oscillations about the closed orbit. The oscillation can be observed with beam position monitors, which give the centroid of the particles in the beam. If the particles all have the same betatron tune, the observed centroid motion is harmonic. However, if the beam contains a spread of tunes, the motion will decohere as the individual betatron phases of the particles disperse. The phase space distribution of the beam spreads from a localized bunch to an annulus which occupies all betatron phases, and the observed centroid of the beam will show a decaying oscillation.

This note will consider decoherence due to two sources of betatron tune spread: The beam bunch may have an intrinsic betatron tune spread due to transverse nonlinearity, and there may be an additional tune spread due to the energy spread of the beam which is coupled to betatron tune through the chromaticity.

Both of these problems can be solved exactly, using appropriate assumptions. In the case of transverse nonlinearity, we shall assume that the transverse distribution is Gaussian. This implicitly assumes that the distortion of phase space trajectories due to the nonlinearity is small. Also assume that the tune shift with betatron amplitude is a quadratic function.

For the case of decoherence due to chromaticity, we shall assume that the synchrotron motion is linear and that the energy distribution is Gaussian. Also assume that the energy distribution is uncorrelated with the transverse distribution, so that the chromaticity decoherence acts on each small cell of betatron

1

Meller, et al., SSC-N-360 (1987)

Gaussian beam, is $b = (a/\sigma)^2/2$, and ν_p is the nonlinear detuning parameter $\nu_p = 4\pi\mu$, where the particle tune varies with betatron amplitude according to $\nu = \nu_0 - \mu(a/\sigma)^2$.

This form (though including synchrotron motion) was used in 2005 to analyze Tevatron BPM data of a kicked beam

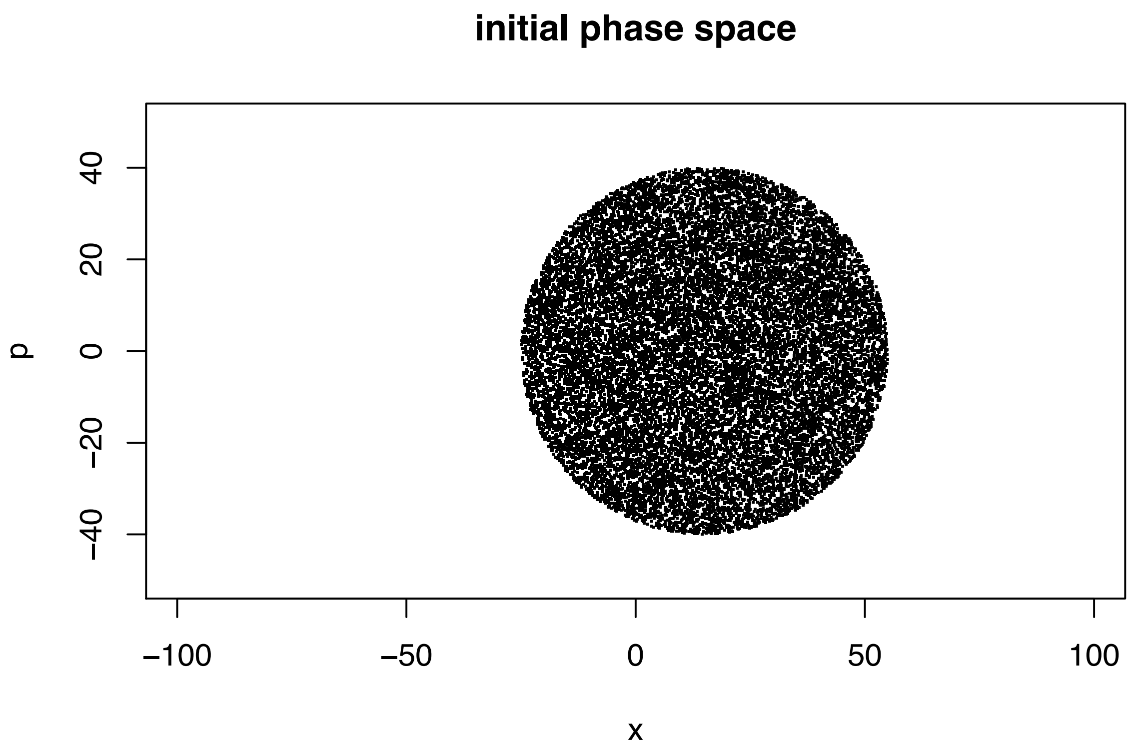
So, in summary, the observed amplitude of the average particle motion will decrease according to

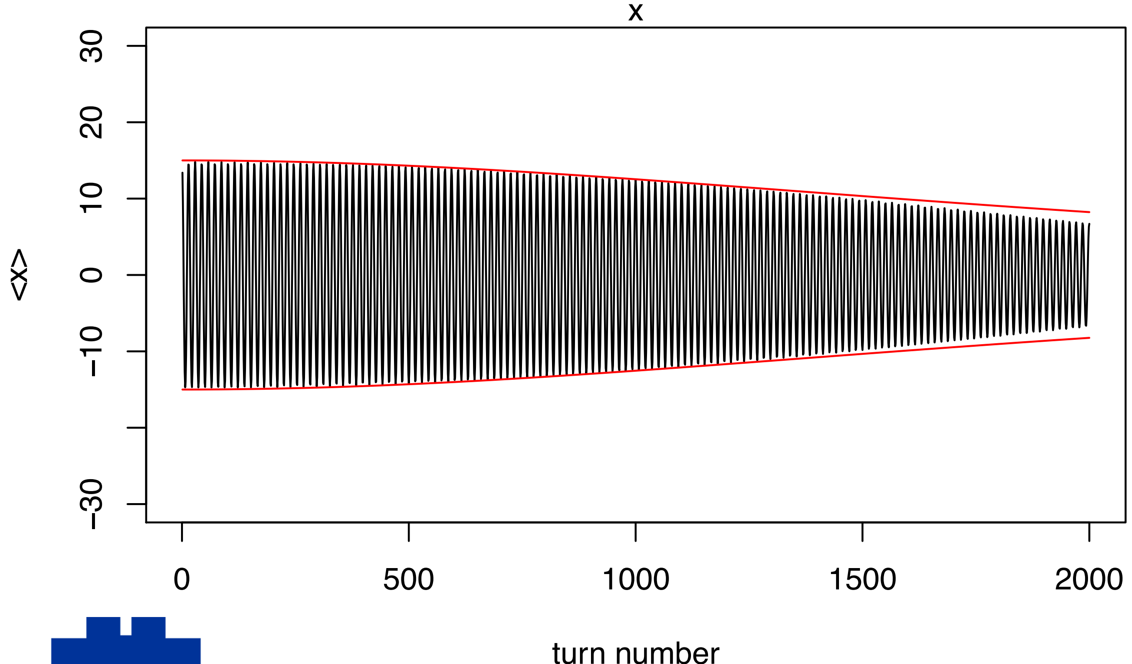
$$|\langle \Delta x \rangle|(n) \approx a \cdot \frac{1}{1+(\nu_p n)^2} e^{\left[-\frac{1}{2}\left(\frac{a}{\sigma_x}\right)^2 \frac{(\nu_p n)^2}{1+(\nu_p n)^2} \right]}$$

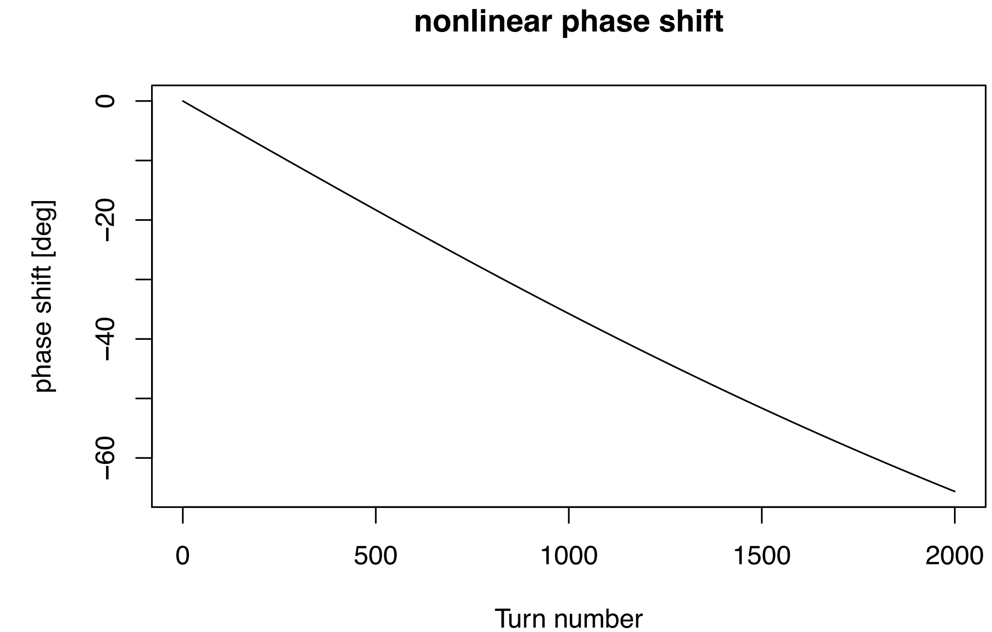
and a phase shift that develops according to

$$\Delta\phi(n) = -\frac{1}{2}\left(\frac{a}{\sigma_x}\right)^2 \frac{\nu_p n}{1+(\nu_p n)^2} - 2\tan^{-1}(\nu_p n).$$

Below, we choose $\mu = 0.2/\text{m}^2$
and go for ~2000 turns



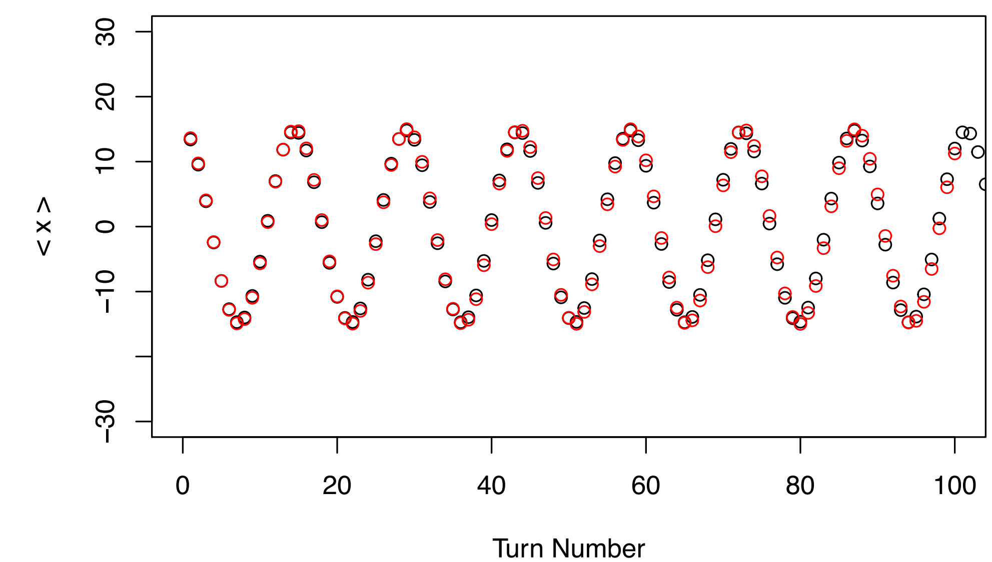




In our example above we would estimate the phase shift after 2000 turns to be $\Delta\phi(n) = -\frac{1}{2}\left(\frac{a}{\sigma_x}\right)^2 \frac{\nu_p n}{1+(\nu_p n)^2} - 2\tan^{-1}(\nu_p n) \approx$ -88.22 °.

Let's see if it works…

```
plot(xbar,xlim=c(0,100),ylim=c(-1,1)*30,typ="p",
   xlab="Turn Number",ylab="< x >")                # simulation data
points(del*cos(2*pi*nu0*c(1:100)), col="red")      # pure sine wave at base betatron tune
```



black = simulation
red = pure sine wave

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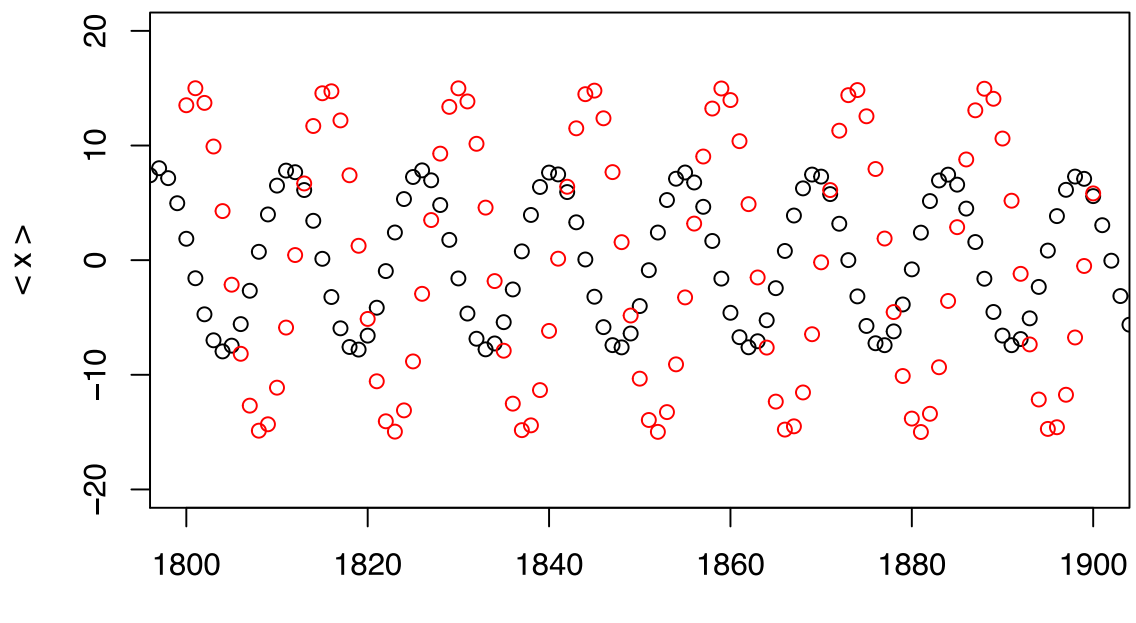
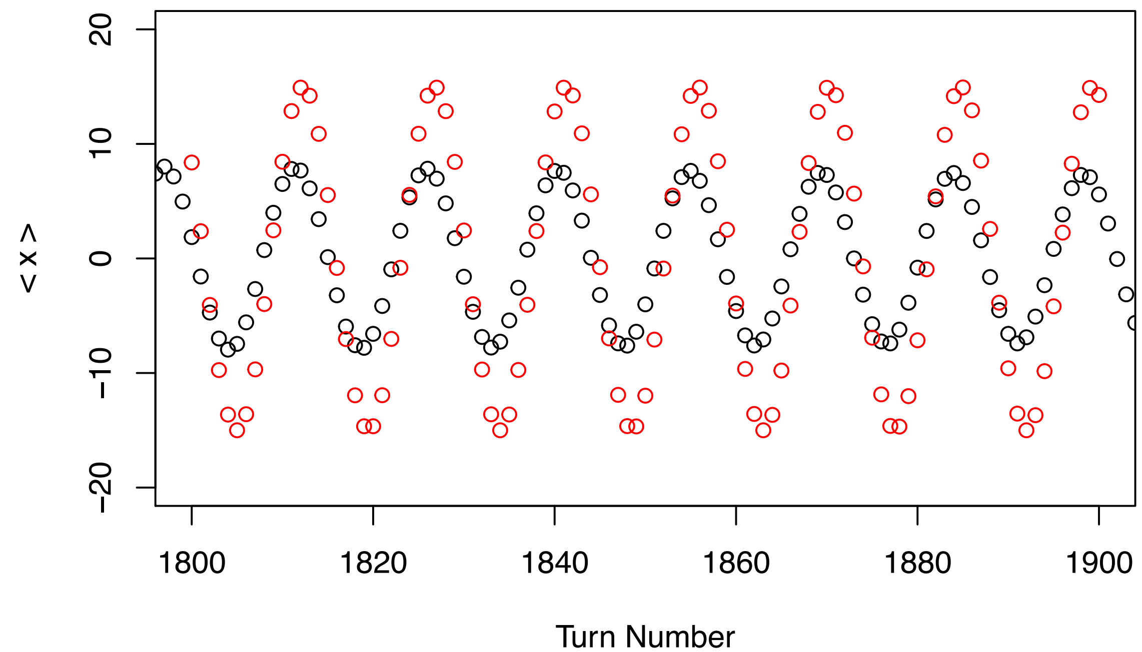
In the above plot we can see the data slipping in phase with respect to a pure sine wave oscillating at the base betatron tune (red points). Now, lets compare at a much later time; first, without a phase shift correction. . .

```
plot(xbar,xlim=c(1800,1900),ylim=c(-1,1)*20,typ="p",
   xlab="Turn Number",ylab="< x >")
points(c(1800:1900),del*cos(2*pi*nu0*c(1800:1900)), col="red")
```



We see the phase shift, as well as the amplitude having decreased as well. Next, we add the phase shift into the red points, using our equation in the text above. Not a perfect match, but pretty close. . .

```
plot(xbar,xlim=c(1800,1900),ylim=c(-1,1)*20,typ="p",
   xlab="Turn Number",ylab="< x >")
points(c(1800:1900), del*cos(2*pi*nu0*c(1800:1900) -
      1/2*(del/asc)^2*nup*c(1800:1900)/(1+(nup*c(1800:1900))^2) -
      2*atan(nup*c(1800:1900))), col="red")
```



black = simulation
red = pure sine wave

here, add phase shift:

$$\Delta\phi(n) = -\frac{1}{2}\left(\frac{a}{\sigma_x}\right)^2 \frac{\nu_p n}{1+(\nu_p n)^2} - 2\tan^{-1}(\nu_p n).$$

Detector Acceptance

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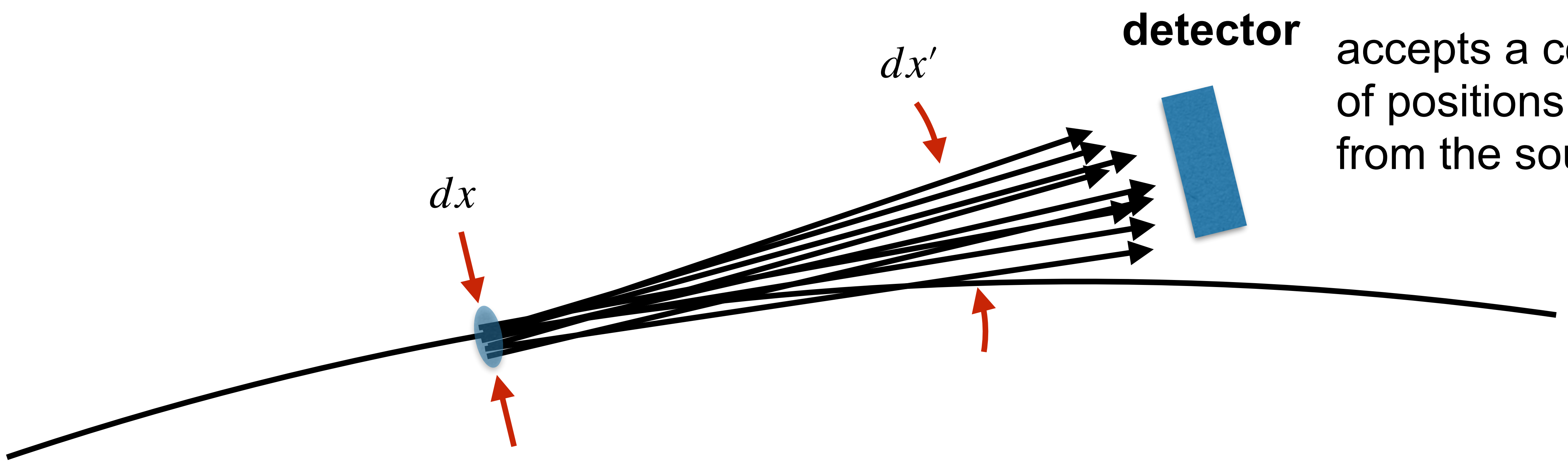
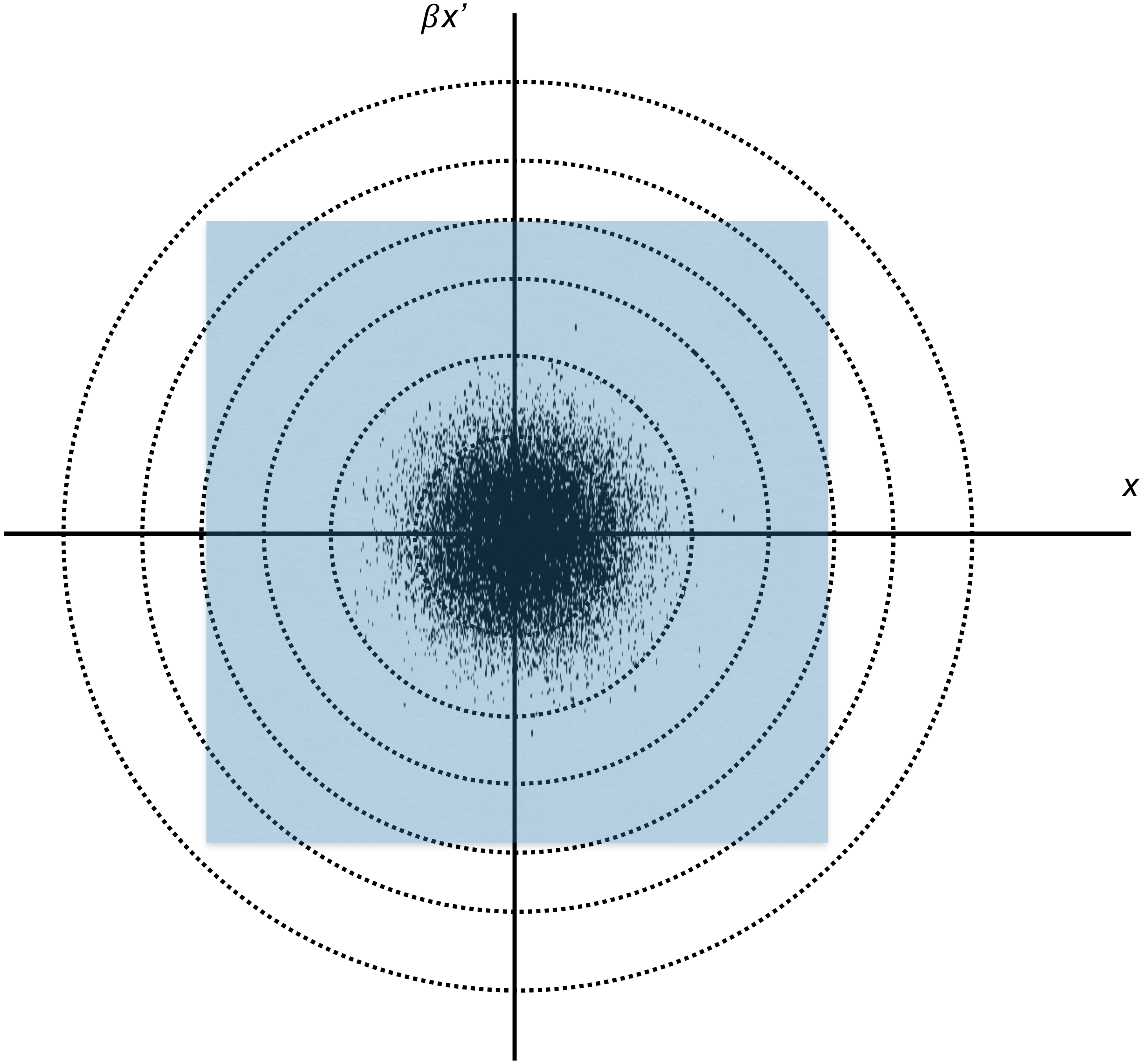


accepts a certain range of positions and slopes from the source (beam)

Detector Acceptance

- Why are the betatron tunes important in the actual g-2 measurement?

