Momentum distribution

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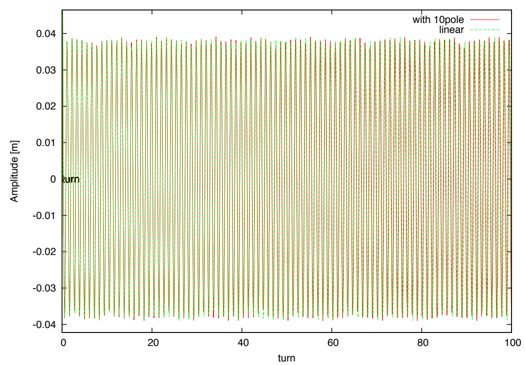


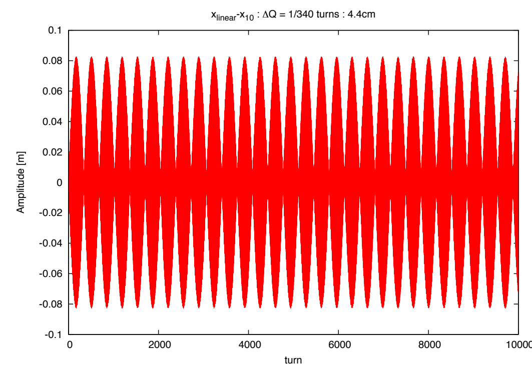


Figure 2: Horizontal displacement vs turn number. The green line is with purely linear quadrupole fields, (no multipoles). The red line is with all quad multipoles included. The amplitude of the oscillation is about 3.9 cm.

Figure 3: Sum of displacements with and without quad multipoles when the oscillation amplitude is 4.4 cm. The beat frequency corresponds to the tune difference.

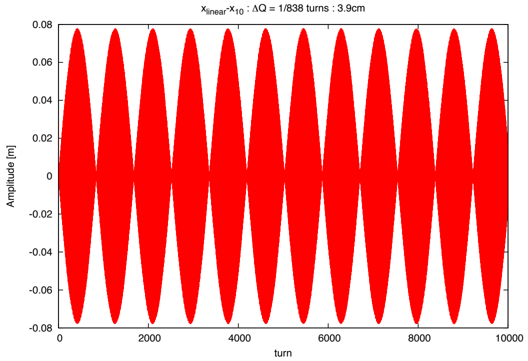


Figure 4: Sum of displacements with and without quad multipoles (nonlinearities) when the oscillation amplitude is 3.9 cm as in Figure 2. $\Delta Q = 1/838$ turns.

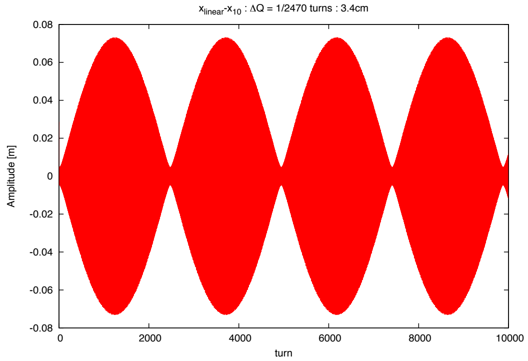


Figure 5: Sum of displacements with and without quad multipoles when the oscillation amplitude is 3.4 cm. $\Delta Q = 1/2470$ turns.

