Normal Cryogenic Fluid Behavior
(emphasis on helium, but not superfluid, which will be covered later)

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Outline

• Cooling modes for superconducting devices
• Forced flow cooling
• Two-phase flow and pool boiling
• Fluid dynamics
• Saturated bath thermodynamics
Large-scale cooling of superconducting devices

- Physicists and engineers designing a large-scale liquid helium system typically must design the cooled components (magnets or RF cavities, their containers, and the interfaces to them).
- Cooling mode, heat transfer, pressure drops, cooldown, warm-up and non-steady or upset system operations all must be considered as part of the component design.
- The cooled devices must be viewed as part of the cryogenic system.
Cooling modes in large-scale cryogenic systems

- Pool boiling helium I used in superconducting RF for HERA (DESY), LEP (CERN), KEKB (KEK, Japan), CESR (Cornell)
- Forced flow of subcooled or supercritical helium I for cooling superconducting magnets (Tevatron, HERA, SSC, RHIC, ITER)
- Stagnant, pressurized helium II (the Tore Supra tokamak in France demonstrated the technology, LHC magnets)
- Saturated helium II (CEBAF, FLASH and Eu-XFEL at DESY, SNS at Oak Ridge, FRIB at MSU, LCLS-II at SLAC, and more)
- This list also illustrates the extent to which superconductivity and cryogenics have become standard technology for accelerators
Helium phase diagram
(S. W. VanSciver, Helium Cryogenics, p. 54)

- Critical point
  - 5.2 K, 2.245 atm
- Lambda transition at 1 atm
  - 2.172 K

- SRF -- HERA, LEP, KEKB, CESR
- Magnets -- HERA, Tevatron
- Magnets -- SSC
- Magnets -- Tore Supra, LHC
- SRF -- CEBAF, XFEL, SNS, LCLS-II

Fig. 3.1. ⁴He phase diagram.
Cooling modes -- magnets vs RF

- Accelerator magnets are often cooled with subcooled liquid
  - Typically working near the limit of the superconductor with large stored energy
  - Ensure complete liquid coverage and penetration
- Superconducting RF cavities are generally cooled with a saturated bath
  - Large surface heat transfer in pool boiling for local “hot spots”
  - Very stable pressures, avoid impact of pressure variation on cavity tune
Cooling modes--surface heat flux

- Boiling helium I (normal helium)
  - 1 W/cm$^2$ in nucleate boiling with 0.5 K temperature rise to the object surface so equivalent to 2 W/cm$^2$K
- Forced convection helium I
  - Convection coefficients on the order of 0.1 W/cm$^2$K
- Saturated helium II (superfluid helium, SF)
  - 1 W/cm$^2$ heat transport to the surface without bubbles
- Pressurized helium II
  - Kapitza conductance about 0.6 W/cm$^2$K
Pressurized versus pool boiling

- Pressurized helium (normal or superfluid) gives maximum penetration of helium mass in magnet coils, which may be a factor in stability if not also heat transfer. But heat flow results in a temperature rise.

- Pool boiling gives pressure stability (important for superconducting RF), provides maximum local heat transfer, and provides nearly isothermal cooling.
Heat transport through channels--pressurized normal helium in SSC

- SSC dipole nominal operating temperature was to be 4.35 K, tightly constrained for magnet quench performance
- Allowable temperature rise of only 0.050 K allowed heat absorption of about 4 J/gK x 0.050 K = 0.20 J/g and forced high flow rate (100 g/s) as well as use of recoilers
- Forced flow of supercritical helium periodically recooled by heat exchange with a saturated bath
Recooler flow scheme

- Thermal shield return
- Helium return to refrigerator (1.2 bar, 4.4 K)
- Liquid helium supply from refrigerator (3 bar, 4.5 K)
- Thermal shield supply (80 K nitrogen or helium thermal shield)

String of magnets or other cooled devices

- Devices cooled with forced flow 4.5 K subcooled liquid helium

Recooler Assembly

- 4.4 K return vapor
- Subcooler liquid level control valve
- 4.4 K vapor
- 4.4 K liquid

String of magnets or other cooled devices

- Devices cooled with forced flow 4.5 K subcooled liquid helium
Heat transport through channels--pressurized normal helium

- This plot of helium enthalpy versus $T$ illustrates the large amount of heat absorbed ($20+ \text{ J/g}$) if one can tolerate 6.5 K or even more.
- Nominally “5 K” thermal intercept flow may take advantage of this heat capacity.
Convective heat transfer

• Convective heat transfer – heat transfer from a solid surface into a moving fluid
  – A complex sequence of heat transfer from the surface to a boundary layer and into the bulk fluid, a combination of conduction and mass transport

• We analyze convection with the equation

\[ Q_{\text{surface-fluid}} = h_c A_c (T_{\text{surface}} - T_{\text{fluid}}) \]
Evaluating $h_c$

$$Q_{\text{surface-fluid}} = h_c A_c (T_{\text{surface}} - T_{\text{fluid}})$$

- This equation defines the convection coefficient, $h_c$
- Empirical and semi-empirical methods are used to find approximate $h_c$
- Formulations for liquids and gases work reasonably well for normal helium
- Several dimensionless parameters are commonly used
Reynolds number, $Re$

$$Re = \frac{\rho v D}{\mu}$$

where $\rho$ is fluid density,
$v$ is fluid velocity,
$D$ is some characteristic dimension
(like pipe inner diameter, or rod outer diameter)
and $\mu$ is viscosity.