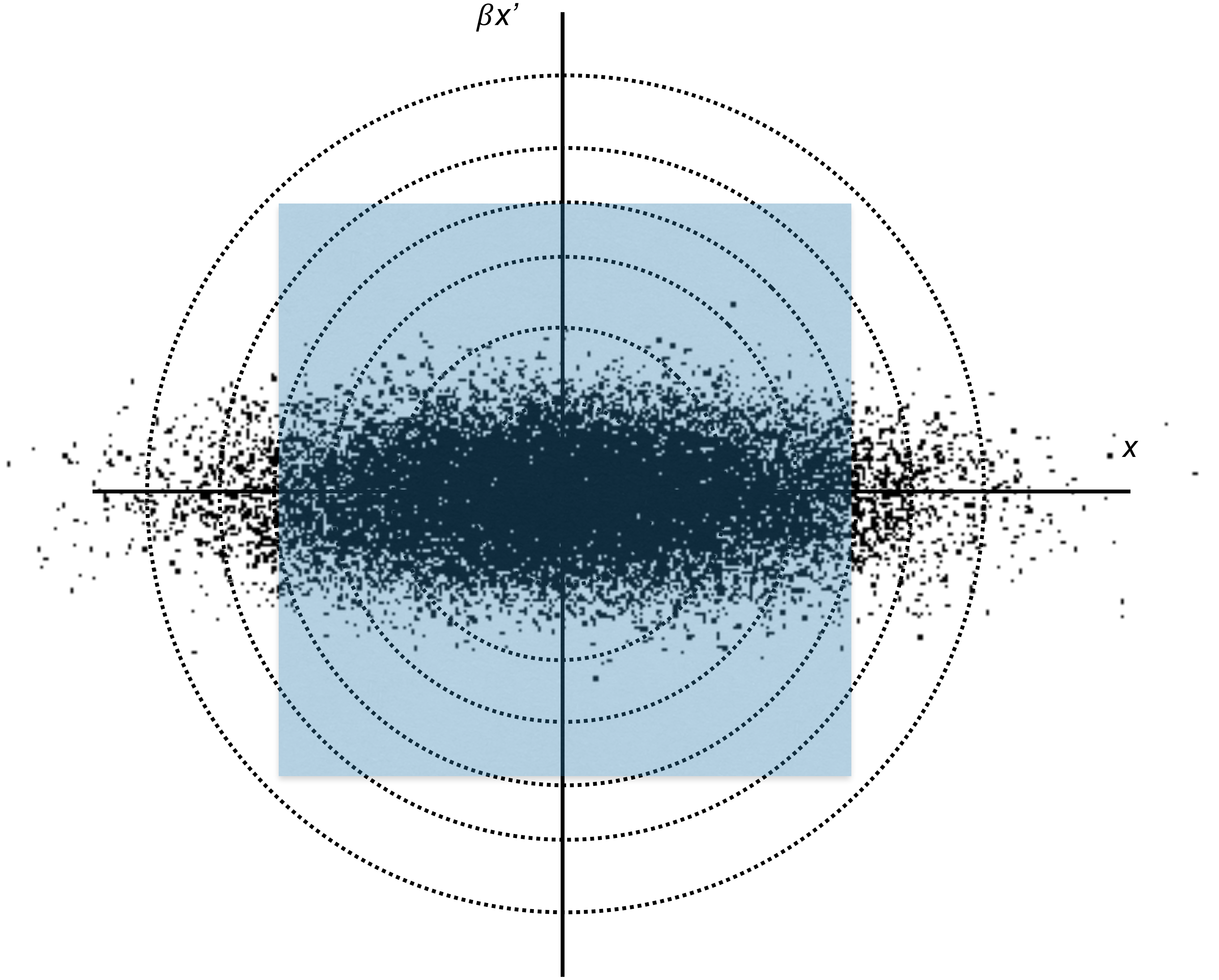
- ideal



Detector Acceptance

- Why are the betatron tunes important in the actual g-2 measurement?

- more realistic



Detector Acceptance

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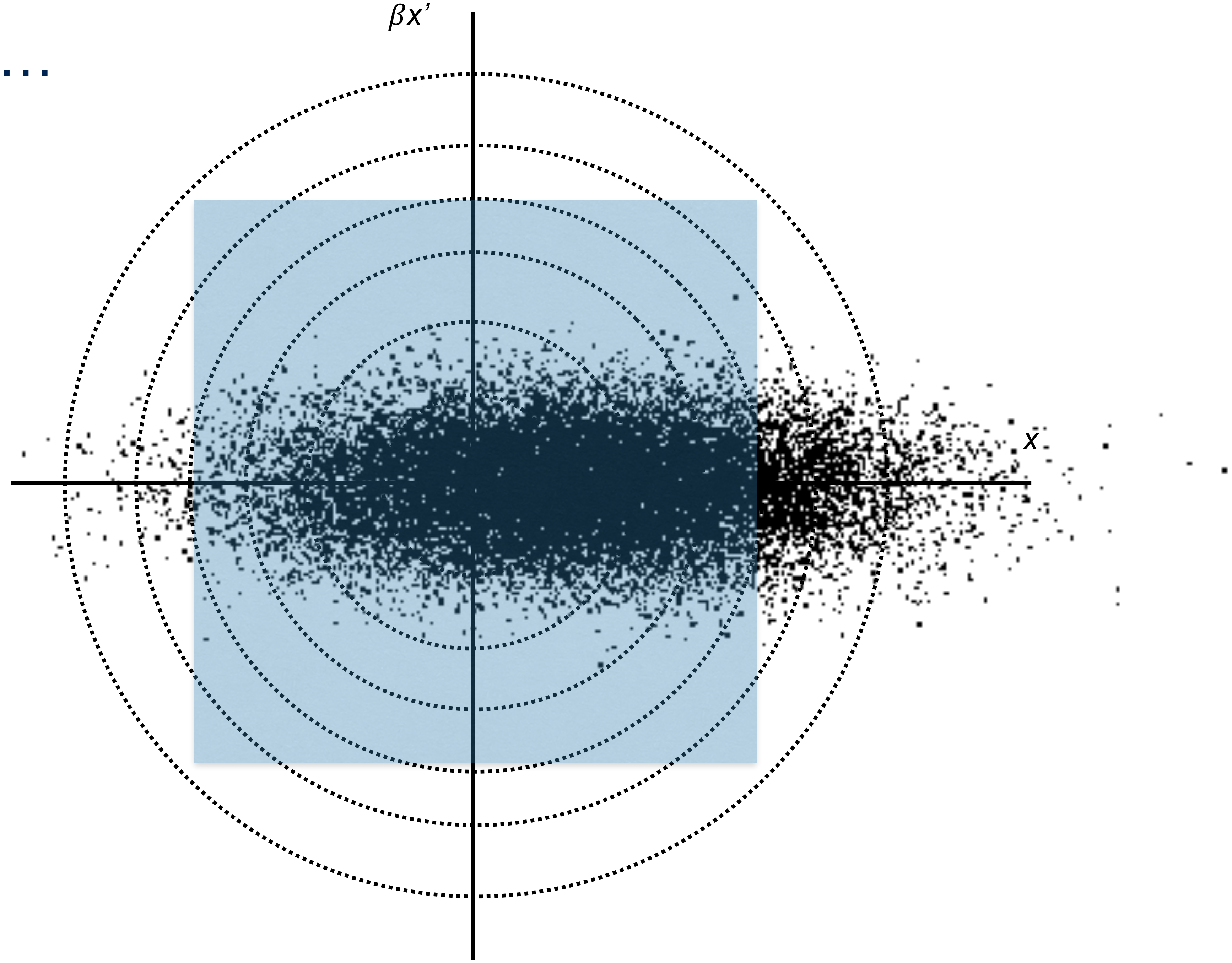
- Why are the betatron tunes important in the actual g-2 measurement?

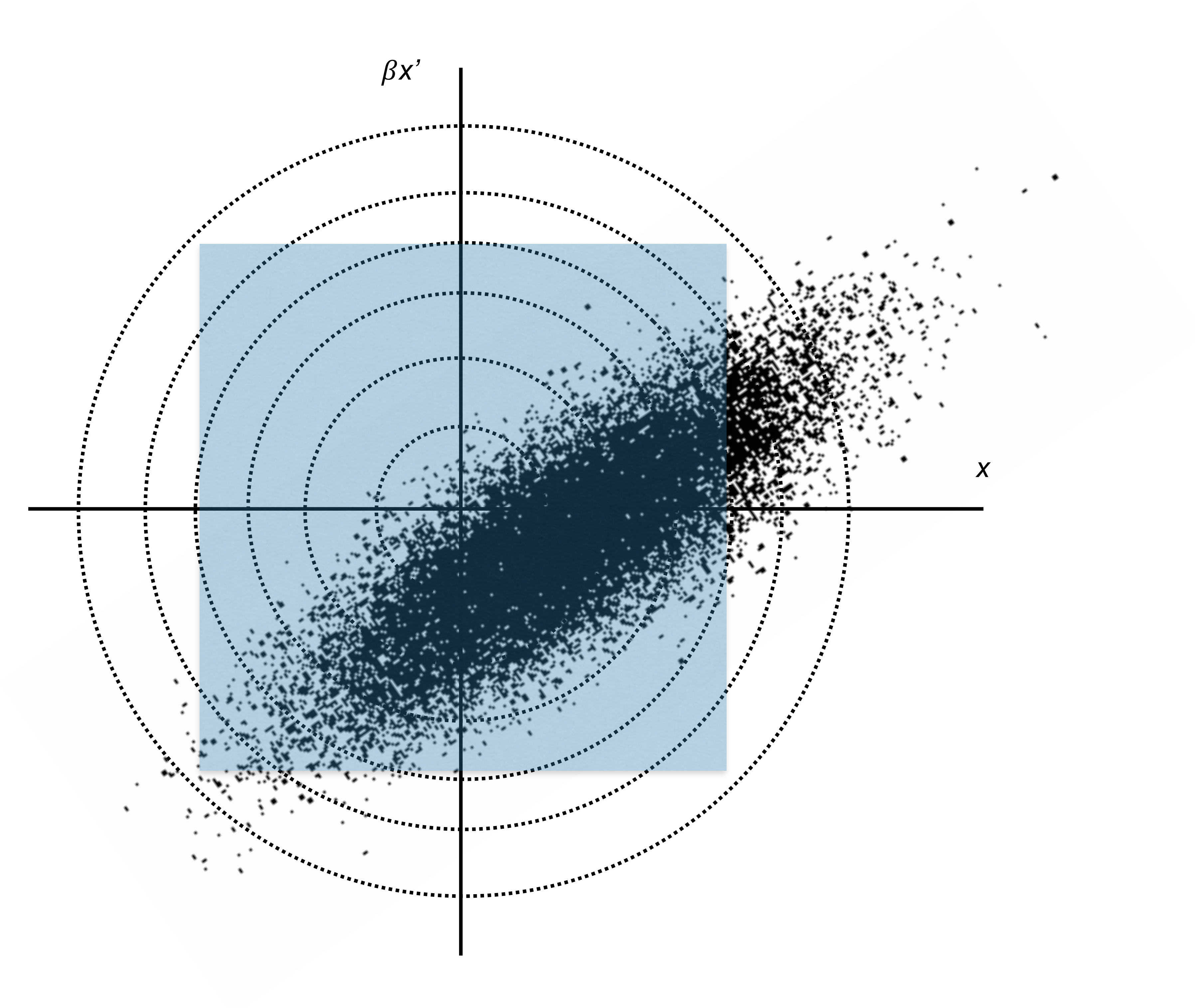
- add more realism…



Detector Acceptance

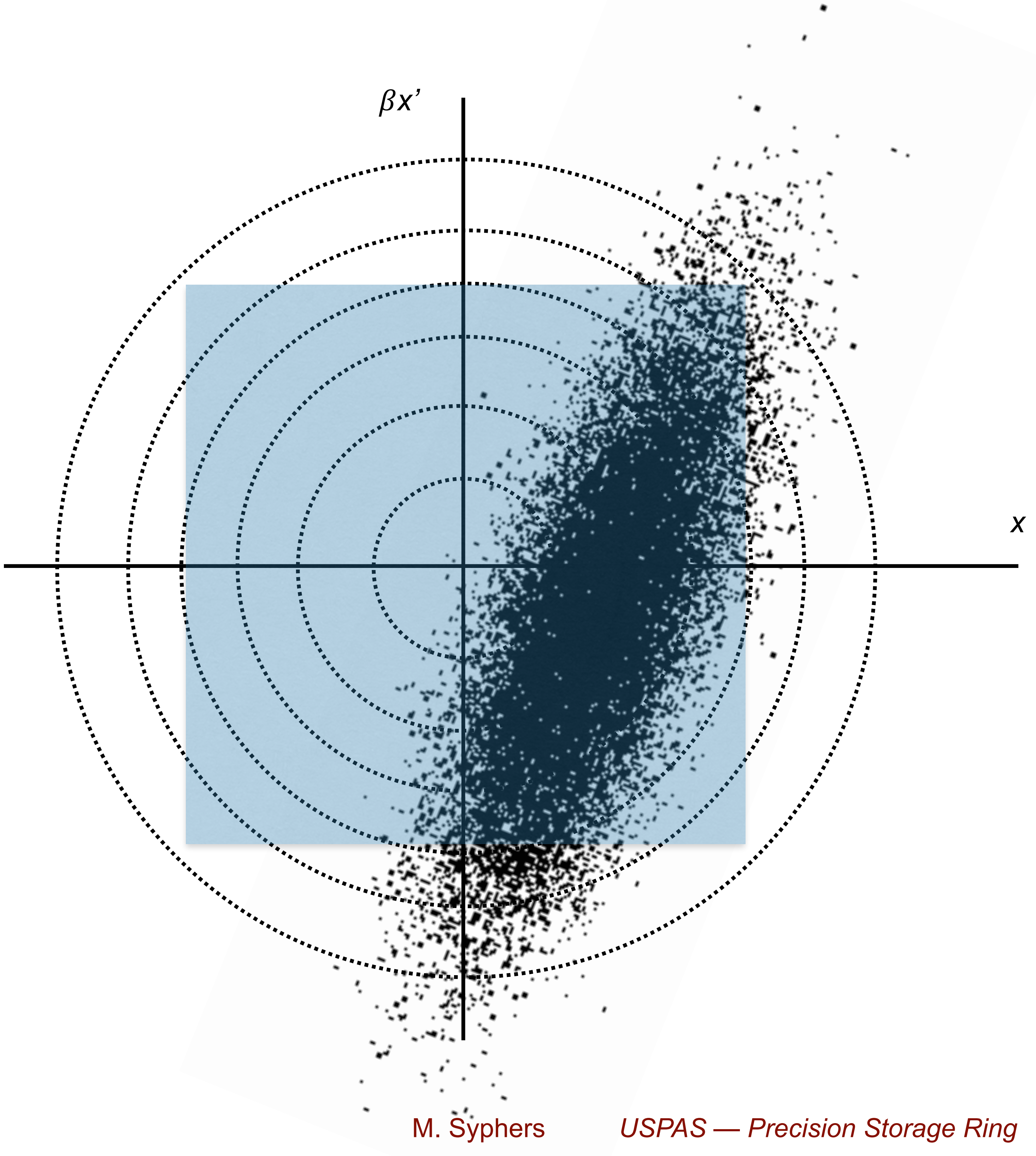
- Why are the betatron tunes important in the actual g-2 measurement?