Fast rotation signal on an imaginary detector plane for a single muon

$$S(t) = \sum_{n=0}^{\infty} \delta(t - nT)$$

And for an off momentum muon

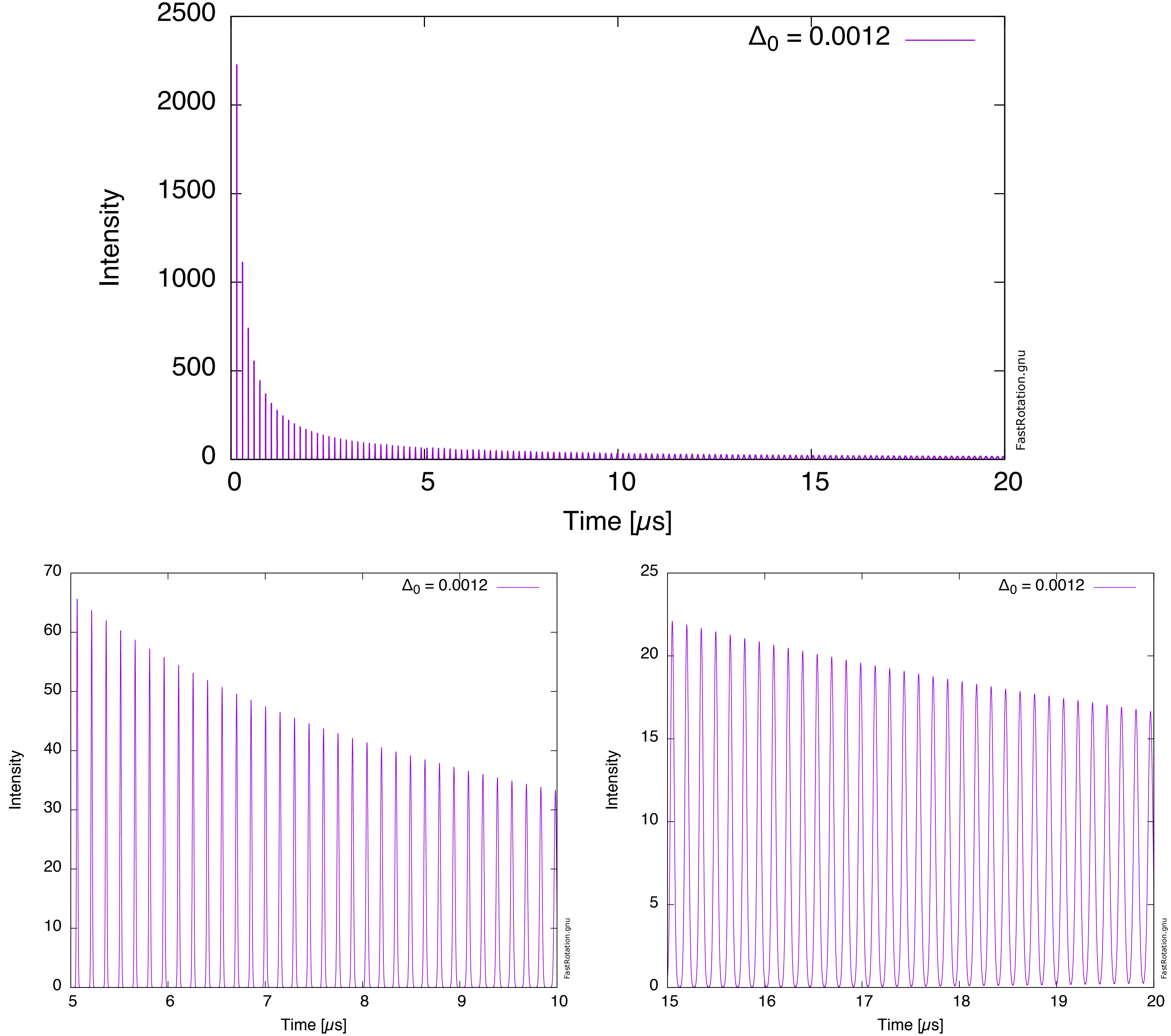
$$S(t) = \sum_{n=0}^{\infty} \delta(t - nT(1+\Delta))$$

For a distribution of momenta

$$S(t) \quad = \quad \sum_{n=0}^{\infty} \int \rho(\Delta)\delta(t - nT(1+\Delta))d\Delta$$