- $Re$ provides a ratio of fluid inertia to viscosity
  - $Re < 2000$ in a tube is generally laminar flow
Nusselt number, $Nu$

\[ Nu = \frac{h_c D}{k_f} \]

where $h_c$ is convection coefficient, $D$ is some characteristic dimension, and $k_f$ is fluid thermal conductivity.

- Correlations of $Nu$ with other parameters have proven useful in evaluation of convection coefficients.
Prandtl number, Pr

Prandtl number is the ratio of kinematic viscosity to thermal diffusivity

\[ \text{Pr} = \frac{\mu / \rho}{k_f / \rho c_p} = \frac{c_p \mu}{k_f} \]

where \( \mu \) is viscosity, \( \rho \) is fluid density, \( c_p \) is fluid heat capacity, and \( k_f \) is fluid thermal conductivity.

- Prandtl number is a ratio of fluid properties
  - Relates velocity profile (kinematic viscosity is a sort of momentum diffusivity) to the temperature profile (thermal diffusivity) from the surface into the fluid
Correlations

\[ Nu = function(Re, Pr) \]

- For gases (0.6 < Pr < 0.8, for example Pr for helium gas = 0.66) in a long pipe with fully developed velocity profile

\[ Nu = 0.022 Re^{0.8} Pr^{0.5} \]
Convection summary

• Far too large a topic to cover here
• Many correlations, depending on whether free convection, laminar, or turbulent, fluid properties, etc.
• Entrance effects, surface and boundary layer effects
• Nevertheless, the classical correlations generally work well for normal helium
More about 2-phase helium flow

- “Baker plot” published in 1954 based on data for air and water and applied to oil and gas in pipes
- In 1960 - 1961, work at Los Alamos suggested the diagram could be applied to 2-phase hydrogen
- Papers published in 1985 and 1987 at the CEC described experimental results showing that the Baker plot does not apply to 2-phase helium flow
- For practical pressure drops and flow velocities with normal helium, one may assume that 2-phase helium flow is separated
- CEA Grenoble studies of 1.9 K 2-phase flow for CERN found that a vapor flow of about 5 m/sec begins to entrain liquid droplets
Plot from “Simultaneous Flow of Oil and Gas,” by Ovid Baker (1954) -- Do not use for helium!
Do not use Baker Plot for helium

Pool boiling and 2-phase flow

• Considerations for pool boiling systems
  – Control of liquid levels, long time constants, inventory management
  – Forced convection for warm-up and cool-down

• Two-phase flow
  – Liquid and vapor phases separate with any acceptably low pressure drop
  – Baker Plot does not apply!

**FIGURE 2.4**

Experimental Nucleate and Film Pool Boiling of Helium Compared with the Predictive Correlation of Kutateladze and Breen and Westwater
Helium boiling curves

• Note the transition from nucleate to film boiling at about 1 K delta-T

• Working delta-T for nucleate boiling such as in a helium subcooler (pressurized helium cooled by boiling helium) is ~0.1 K.

FIGURE 2.2
Experimental Nucleate and Film Pool Boiling of Nitrogen at One Atmosphere Compared with the Predictive Correlations of Kutateladze and Breen and Westwater

(The points of minimum film boiling are given by either the correlation of Lienhard & Wong or of Zuber, et al.)
Some simple analytic formulas for fluid flow

• In designing cryogenic piping, we generally have low pressure drop and may assume steady-state conditions for normal operational conditions
  – Emergency venting may be very dynamic, non-steady, but we often do conservative analyses assuming worst-case as if a steady-state condition

• Cryogenic liquids and vapors (except for Helium II) behave like normal liquids and gases

• Standard engineering pressure drop and heat transport equations may be used
A general form of the Bernoulli equation.

Pressure drop for isothermal turbulent flow of a compressible fluid in a circular or non-circular conduit, neglecting gravitational effects, is

\[
\int_{P_1}^{P_2} \frac{dP}{\rho} + \sum_i \left(\frac{v_i^2}{2} \frac{L}{R_h} f_i \right) + \sum_i \left(\frac{v_i^2}{2} e_{vi} \right) = 0
\]

where \(P_1\) is pressure in, \(P_2\) is pressure out,
\(\rho\) is fluid density, a function of pressure (temperature is assumed constant),
\(v\) is average fluid velocity within the \(i\)-th section of conduit or downstream of the \(i\)-th fitting,
\(L\) is conduit section length,
\(R_h\) is channel hydraulic radius (defined as flow area divided by wetted perimeter, which is \(D/4\) for circular pipes),
\(f\) is friction factor based on hydraulic radius,
and \(e_{vi}\) represents the resistance factor for fittings, elbows, tees, etc.