-



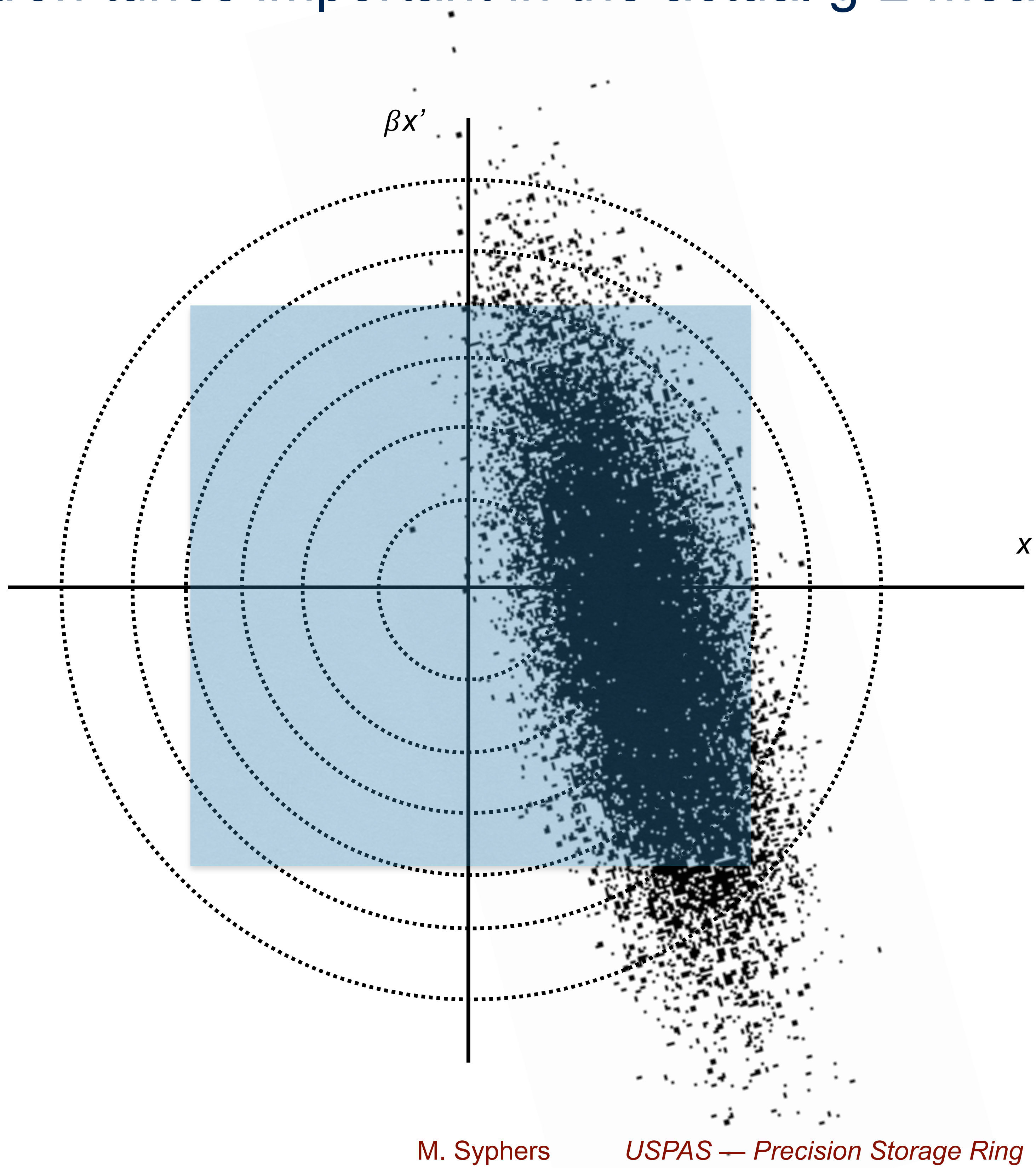
Detector Acceptance

- Why are the betatron tunes important in the actual g-2 measurement?
-



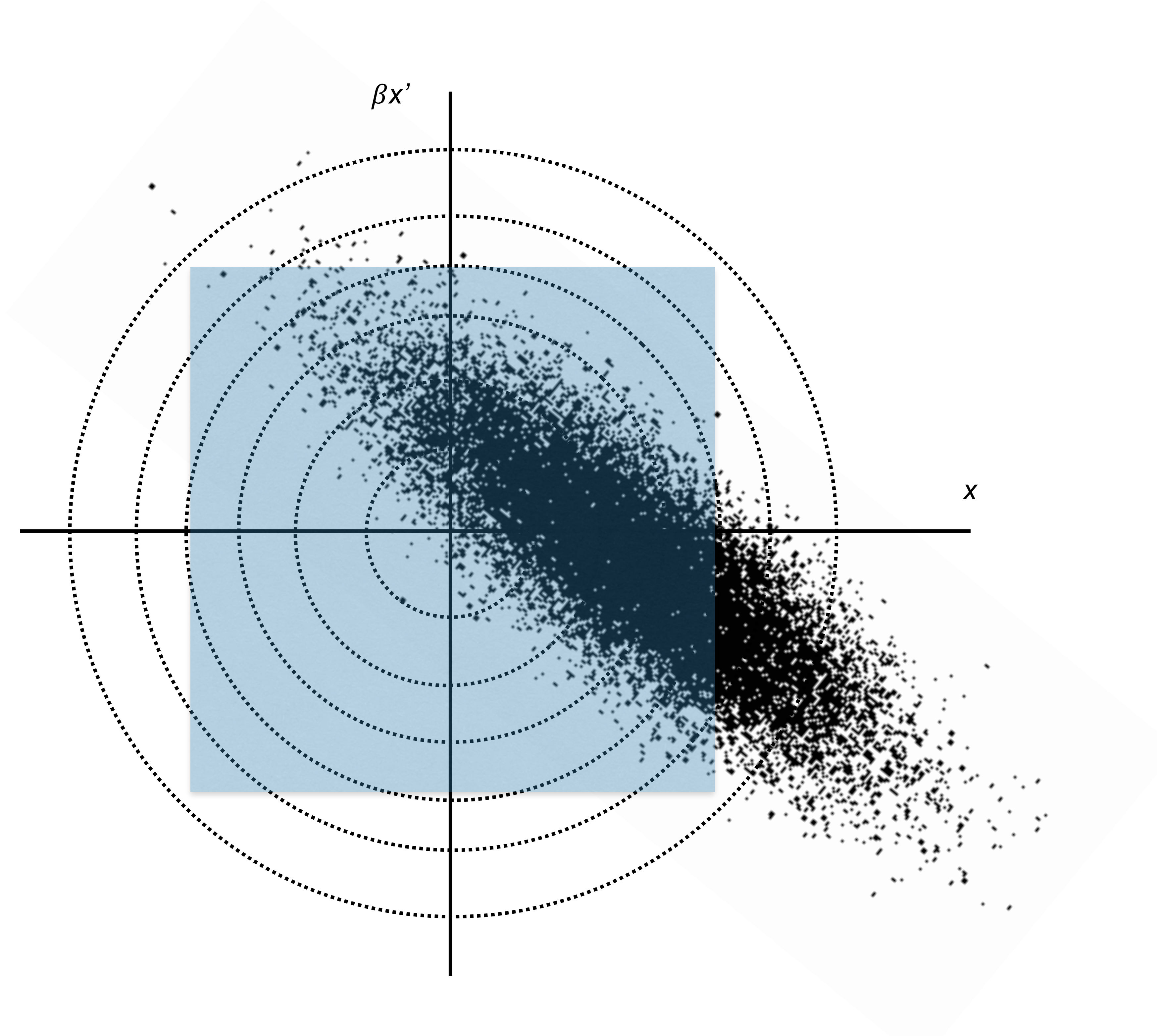
Detector Acceptance

- Why are the betatron tunes important in the actual g-2 measurement?
-



Detector Acceptance

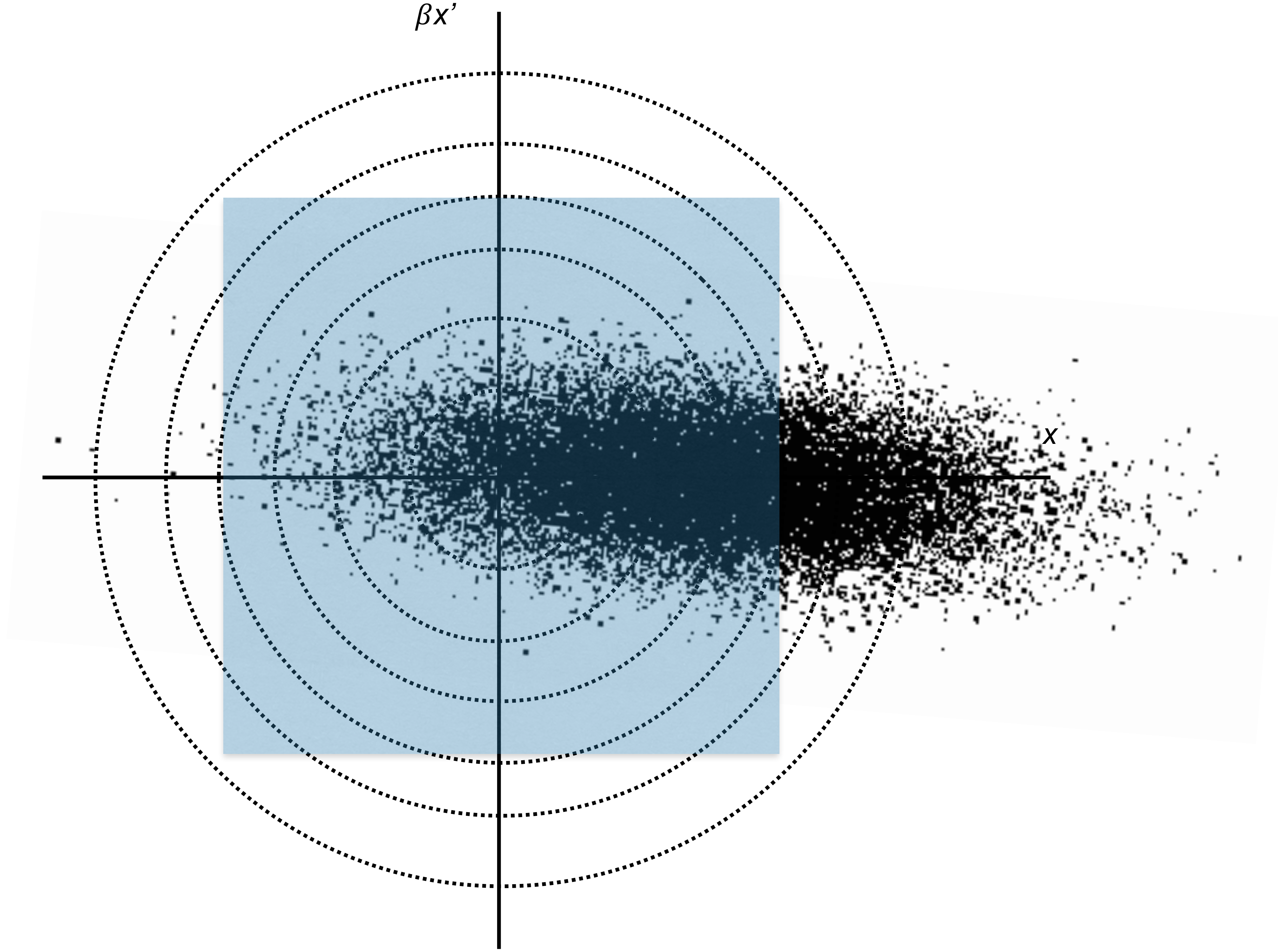
- Why are the betatron tunes important in the actual g-2 measurement?
-



Detector Acceptance

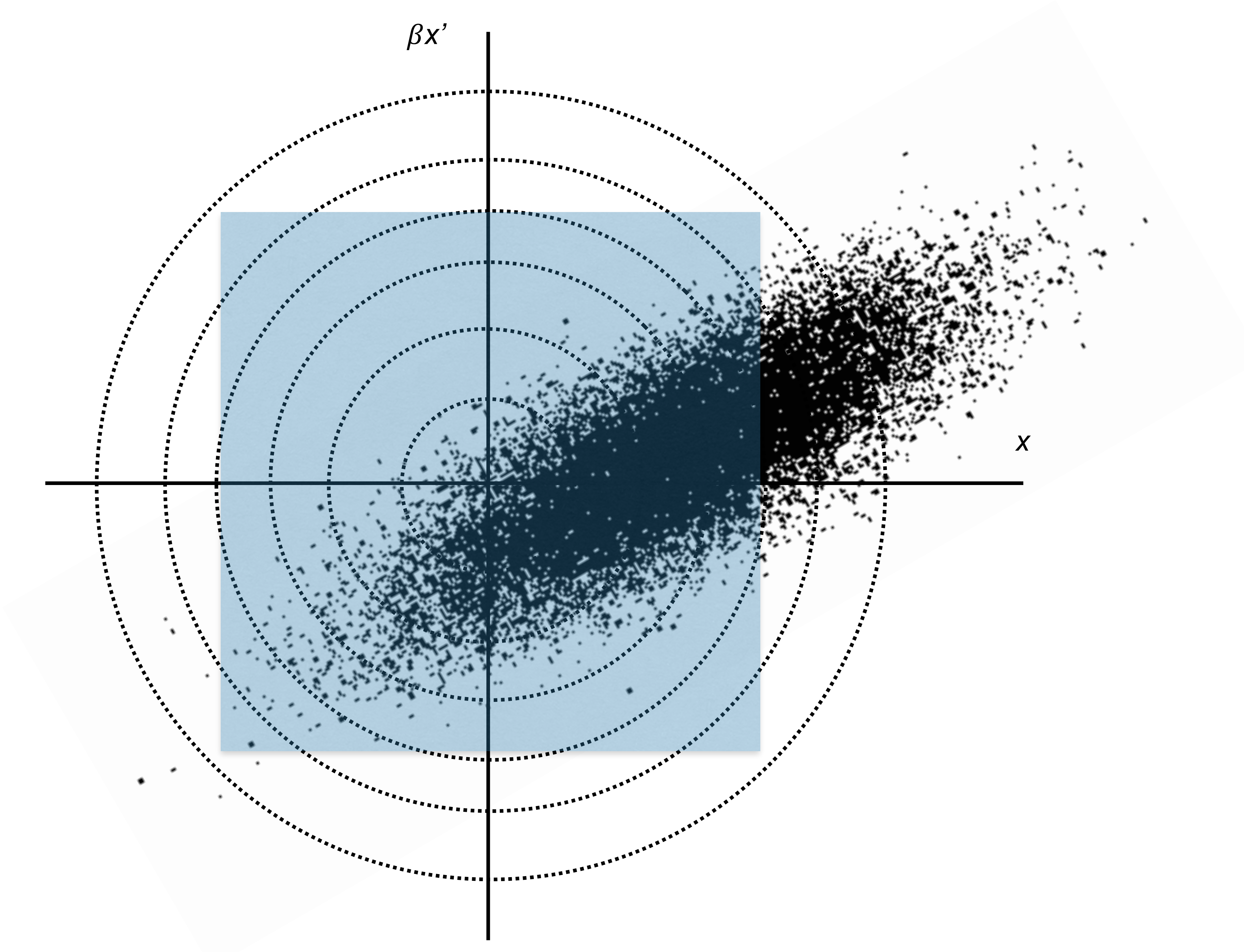
- Why are the betatron tunes important in the actual g-2 measurement?

-



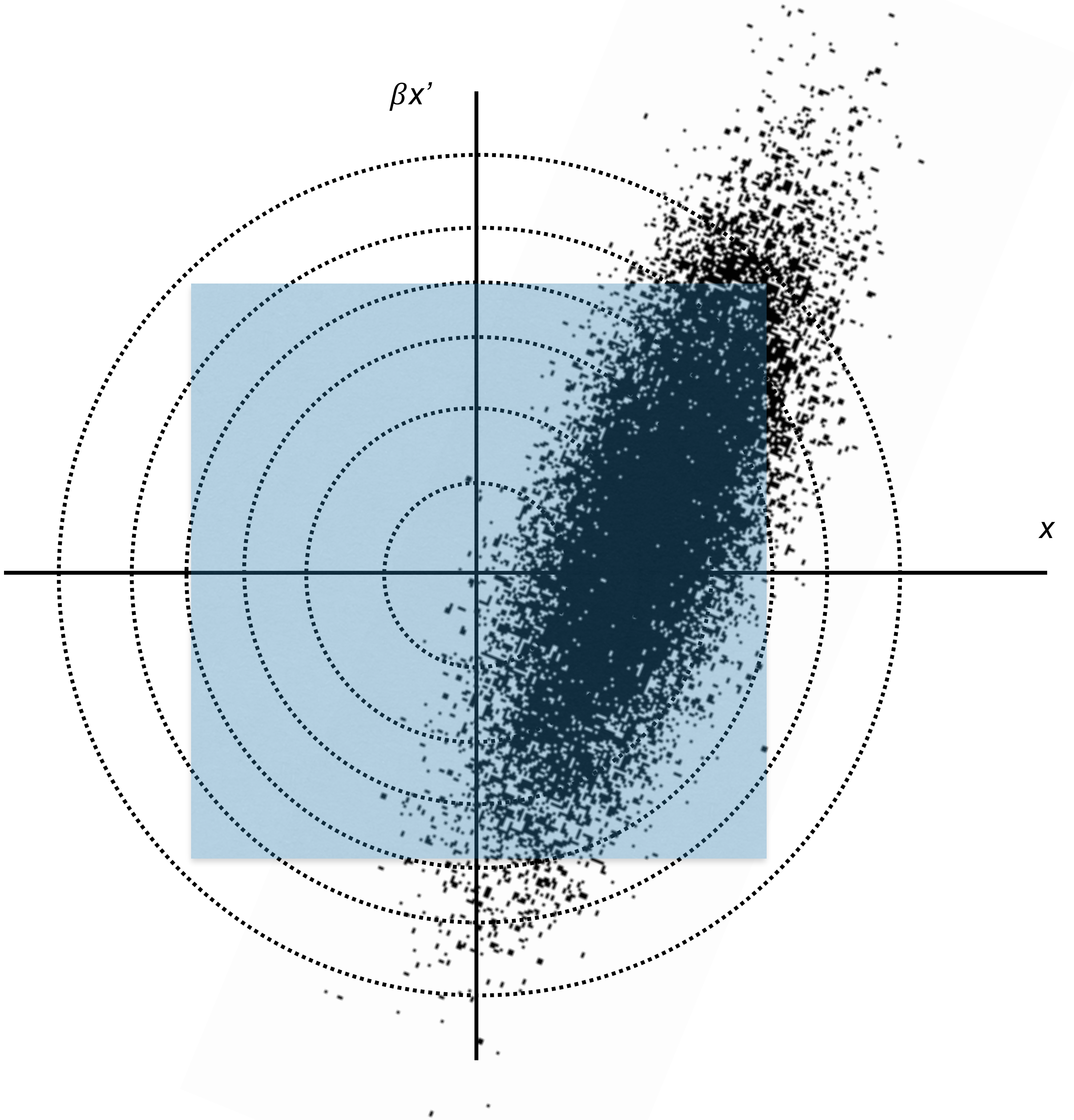
Detector Acceptance

- Why are the betatron tunes important in the actual g-2 measurement?
-



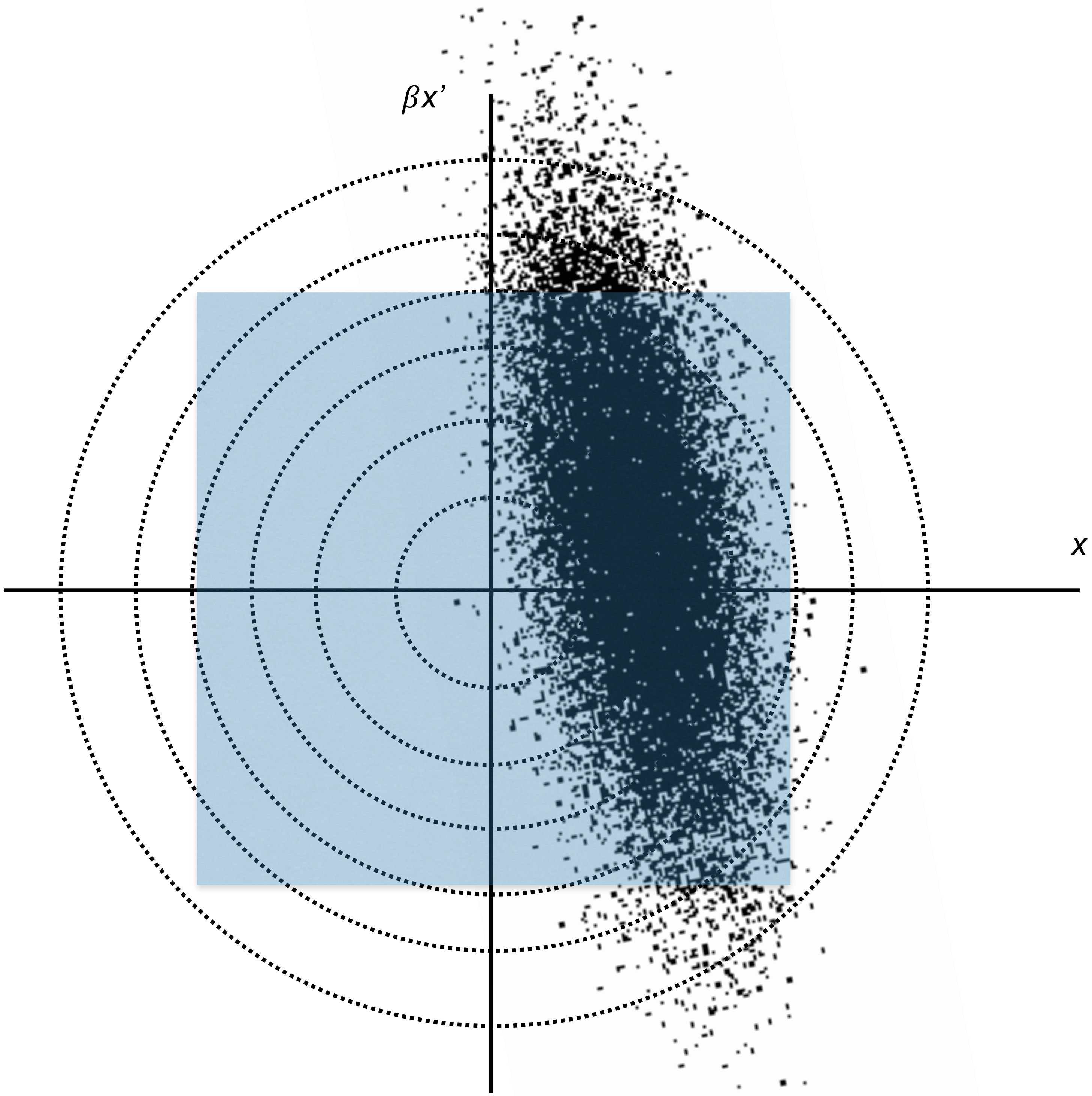
Detector Acceptance

- Why are the betatron tunes important in the actual g-2 measurement?
-



Detector Acceptance

- Why are the betatron tunes important in the actual g-2 measurement?
-



Detector Acceptance

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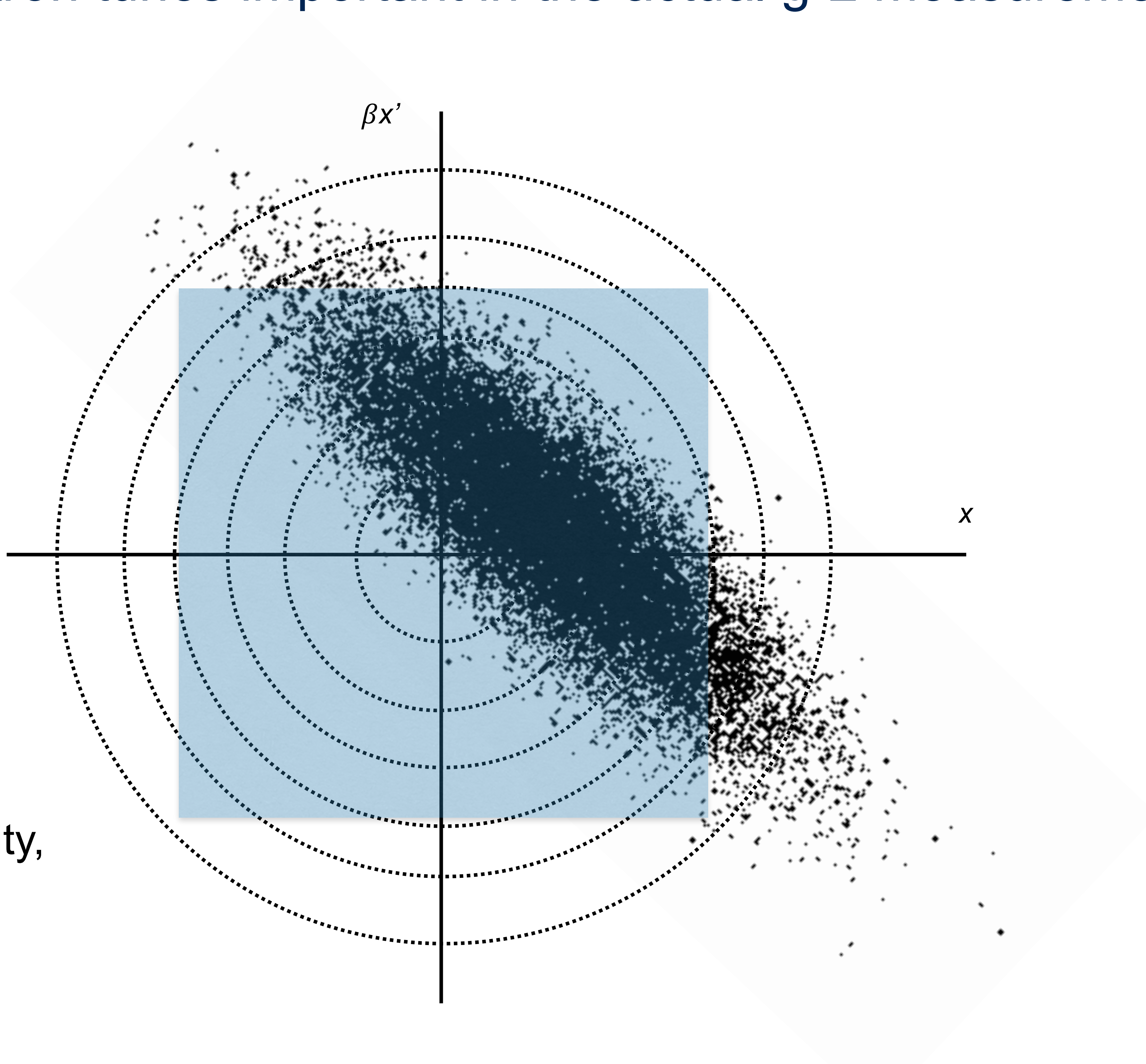
- Why are the betatron tunes important in the actual g-2 measurement?
-