Gaussian distribution of momenta

$$\begin{aligned} S(t) \quad &= \quad \sum_{n=0}^{\infty} \int \frac{e^{-\Delta^2/(2\Delta_0^2)}}{\sqrt{2\pi}\Delta_0} \delta(t - nT(1+\Delta))d\Delta \\ &= \quad \sum_{n=0}^{\infty} \int \frac{e^{-\Delta^2/(2\Delta_0^2)}}{\sqrt{2\pi}\Delta_0} \frac{\delta(\Delta - (\frac{t}{nT} - 1))}{nT} d\Delta \\ &= \quad \sum_{n=0}^{\infty} \frac{e^{-(\frac{t}{nT}-1)^2/(2\Delta_0^2)}}{\sqrt{2\pi}\Delta_0 nT} \end{aligned}$$



Δ_0 = 0.0012
Intensity
Time [μs]
FastRotation.gnu

Our ‘perfect’ signal extends over all time and is periodic

The real measured signal

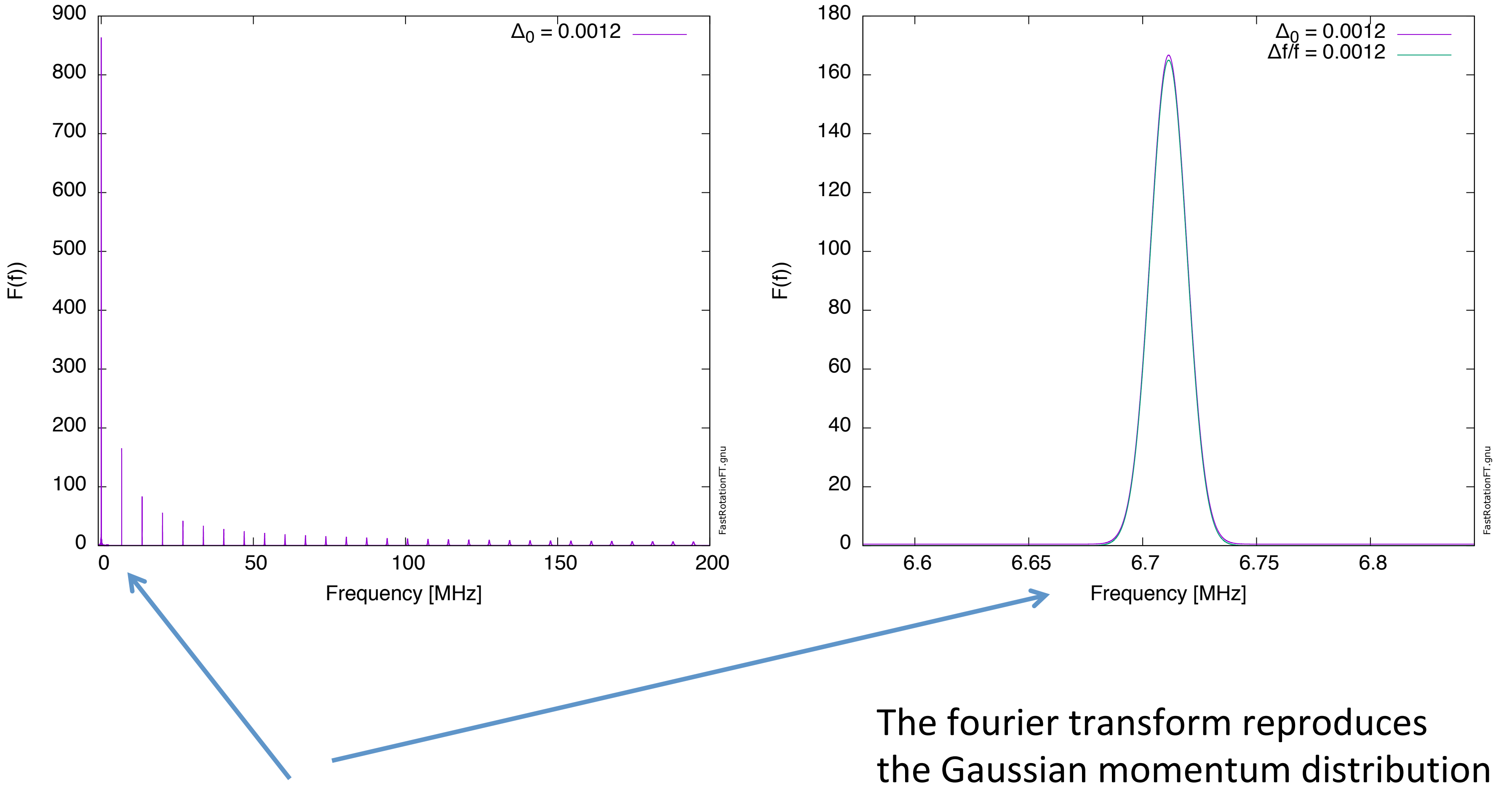
- Starts a few turns beyond t_0
- Muons decay
- Our only signal is when the muon decays (rather than on every turn)