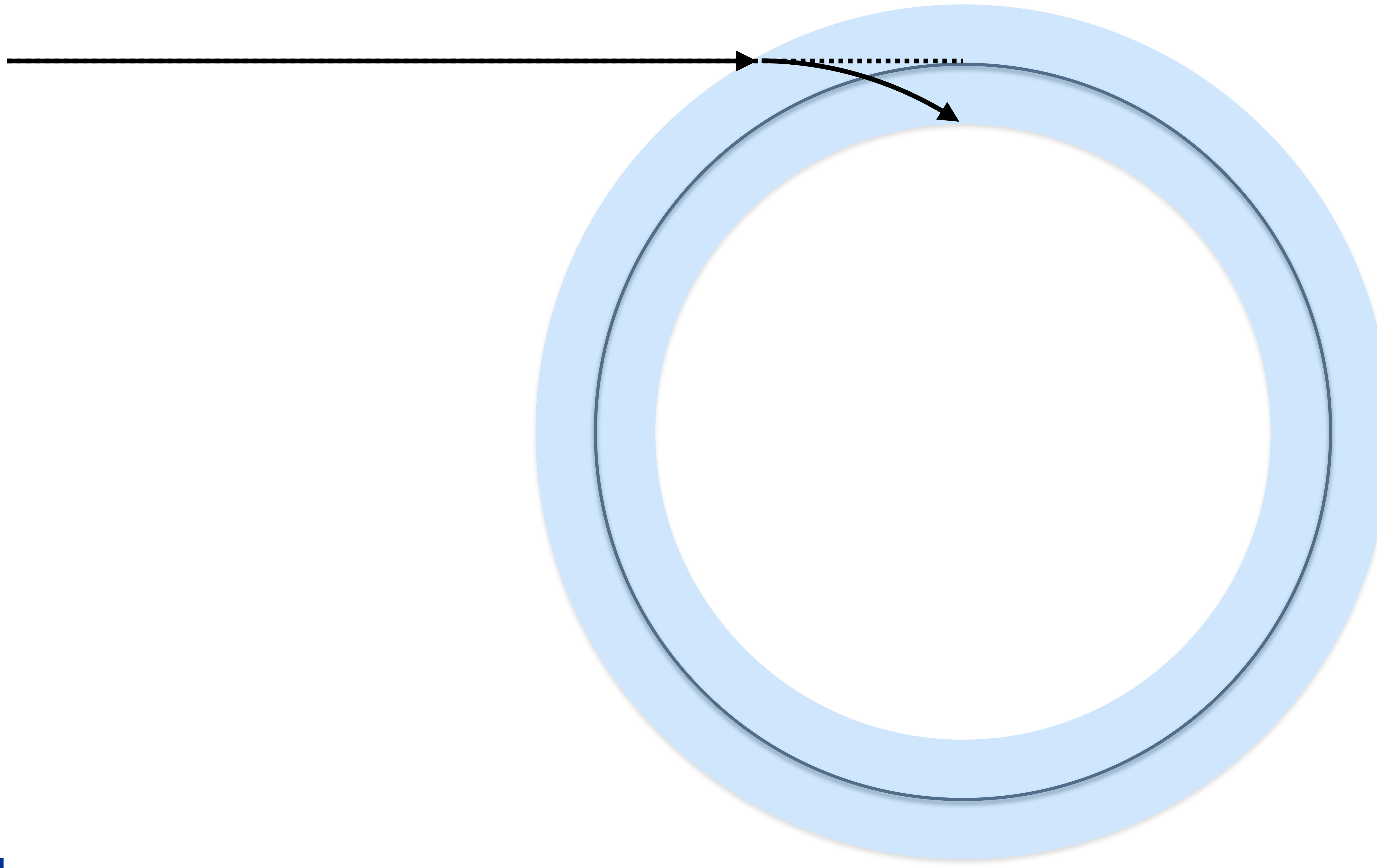
Actual phase space dynamics more complicated, as have dispersion, chromaticity, nonlinearities, etc.

Summary

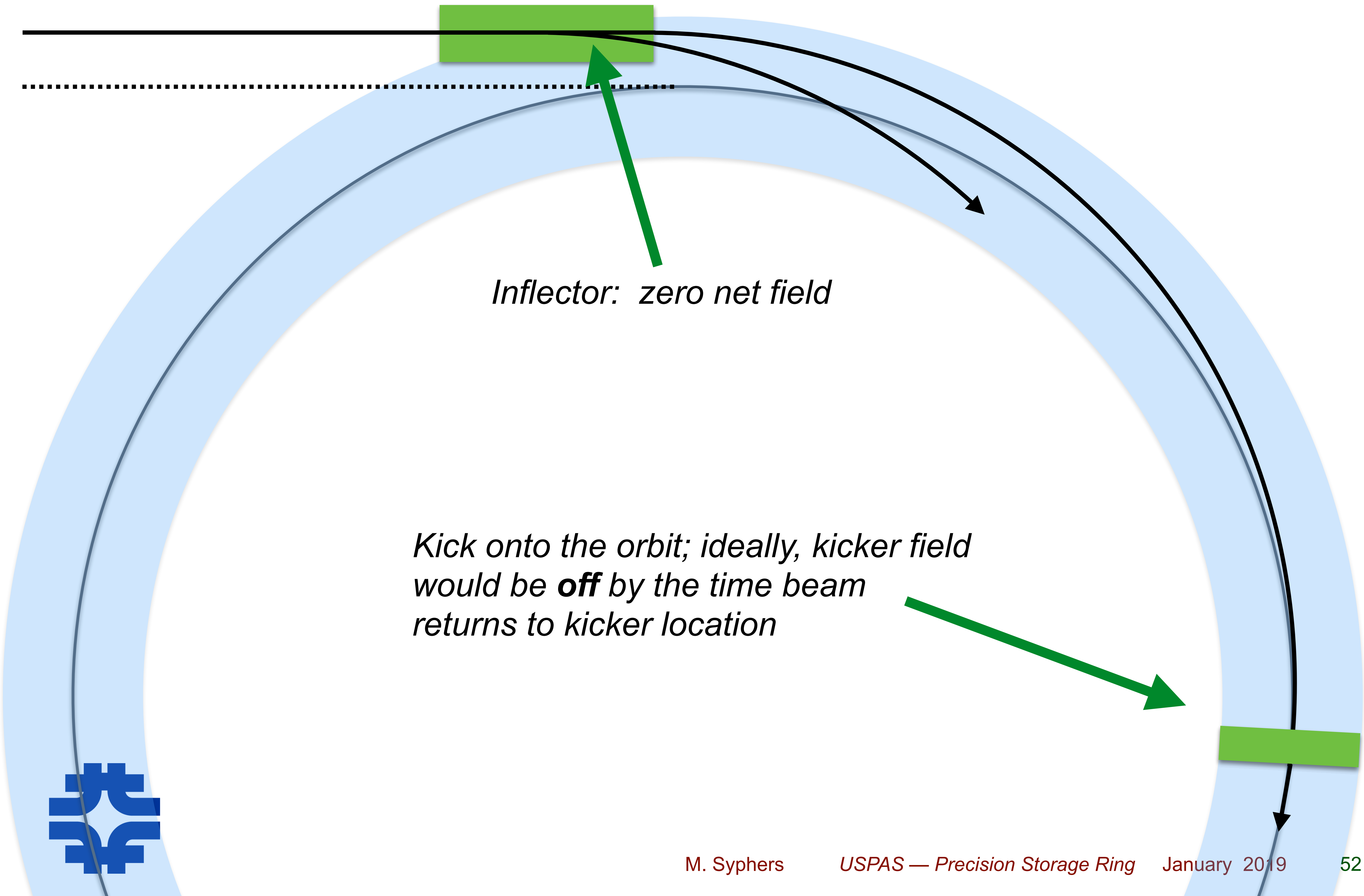
- As what we measure in the experiment is a rate of particles reaching and being detected by the detector system, this rate will not only depend upon the precession that is going on — which is what we want to detect — but will also depend upon the general beam motion
  - closed orbit distortion
  - coherent betatron motion about that closed orbit
    - centroid motion
    - “quadrupole” motion
  - time-varying fields (due to errors in the system — quad resistor problem, etc)
- When averaging over an ensemble of positron hits taken during a single ring fill, must understand that the “coherent” motion will decohere due to
  - chromaticity and the momentum spread of the beam
  - nonlinear fields that affect the betatron tunes as a function of the amplitude of the particle’s motion
  - can result in a phase or frequency slippage, but is an artifact of the
- Rates also affected by particles that did NOT decay, but rather found their way to the detector by other dynamic means

Back-Up

Why We Need a Kicker

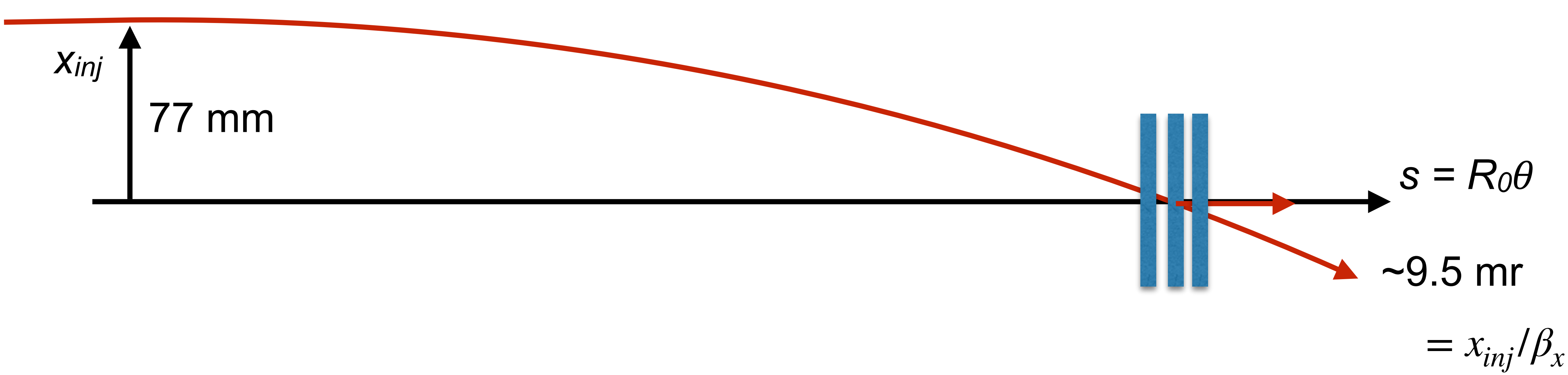


Injection Trajectory and the Kicker



Injection of Ideal Muon with Ideal Kicker

- Proper kicker location — should be centered at the point where the incoming trajectory crosses the desired circulating orbit
- The kicker system should deliver a total kick angle equal to this "crossing angle"



- Kick angle required:

$$\theta_{kick} \approx \frac{eB \cdot L_{kick}}{p} \qquad B \approx \theta_{kick}(p/e)/L_{kick} \approx 190 \text{ gauss}$$

Injection of Ideal Muon with Non-Ideal Kicker

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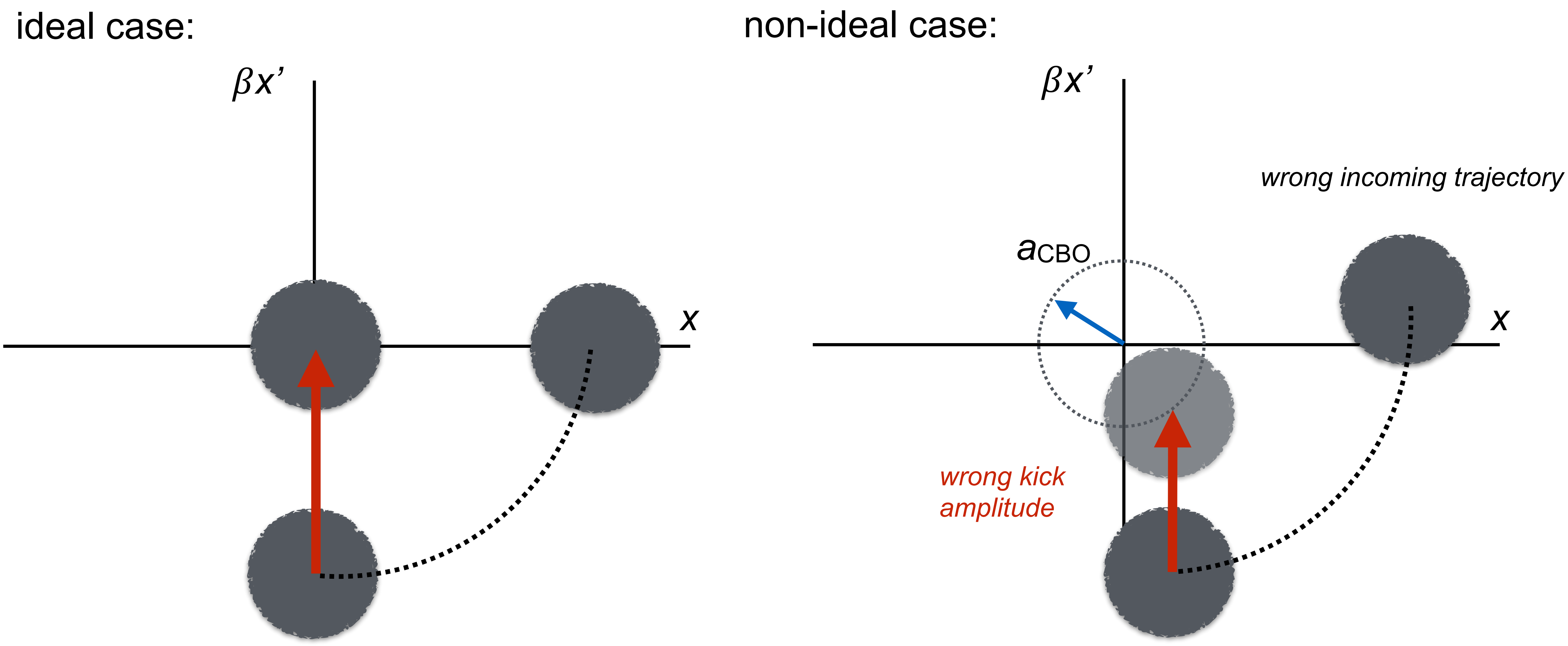
- if non-ideal kicker amplitude, and/or non-ideal incoming trajectory

Phase Space:



CBO = “coherent betatron oscillation”

Injection of a large mismatched, large momentum spread, funny-time-distribution beam with a non-ideal ringing kicker

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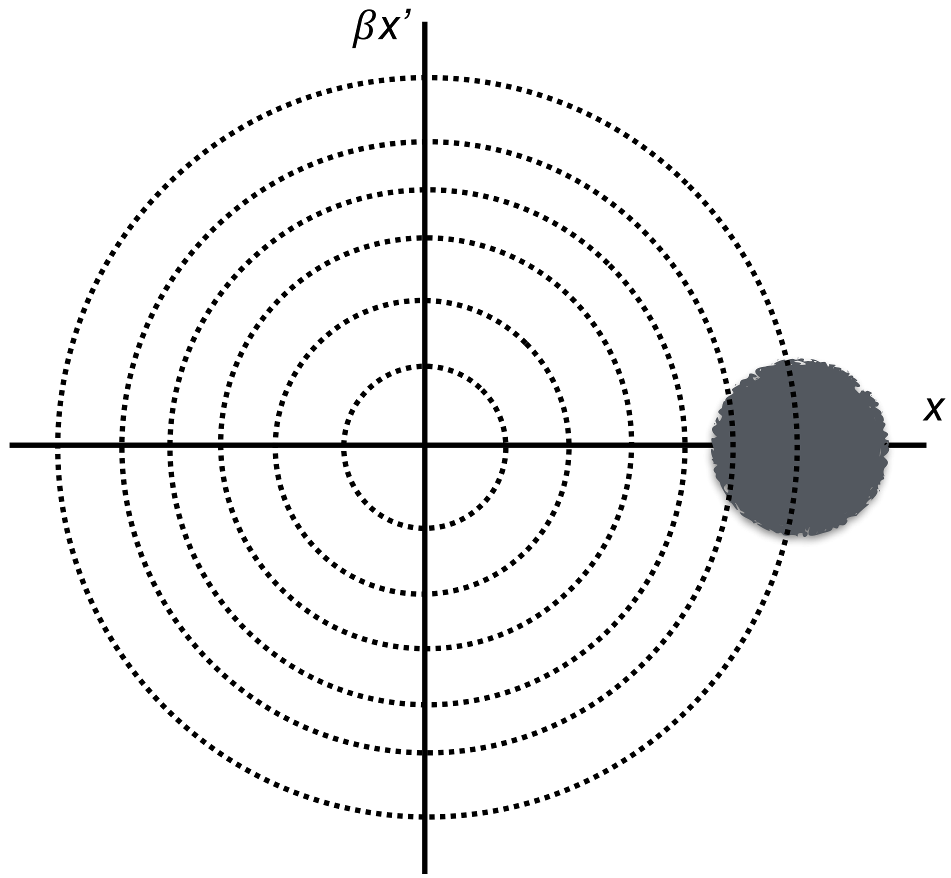
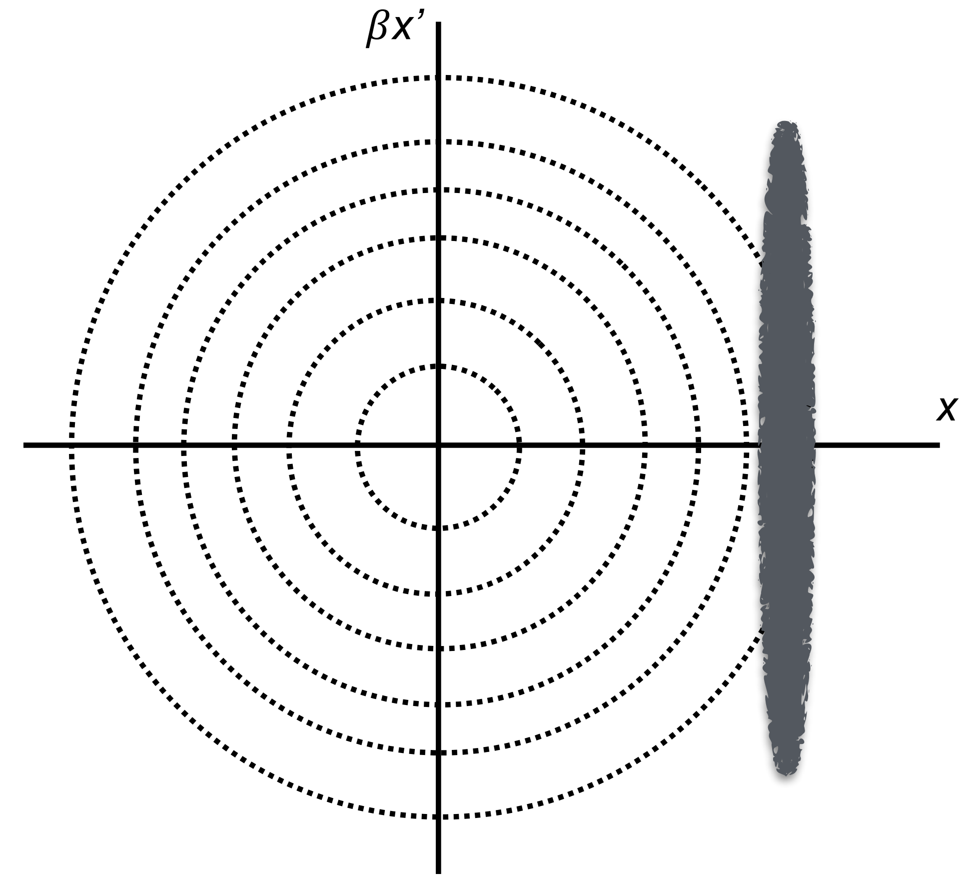
- In reality, we have
  - a beam that does not emerge "round" in our "round" phase space coordinates, due to the fact that we squeeze the beam through the inflector horizontal aperture
  - material at the ends of the inflector that the particles will scatter through, thus affecting their trajectories (and hence phase space coordinates); and their momentum as well. This also occurs on the first turn, as the muons pass through the plates of the first quadrupole (Q1)
  - a *non-uniform* main kicker pulse that does not give all particles the same kick
  - a pulse that "rings" such that particles receive multiple kicks over several revolutions

Incoming Phase Space

- The periodic β of the ring is about 8 m; however, to squeeze through the inflector aperture, the β of the beam is only about 1-2 m:



Here, beam has initial trajectory offset, but it is matched to the ring optics (if it were *centered*, the distribution would be stationary with time)



Here, beam is NOT matched to the ring optics; has same phase space **area**, but is *squeezed* through the inflector aperture: smaller horizontal size, but larger angles

A 1 CBO, and a 2 CBO, and a …

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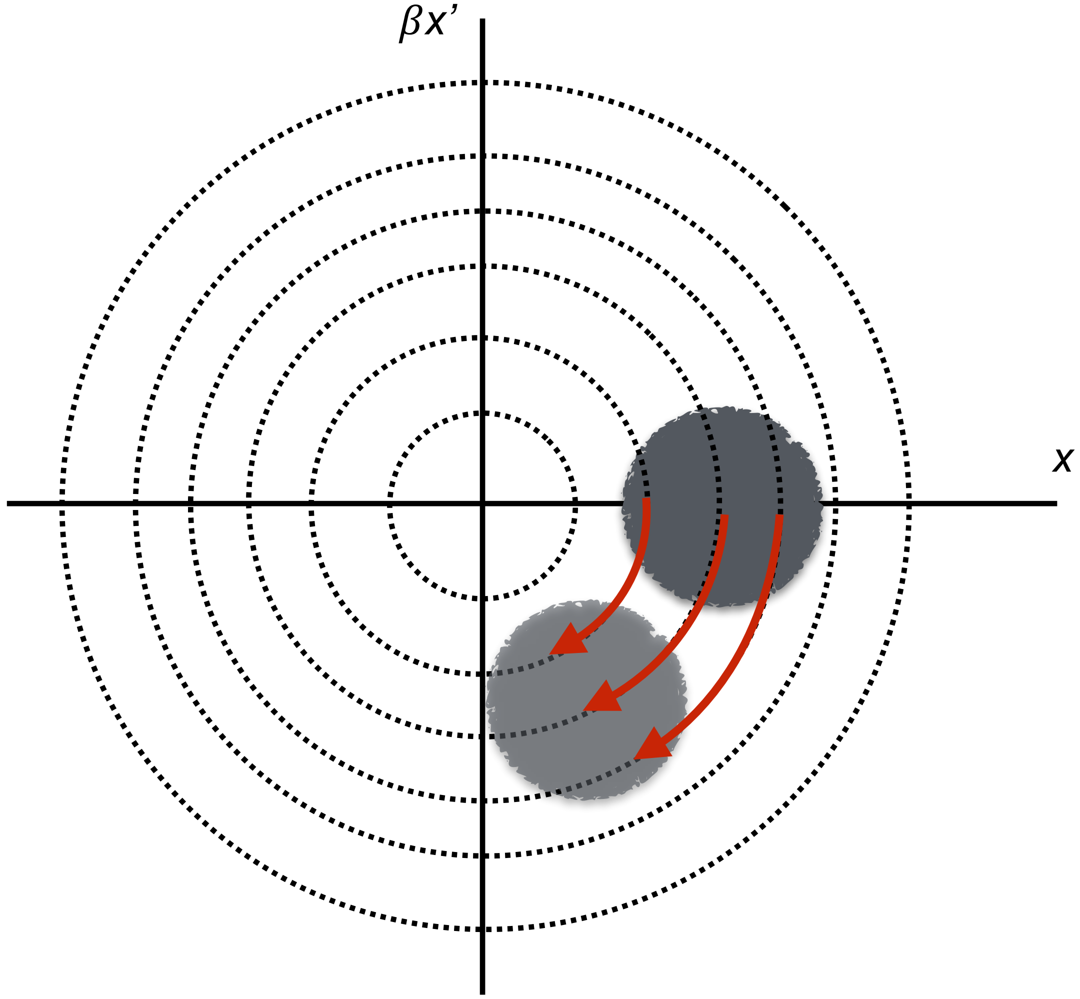
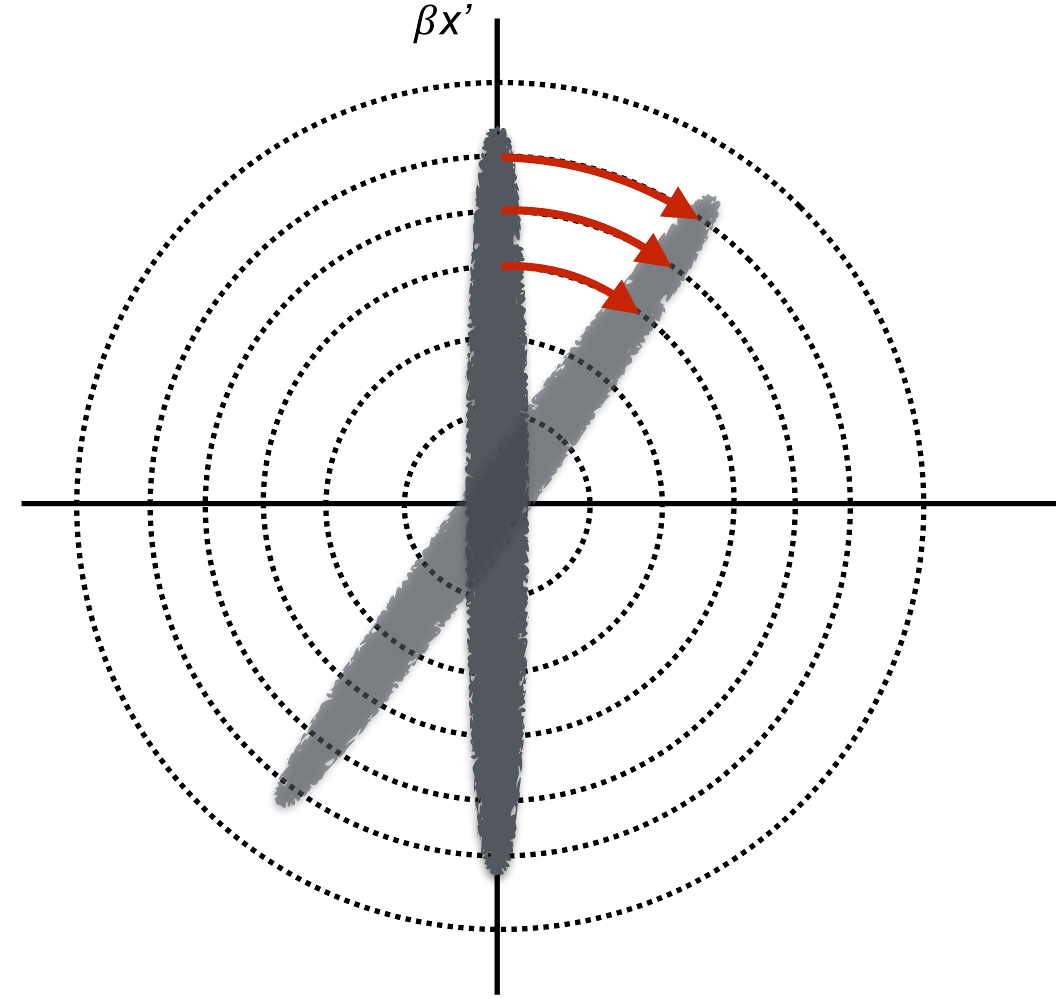


A trajectory offset will create a beam *centroid* oscillation at the betatron frequency (betatron tune, ν)

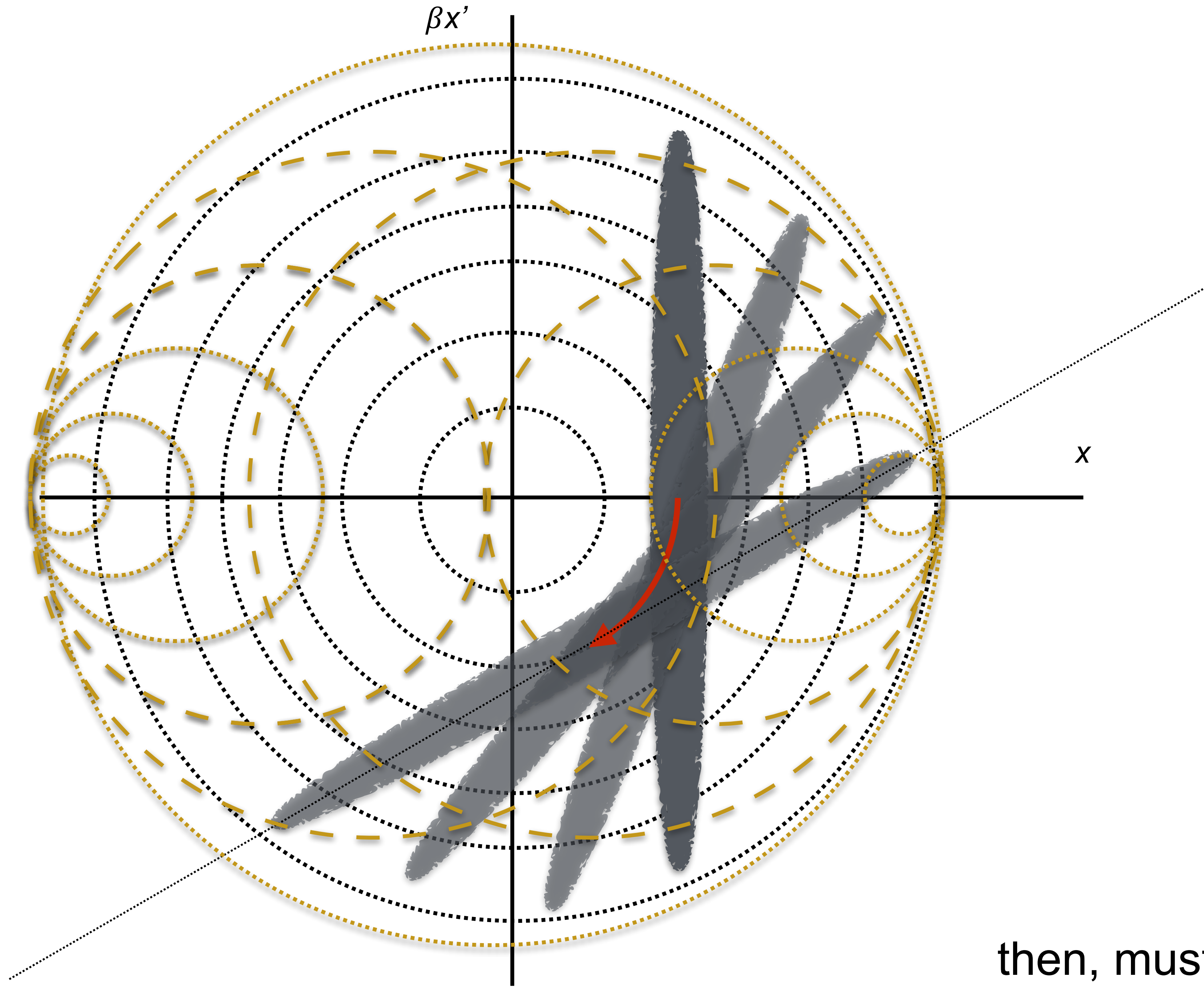
dipole oscillation



An optics mismatch will create a beam *size* oscillation at twice the betatron frequency (2ν)

quadrupole oscillation

All Together Now…



then, must add in
dispersive effects

Injection Kick: Phase Space

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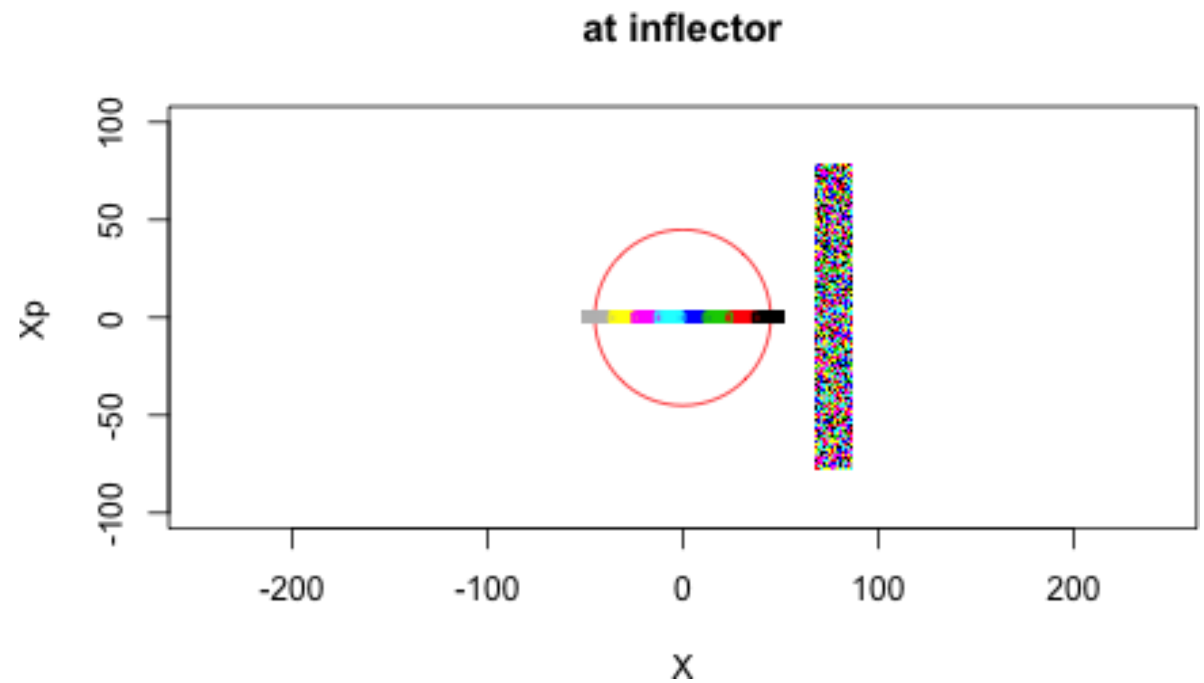
colors represent various momenta

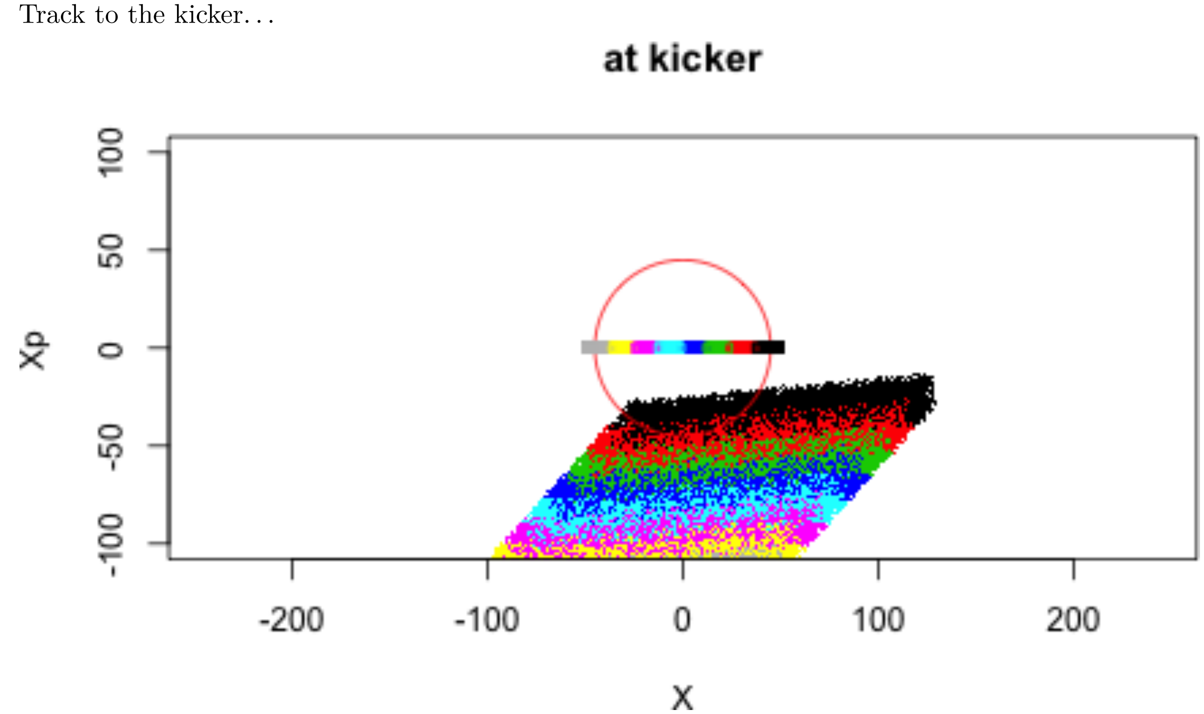
horizontal bar shows equilibrium orbits of various momentum particles

red circle is ring admittance of the central momentum, governed by 45 mm radius aperture




```
# Matrix to transport to kicker:
R4 = matrix(c(cos(pi*nu/2),sin(pi*nu/2),-sin(pi*nu/2),cos(pi*nu/2)),
   nrow=2,byrow=TRUE)
# Matrix to transport nt times around ring, for momentum dp:
R0 = function(nt,dp){
   mu = 2*pi*nu*nt+xi*dp
   matrix(c(cos(mu),sin(mu),-sin(mu),cos(mu)),nrow=2,byrow=TRUE) }
```