To extract momentum distribution from the fast rotation signal
Consider Fourier transform

We use cos assuming symmetry about t=0

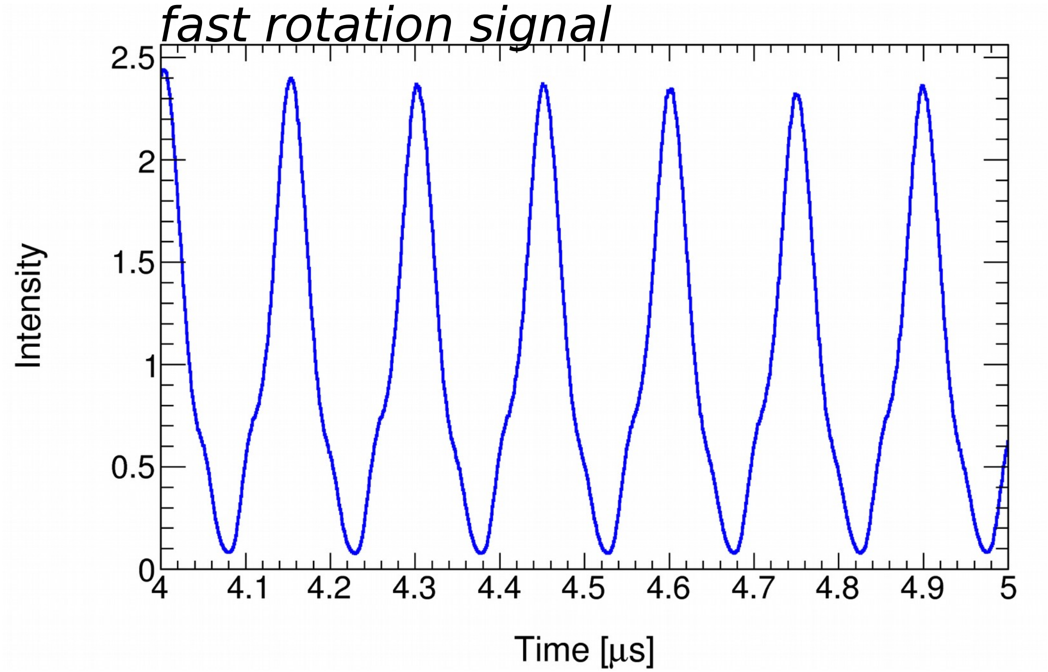
$$\begin{aligned} F(\omega, t_0) &= \int_0^\infty S(t, \Delta_0) \cos\omega(t - t_0) dt \\ &= \sum_{n=0}^{\infty} \int_0^\infty \frac{e^{-(\frac{t}{nT} - 1)^2/(2\Delta_0^2)}}{\sqrt{2\pi}\Delta_0 nT} \cos\omega(t - t_0) dt \end{aligned}$$

$$F(\omega, t_0) = \sum_{n=0}^{\infty} e^{-\omega^2 (nT)^2 \Delta_0^2/2} \cos\omega(nT - t_0)$$



The fourier transform reproduces the Gaussian momentum distribution

The measured fast rotation signal starts and ends



The fourier transform is not entirely characterized by harmonics of the rotation frequency but includes all frequencies