Track to the kicker…



Actual Kicker Pulse

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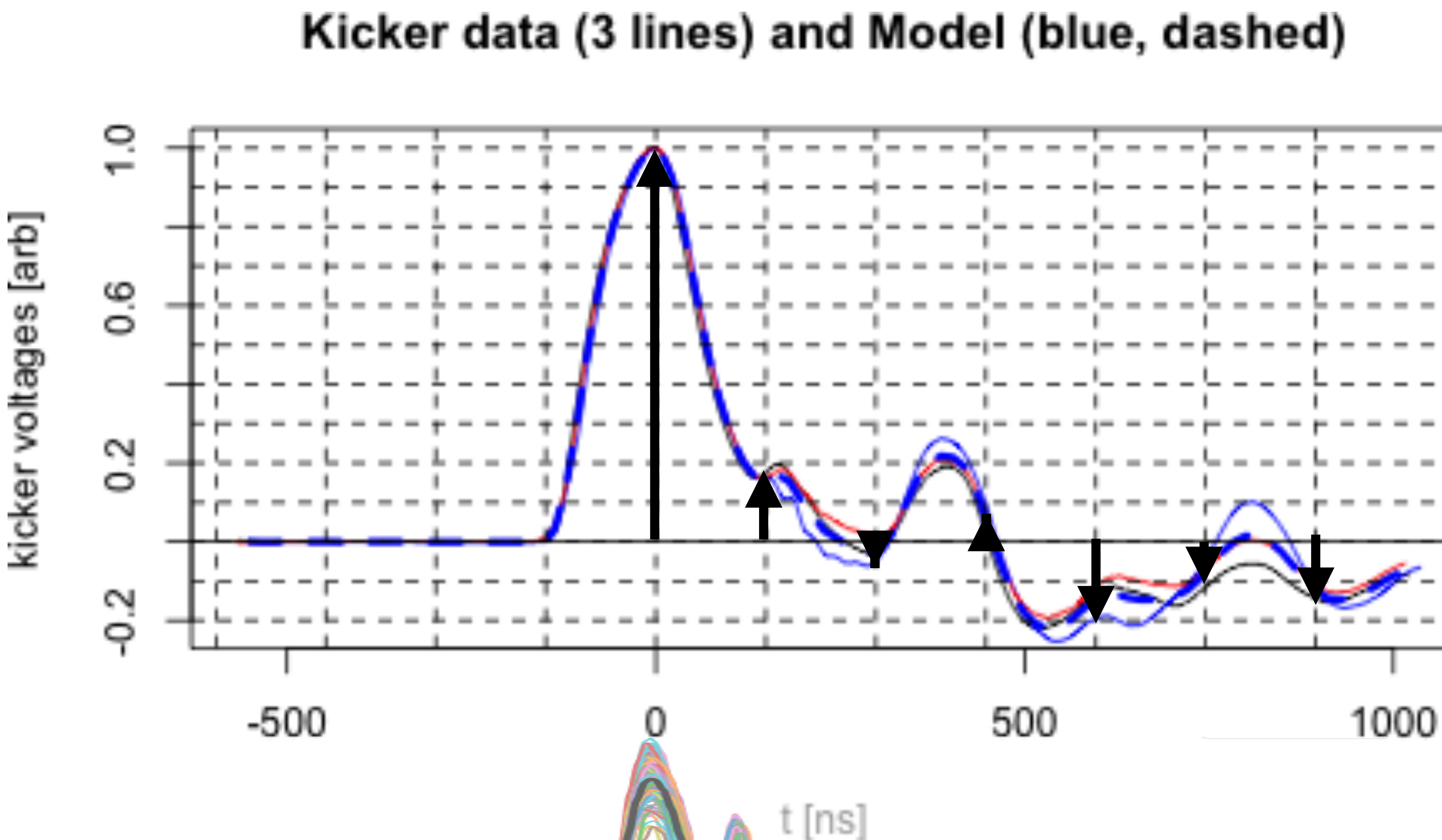
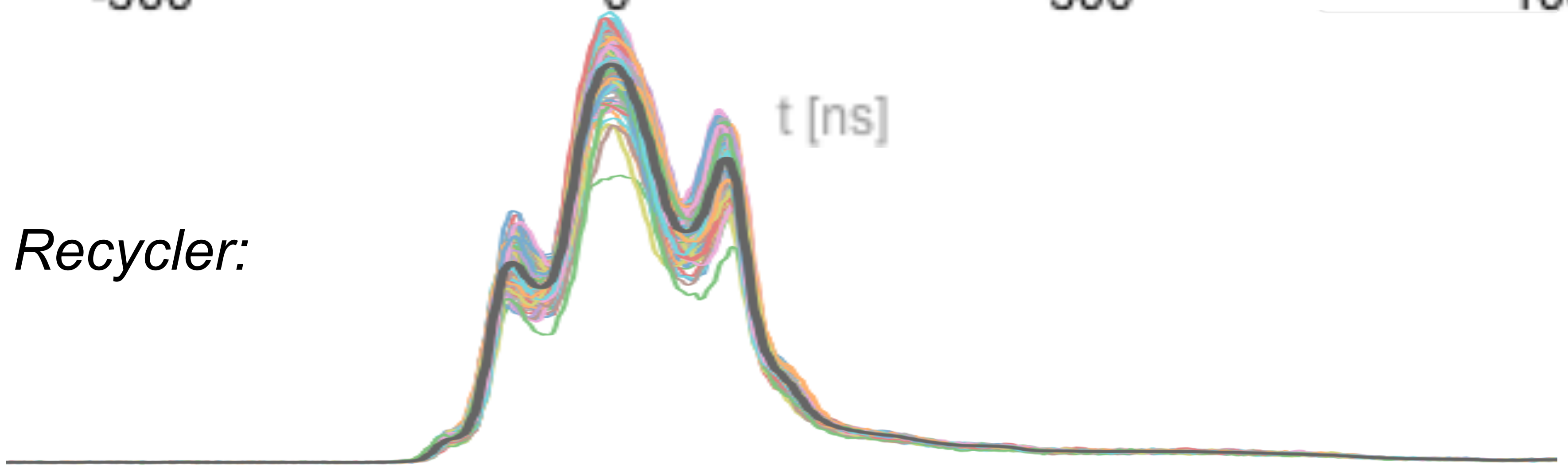


typical incoming beam pulse from Recycler:



Injection Kick: Phase Space

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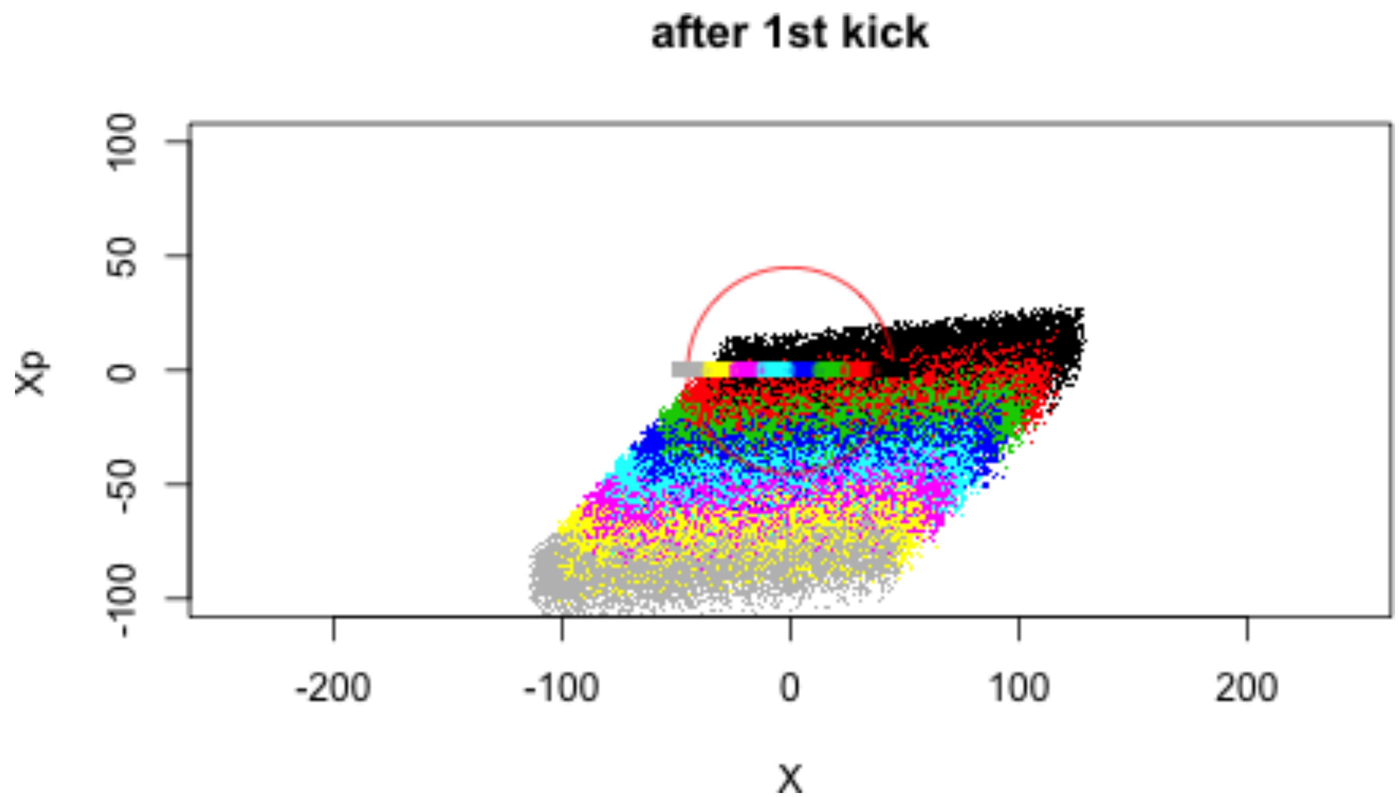
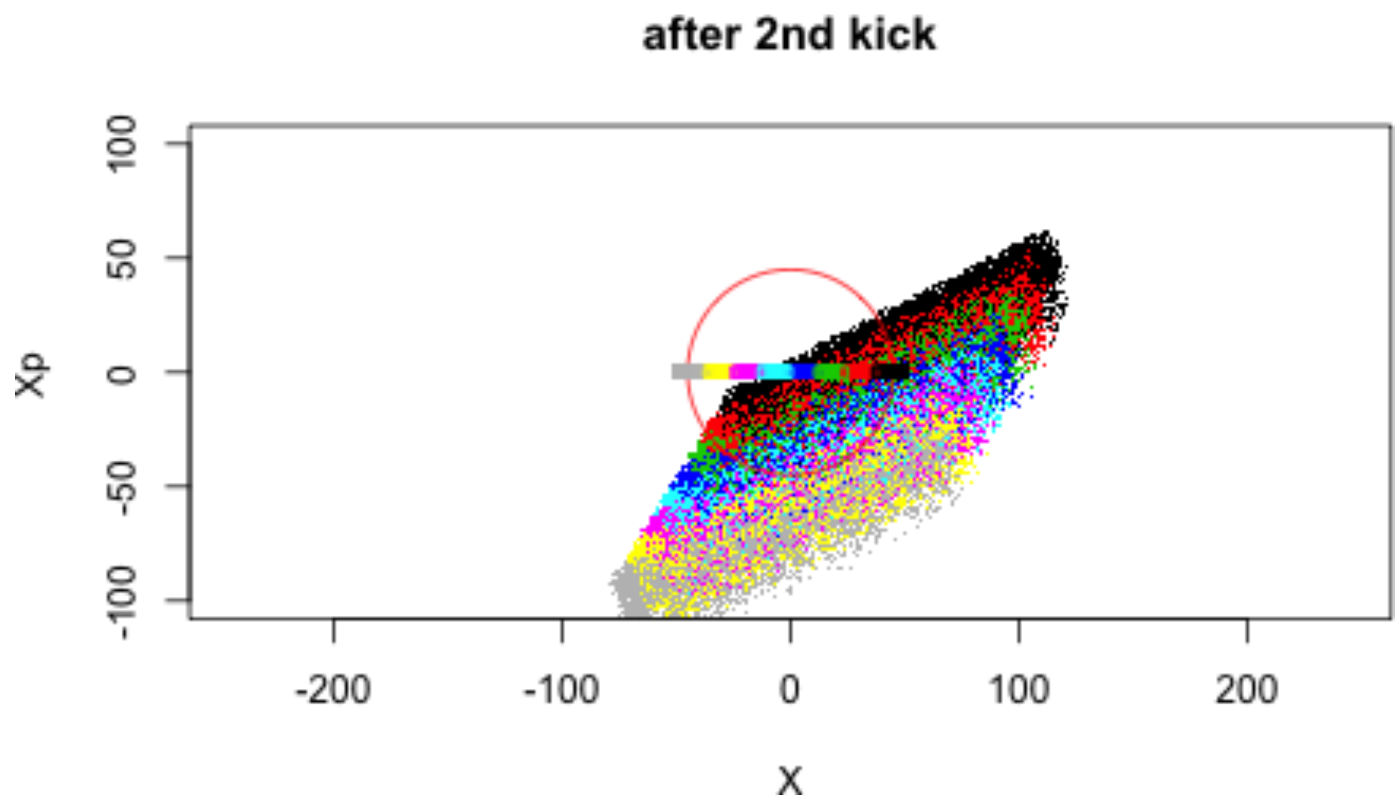
here, a lower-amplitude kick



After a revolution, receive another, even smaller amplitude kick, etc.



Injection Kick: Phase Space

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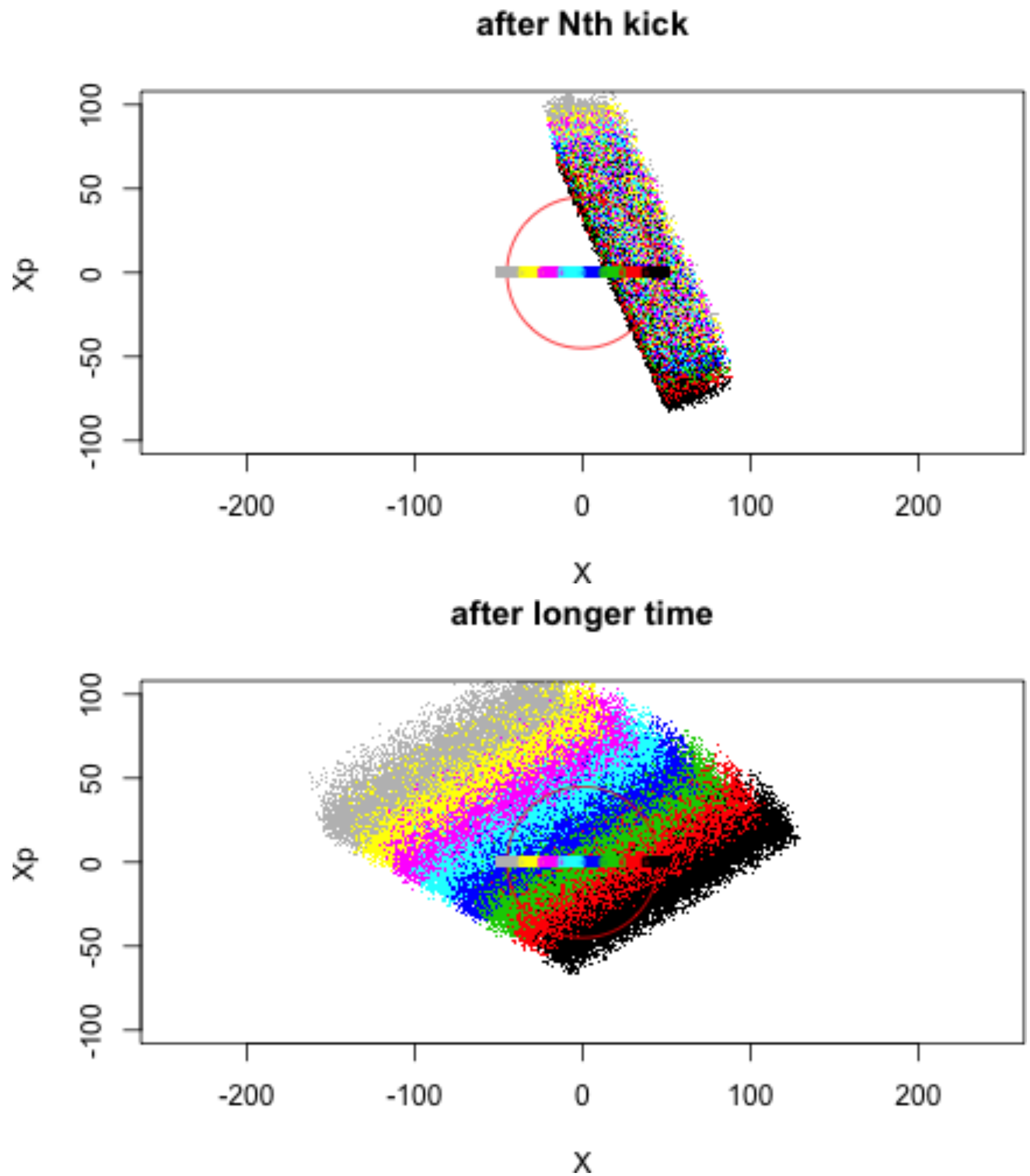
final momentum distribution is determined after the last “kick”



which particles can survive long-term?

betatron amplitude for **each momentum** must satisfy:

$$a_\beta < r_{ap} - D\delta$$

Injection Kick: Phase Space

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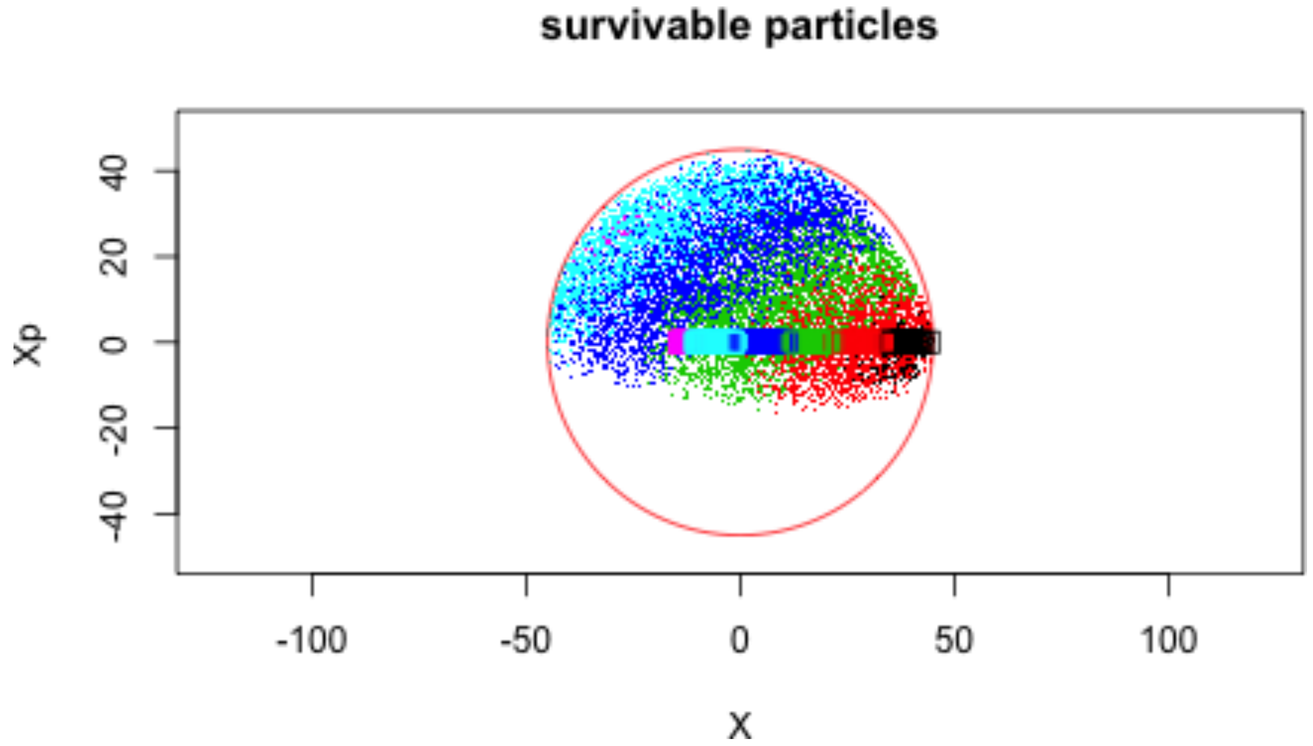
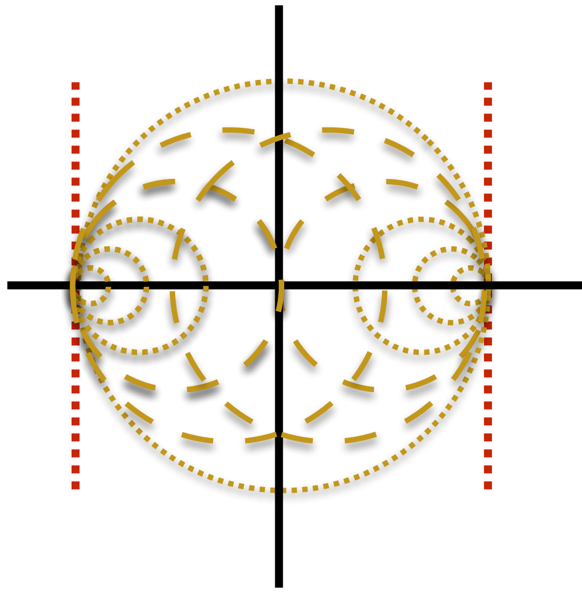


$a_\beta < r_{ap} - D\delta$



look at momentum distribution of the particles that can survive long-term:

tweak: ~55% of full kick makes a “match”

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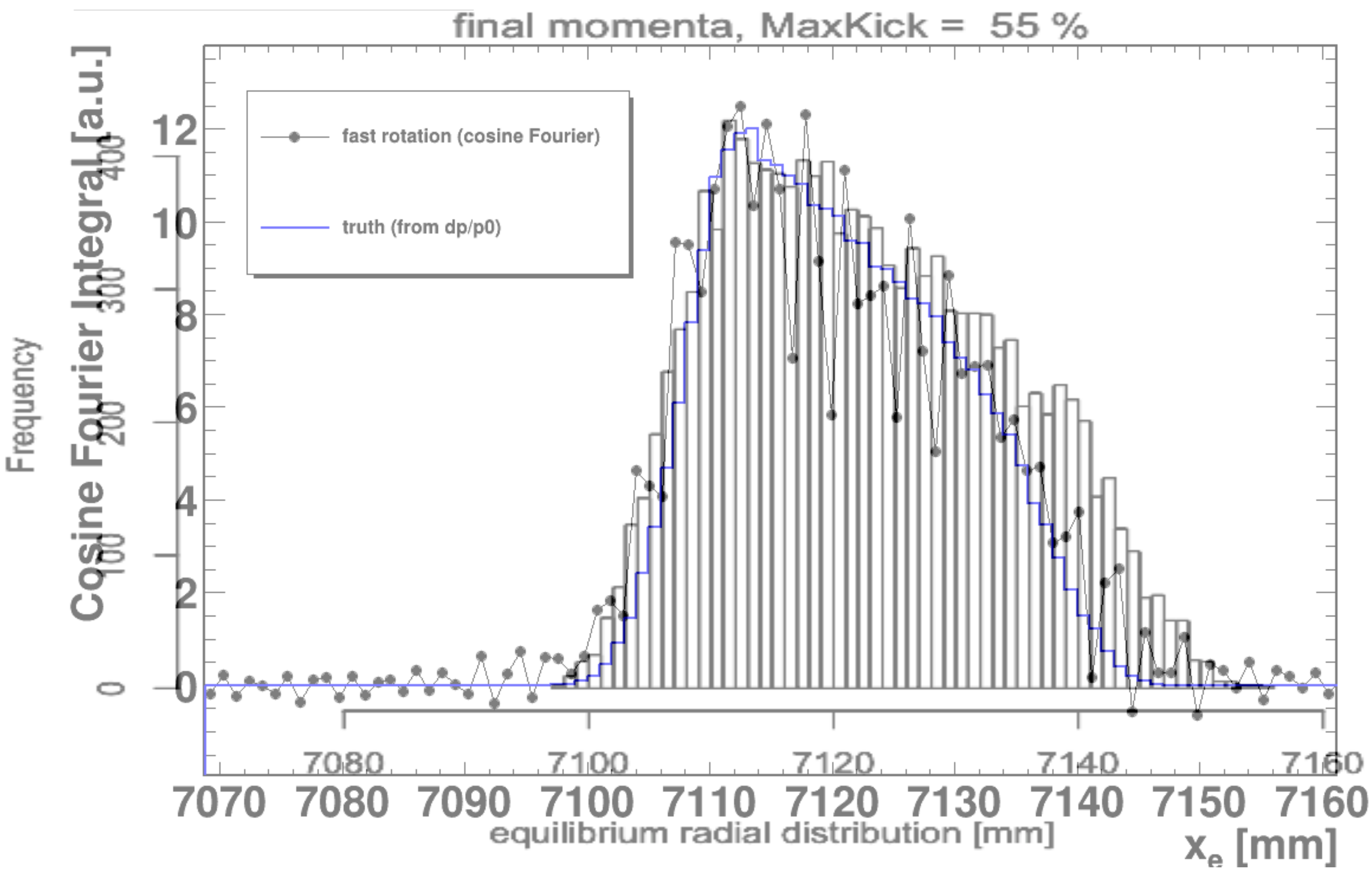
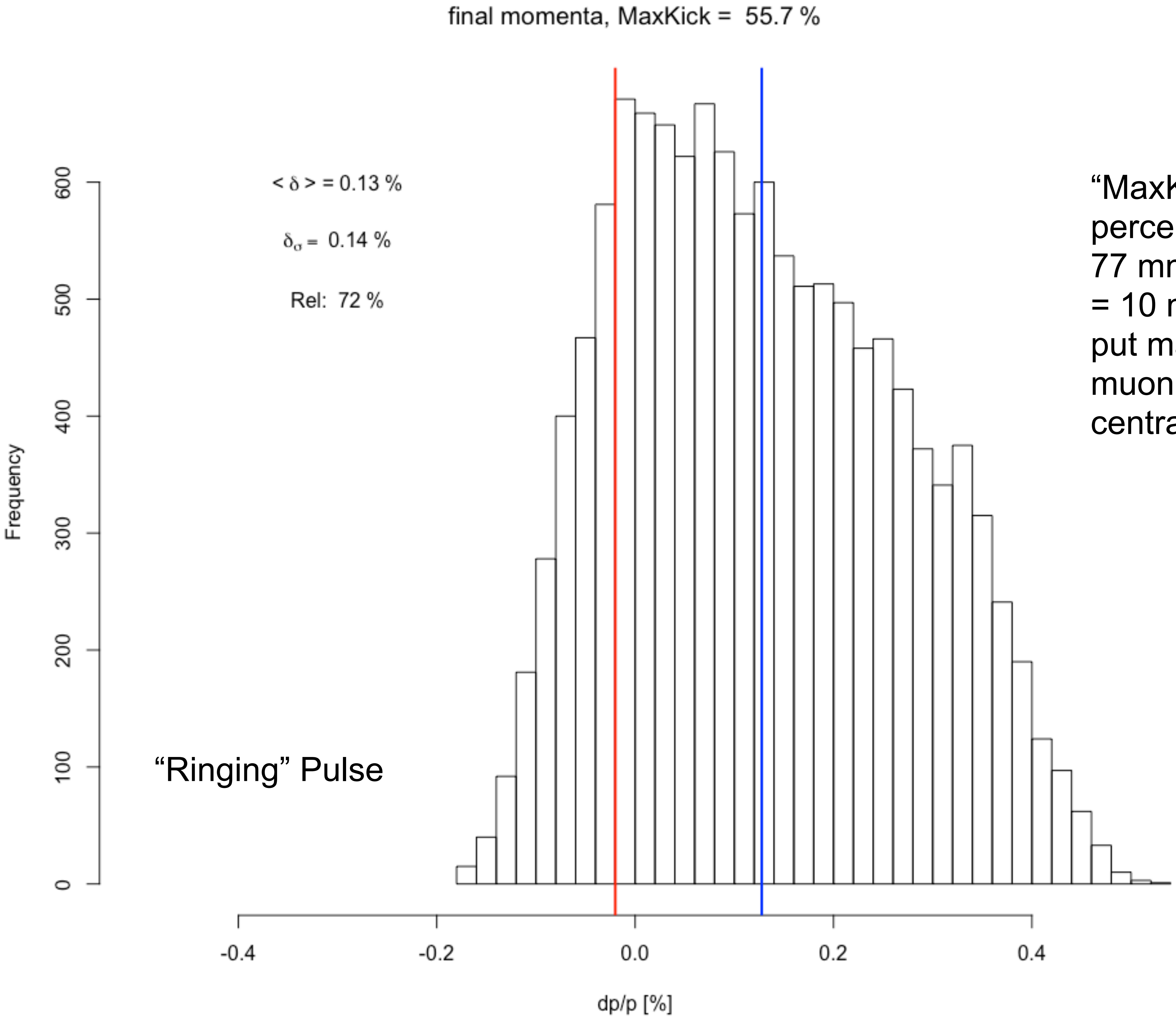
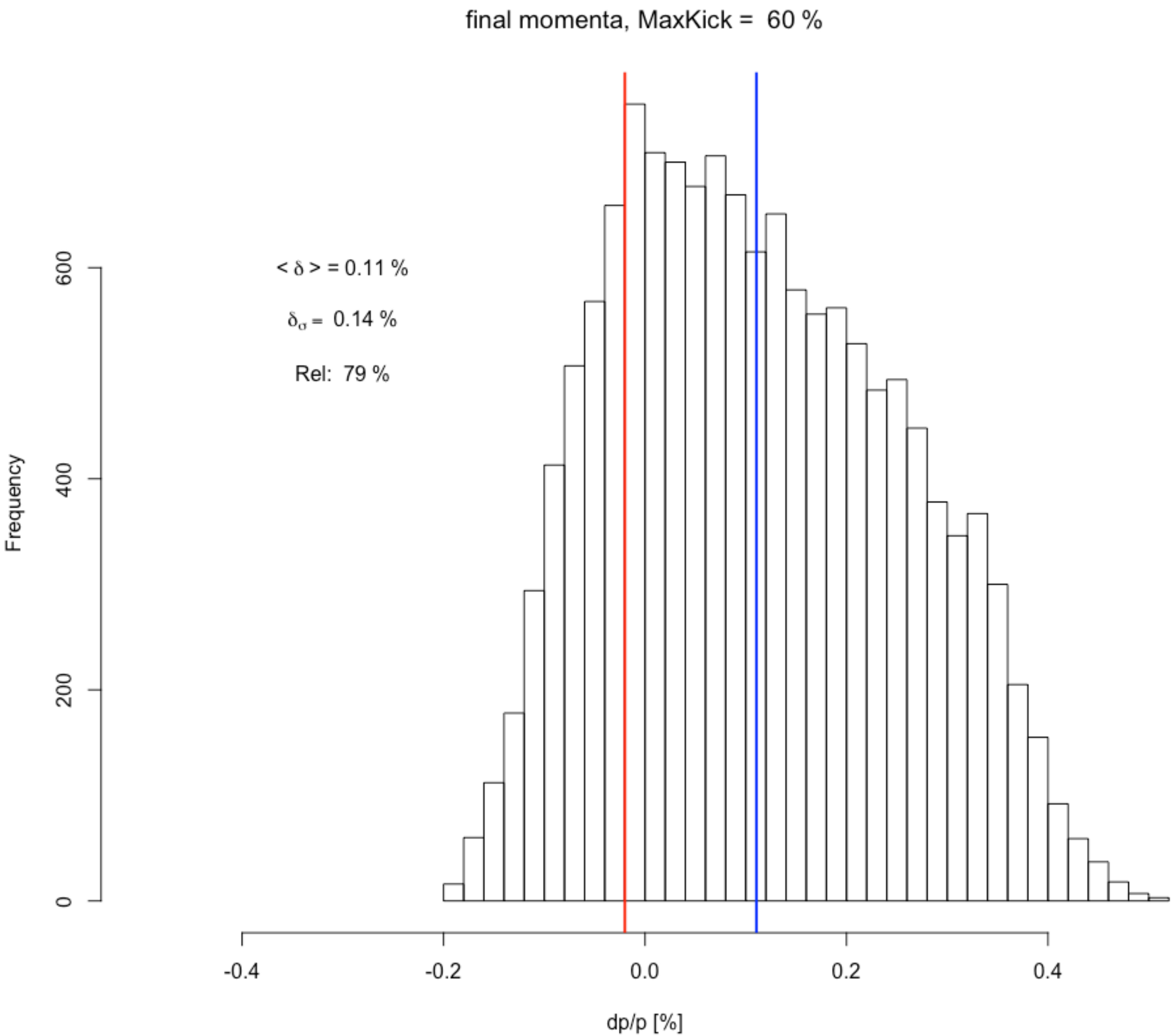


May 3 Kim Siang I BAM Meeting



“MaxKick” = percent of 77 mm/7.8 m = 10 mr to put magic muon onto central orbit

“Ringing” Pulse



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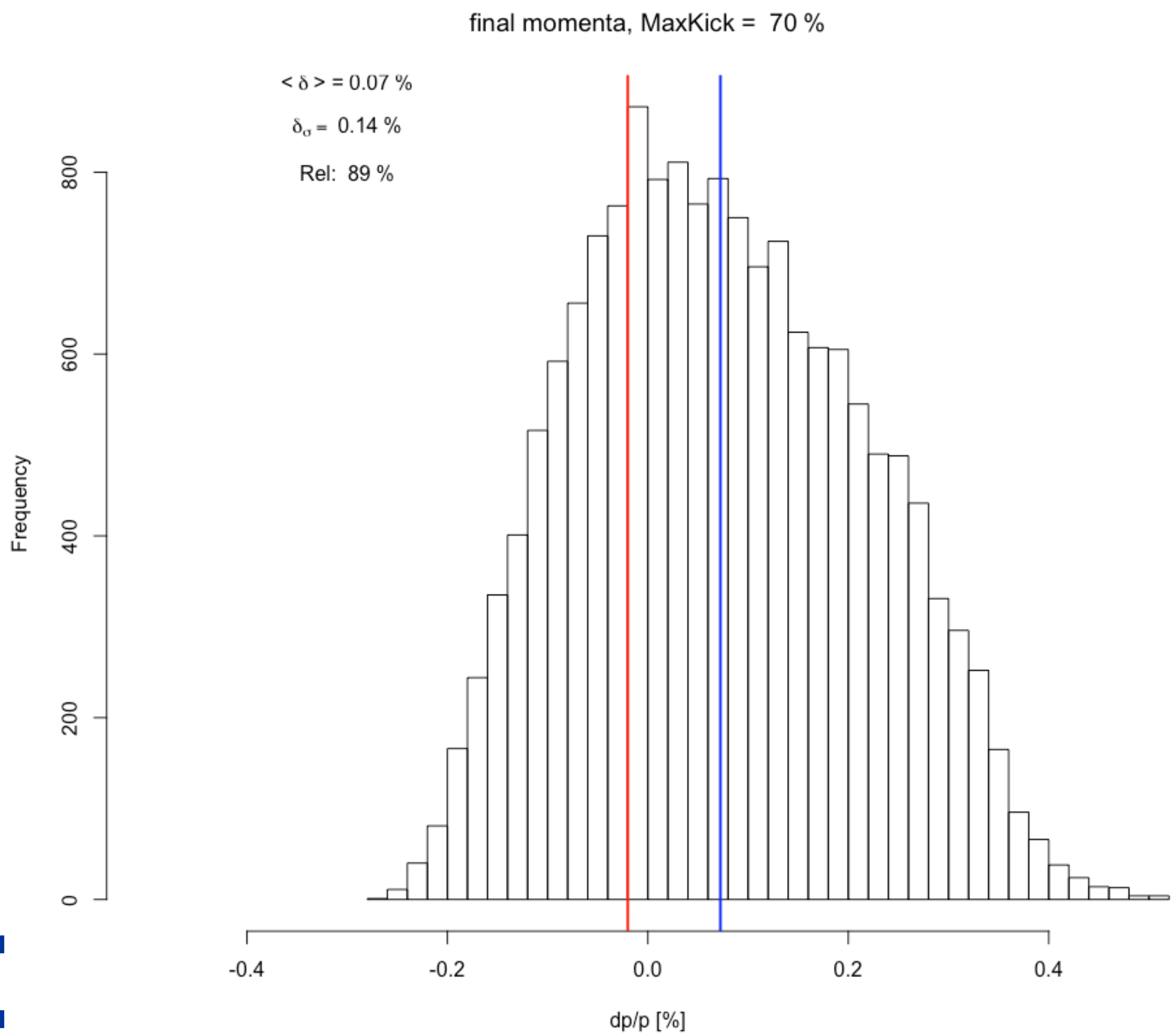
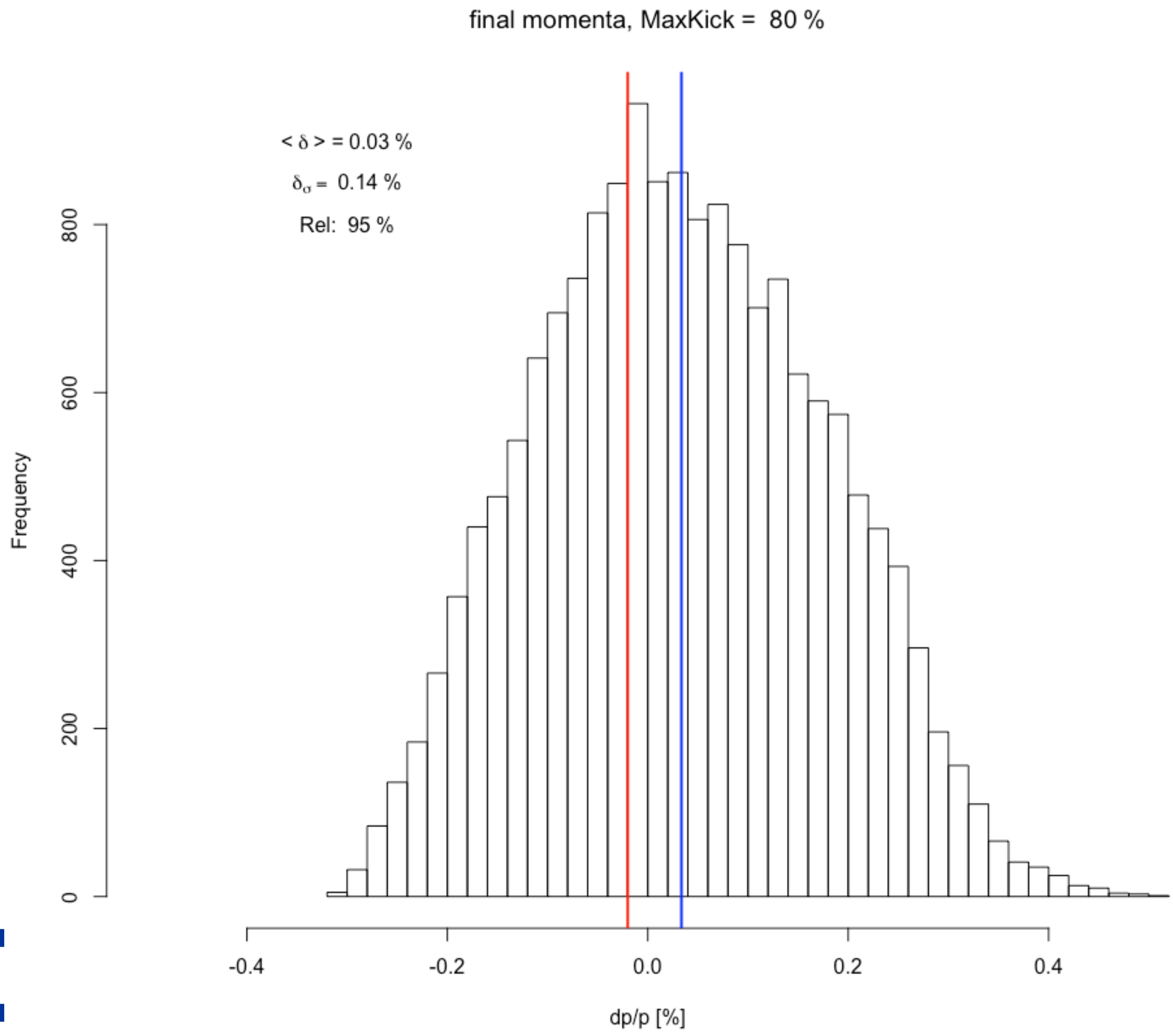
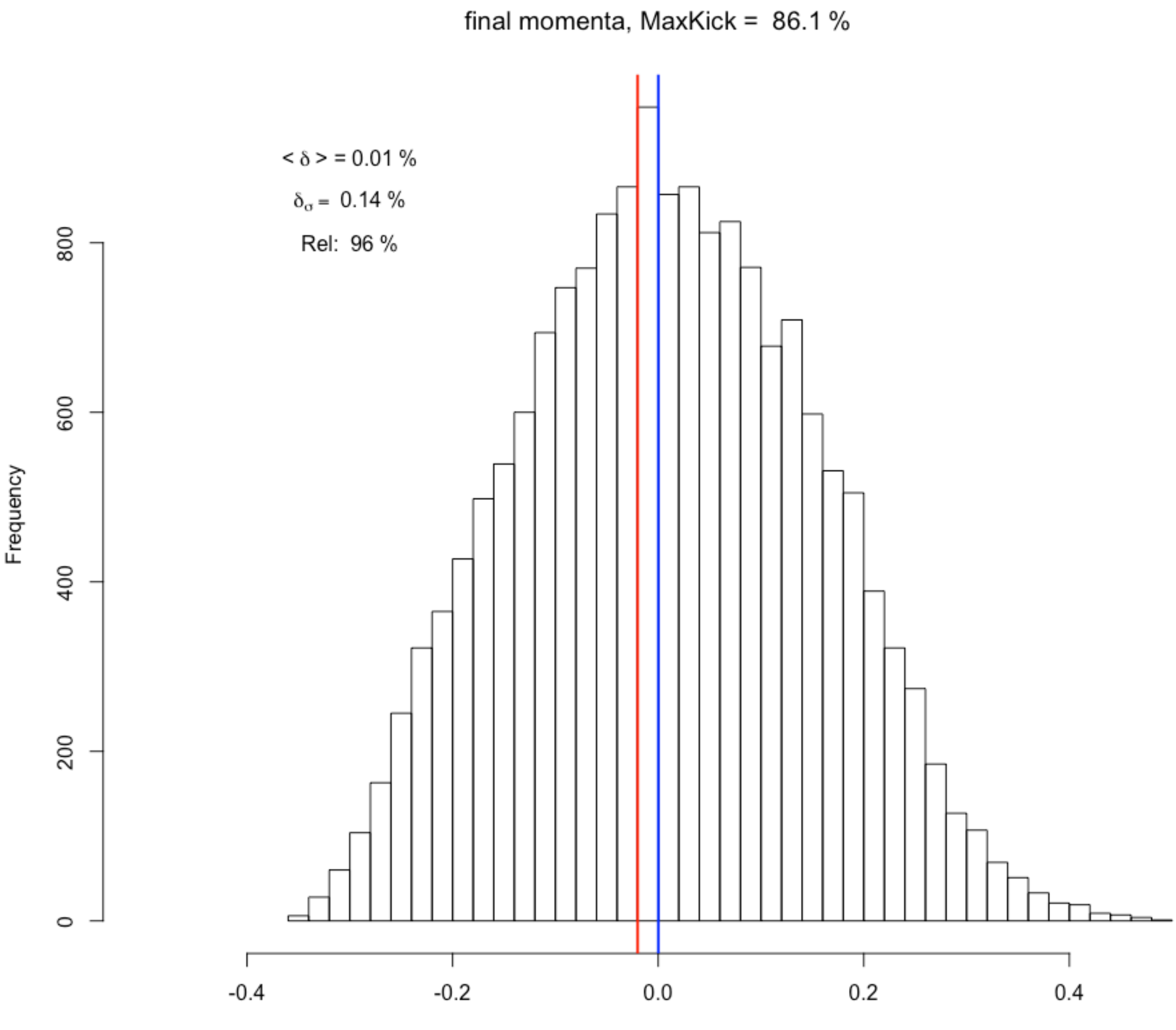
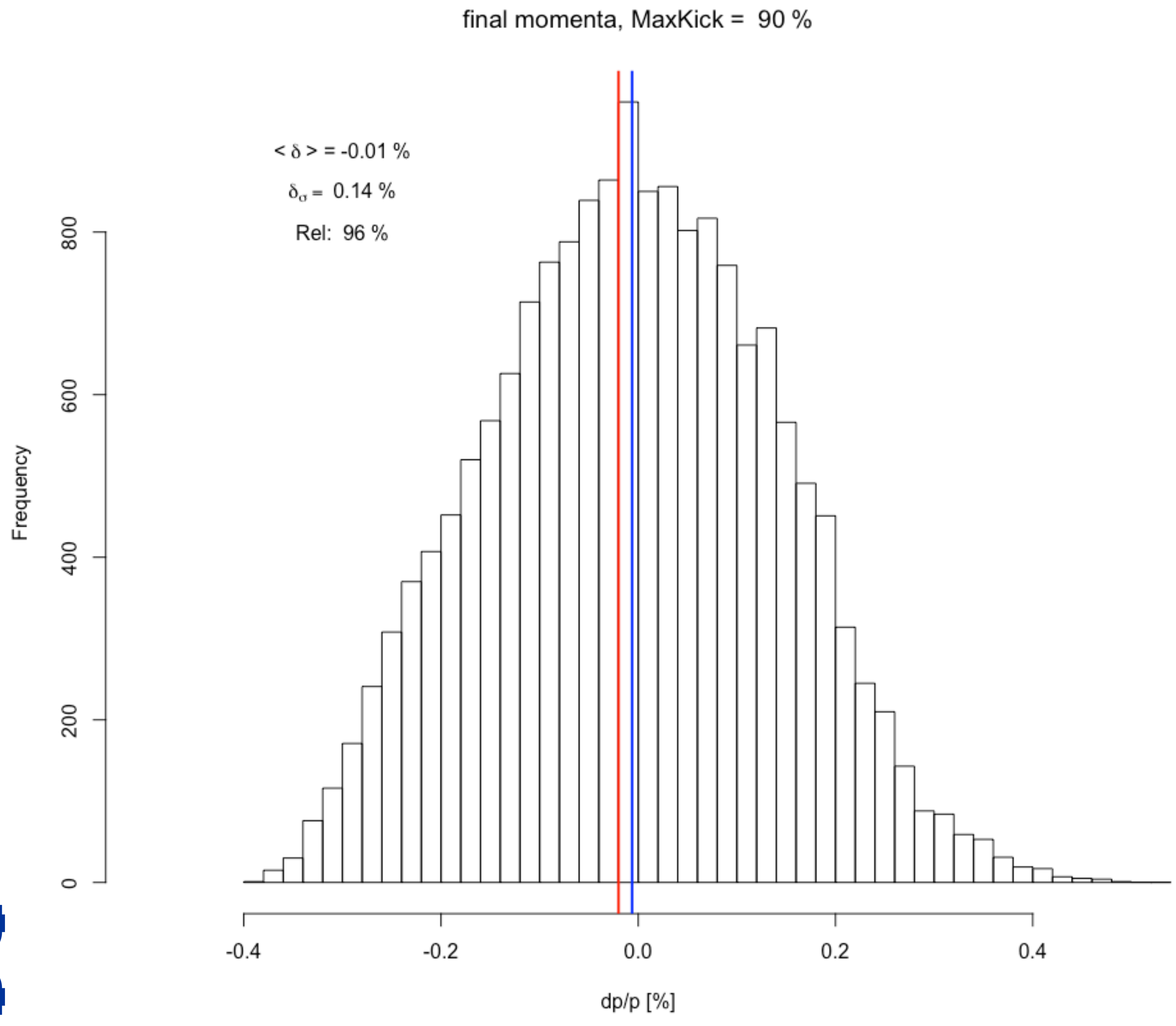