Including frequencies that are unphysical, and outside the aperture

$$\hat{S}(\omega) = \sqrt{\frac{2}{\pi}} \int_{t_0}^{\infty} S(t) \cos \omega (t - t_0) dt$$

$$S(t) = \sqrt{\frac{2}{\pi}} \int_{0}^{\infty} \hat{S}(\omega) \cos \omega (t - t_0) d\omega$$

Start collecting data at t_s

$$\hat{S}_1(\omega) = \sqrt{\frac{2}{\pi}} \int_{t_s}^{\infty} S(t) \cos \omega (t - t_0) dt$$

Missing bit

$$\Delta(\omega) = \sqrt{\frac{2}{\pi}} \int_{t_0}^{t_s} S(t) \cos \omega (t - t_0) dt$$

$$\Delta(\omega) = \sqrt{\frac{2}{\pi}} \int_{t_0}^{t_s} S(t) \cos\omega(t - t_0) dt = \frac{2}{\pi} \int_{t_0}^{t_s} \int_0^{\infty} \hat{S}(\omega') \cos\omega'(t - t_0) \cos\omega(t - t_0) d\omega' dt$$

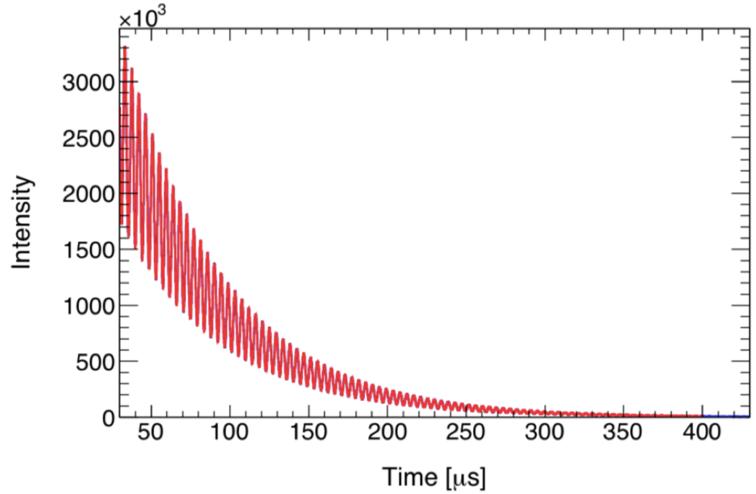
$$\Delta(\omega) = \frac{2}{\pi} \int_{t_0}^{t_s} \sum_{n=1}^{\infty} \int_{\omega_n^-}^{\omega_n^+} S(\omega') \cos\omega'(t - t_0) \cos\omega(t - t_0) d\omega' dt$$

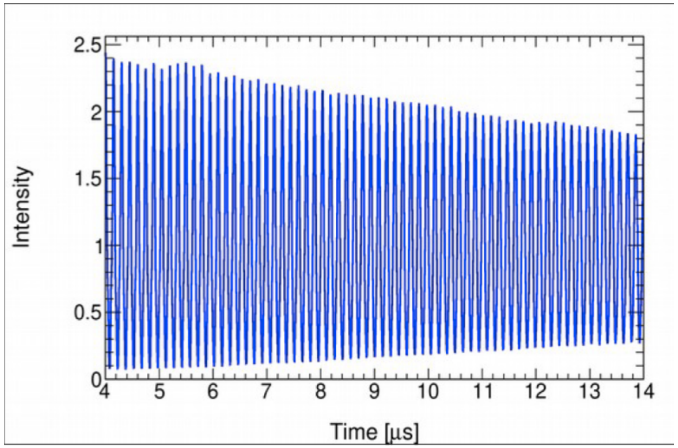
Include only frequencies (or harmonics) within the aperture

$$\hat{S}(\omega) = S_1(\omega) + \Delta(\omega)$$

Fourier Analysis

1) Fit 9-parameter wiggle





2) Divide out background signals → ω_a, τ_μ and CBO