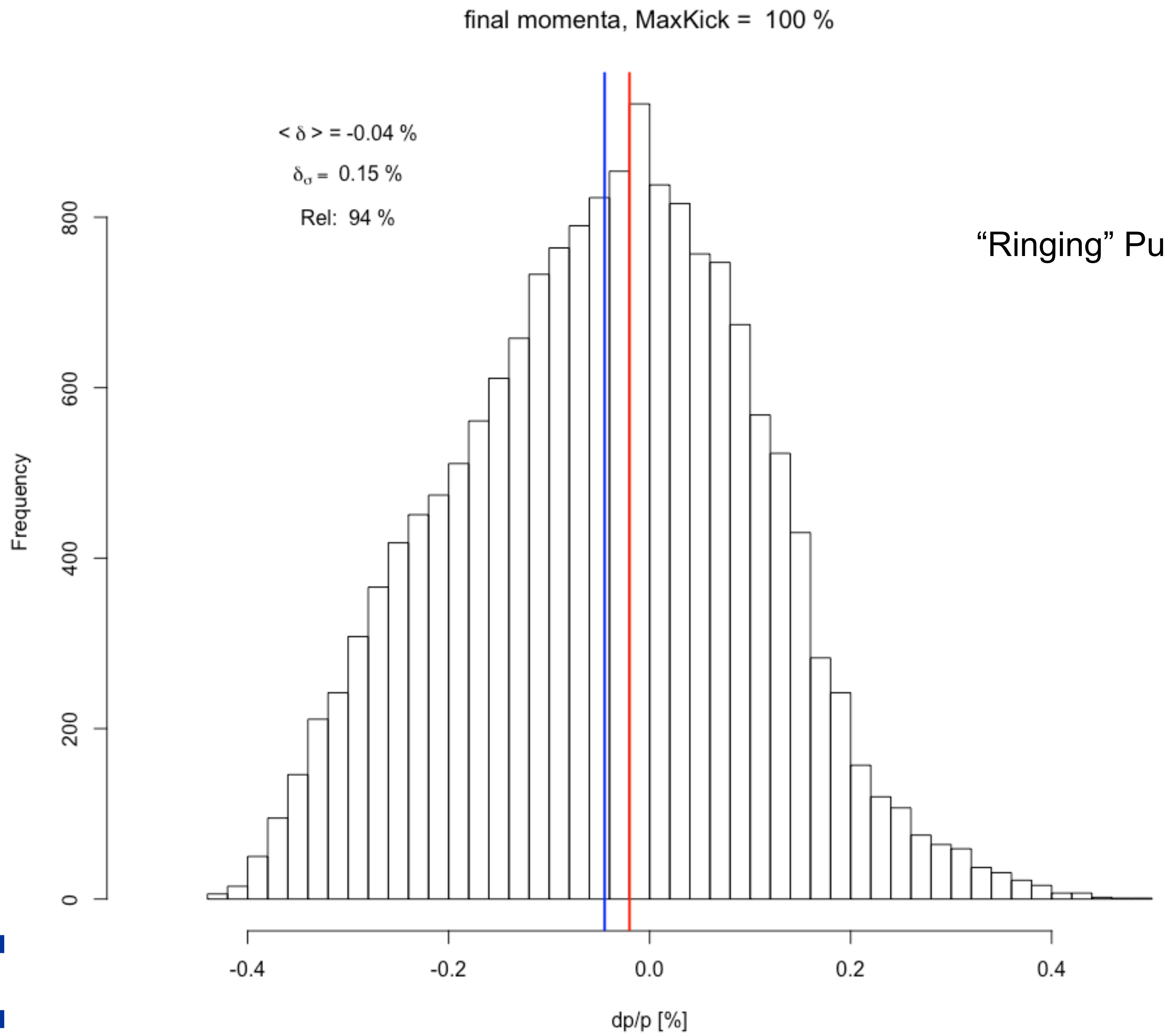
Illinois
rsity



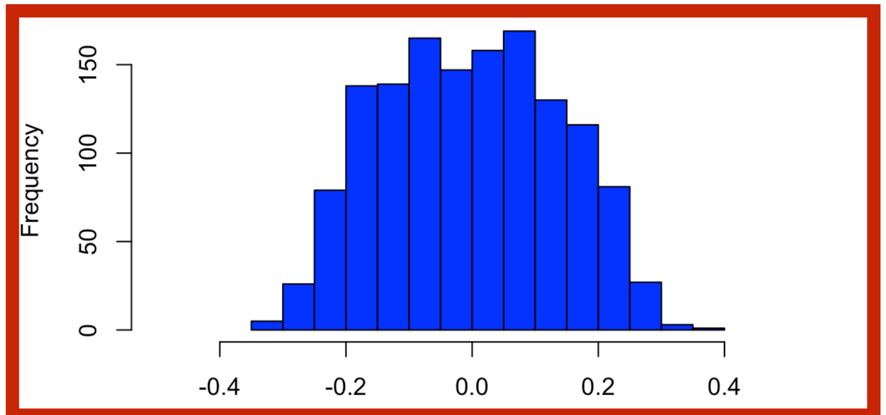
Illinois
rsity


final momenta, MaxKick = 86.1 %
< δ > = 0.01 %
δ_σ = 0.14 %
Rel: 96 %
Frequency
0
200
400
600
800
-0.4
-0.2
0.0
0.2
0.4
dp/p [%]

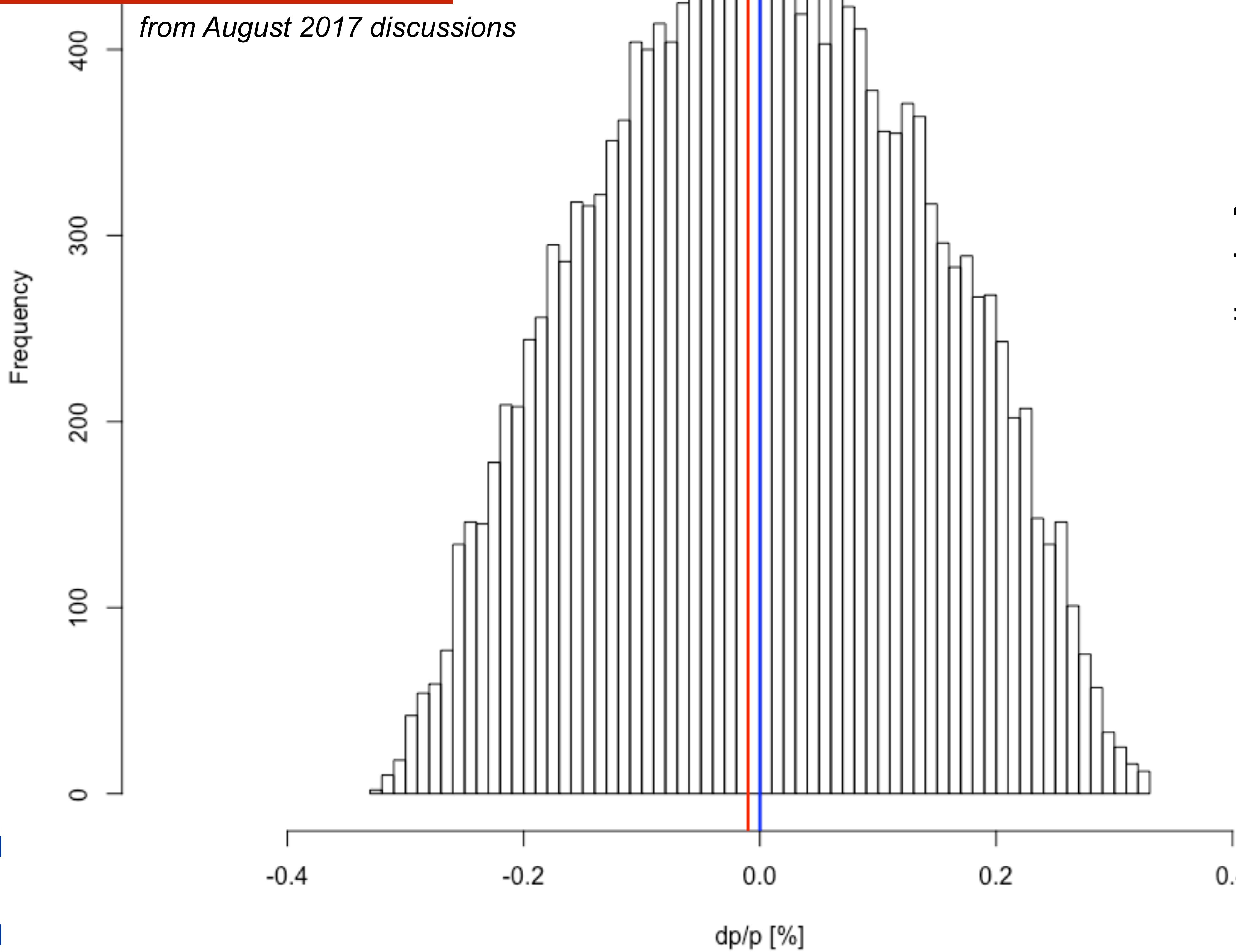

final momenta, MaxKick = 90 %
< δ > = -0.01 %
δσ = 0.14 %
Rel: 96 %
Frequency
0
200
400
600
800
-0.4
-0.2
0.0
0.2
0.4
dp/p [%]



“Ringing” Pulse



from August 2017 discussions



Square Pulse

"MaxKick" =
77 mm/7.8 m
= 10 mr

Momentum - Δt_0 Correlation

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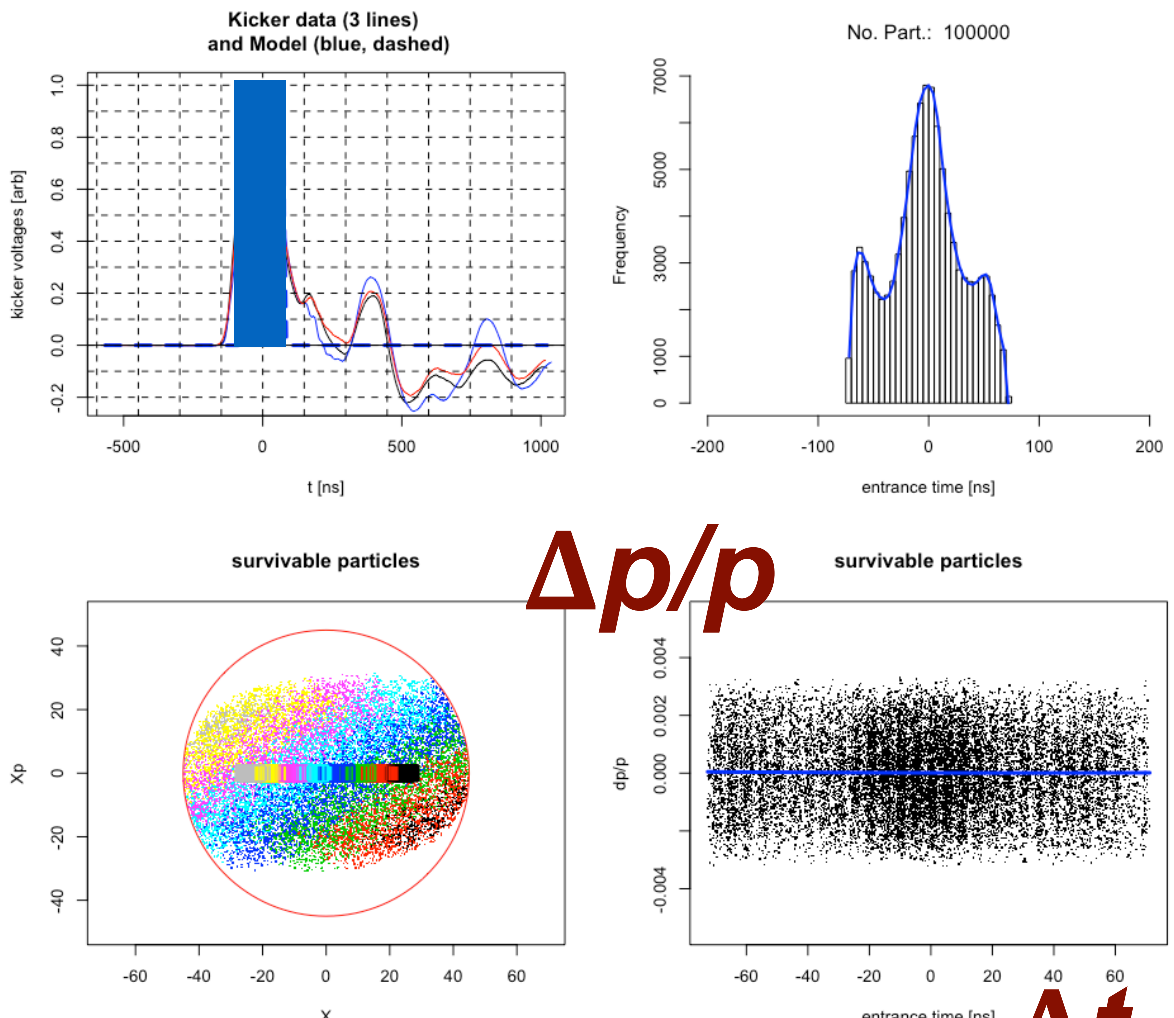
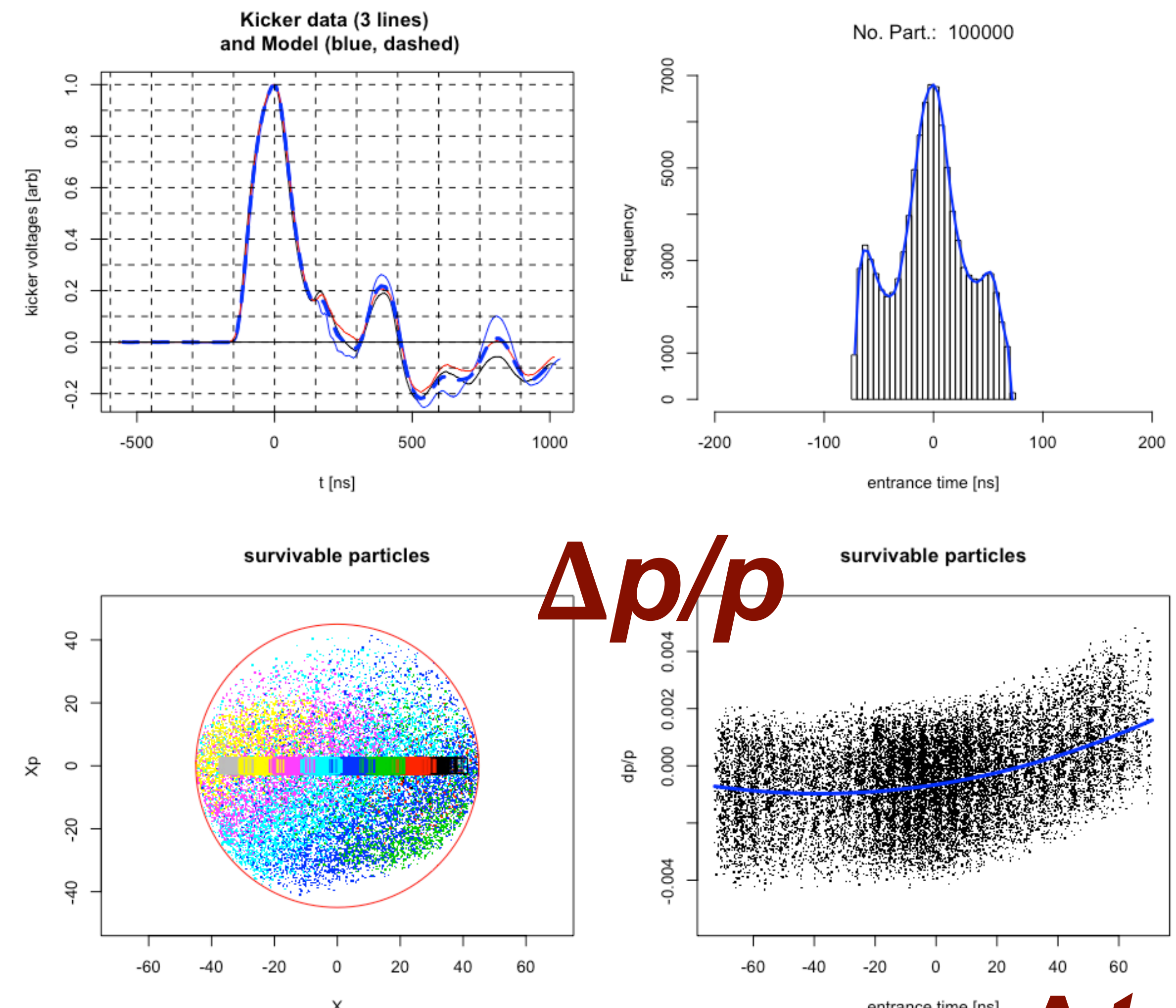
- Square Pulse Ringing Pulse



$\Delta p/p$

Δt_0



$\Delta p/p$

Δt_0