Considering just an increment of conduit where the pressure drop is dominated by pipe friction with constant flow area rather than fittings or cross-section changes, with pressure and density changing over the length \(dx\) of conduit, we have

\[
\frac{dP}{\rho} + \frac{v^2}{2} \frac{f}{R_h} \, dx = 0
\]

Substituting for \(v\) with \(v = \left(\frac{\dot{m}}{\rho A}\right)\) where \(\dot{m}\) is fluid mass flow and \(A\) is flow area (both are assumed constant),

\[
\frac{dP}{\rho} + \frac{1}{2} \left(\frac{\dot{m}}{\rho A}\right)^2 \frac{f}{R_h} \, dx = 0
\]

or,

\[
\rho dP + \frac{1}{2} \left(\frac{\dot{m}}{A}\right)^2 \frac{f}{R_h} \, dx = 0
\]
Thus, integration of the general equation, using the ideal gas approximation for a uniform section of conduit with large pressure drop, isothermal flow, and assuming a conservatively large constant friction factor, we have

$$\rho_{\text{avg}} \Delta P + \frac{1}{2} \left( \frac{m}{A} \right)^2 \frac{f}{R_h} L = 0$$

where $$\frac{(\rho_L + \rho_0)}{2} = \rho_{\text{avg}}$$

The point of this little derivation is to show that for sections of pipe with large enough pressure drop that density and velocity changes are significant, iterating pressure drop calculations to come up with a linear average density through the section of constant cross section gives a good estimate of pressure drop.
Pressure drop analysis, working formula for round pipes

This is a form of the D'Arcy-Weisbach formula. With pressure drop expressed as head loss, this is sometimes called simply the Darcy formula. (Note that delta-P changed signs here, to a positive number.)

Pressure drop for turbulent flow in a pipe is

$$\Delta P = \frac{\rho v^2}{2} \frac{4L}{D} f$$

where $\rho$ is average fluid density, $v$ is average fluid velocity, $L$ is pipe length, $D$ is pipe inner diameter, and $f$ is friction factor based on hydraulic radius (which is $D/4$ for circular pipes).

Substituting $\dot{m} = \rho v \left( \frac{\pi D^2}{4} \right)$ where $\dot{m}$ is mass flow

$$\Delta P = (0.811) \frac{\dot{m}^2}{\rho D^5} L \times 4 \times f$$
Summary of Formulas — continued

- Head loss and pressure drop through valves and fittings

Head loss through valves and fittings is generally given in terms of resistance coefficient $K$ which indicates static head loss through a valve in terms of “velocity head”, or, equivalent length in pipe diameters $L/D$ that will cause the same head loss as the valve.

From Darcy’s formula, head loss through a pipe is:

$$h_L = f \frac{L}{D} \frac{v^2}{2g}$$

and head loss through a valve is:

$$h_L = K \frac{v^2}{2g}$$

therefore:

$$K = f \frac{L}{D}$$

To eliminate needless duplication of formulas, the following are all given in terms of $K$. Whenever necessary, substitute $(fL/D)$ for $(K)$.

$$h_L = 0.003 28 \left( \frac{L}{D} \right) \frac{\rho v^2}{2g}$$

$$h_L = 1.760 \left( \frac{L}{D} \right) \frac{\rho w^2}{2g}$$

$$h_L = 0.000 408 \left( \frac{L}{D} \right) \frac{\rho w^2}{2g}$$

$$\Delta P = 0.000 0597 \left( \frac{L}{D} \right) \frac{\rho w^2}{2g}$$

$$\Delta P = 0.000 0519 \left( \frac{L}{D} \right) \frac{\mu B}{2g}$$

$$\Delta P = 0.000 0284 \left( \frac{L}{D} \right) \frac{\rho w^2}{2g}$$

- Head loss and pressure drop with laminar flow ($Re < 2000$) through valves; Darcy’s formula

$$h_L = 0.000 28 \left( \frac{L}{D} \right) \frac{\rho v^2}{2g}$$

$$\Delta P = 0.000 0257 \left( \frac{L}{D} \right) \frac{\rho w^2}{2g}$$

$$\Delta P = 0.000 0159 \left( \frac{L}{D} \right) \frac{\rho w^2}{2g}$$

$$\Delta P = 0.000 0059 \frac{L}{D} \frac{\rho w^2}{2g}$$

- Equivalent length correction for laminar flow with $Re < 1000$

$$\left( \frac{L}{D} \right) = \left( \frac{L}{D} \right)_e \frac{Re}{1000}$$

See pages 3-11 and A-30. Minimum $(L/D)_e$ = length of center line of actual flow path through valve or fitting. Subscript $e$ refers to equivalent length with $Re < 1000$. Subscript $i$ refers to equivalent length with $Re > 1000$.

- Discharge of fluid through valves, fittings, and pipe; Darcy’s formula

$$q = 0.0438 \frac{D^2}{K} \frac{\Delta P}{K}$$

$$Q = 19.85 \frac{D^2}{K} \frac{\Delta P}{K}$$

$$w = 0.0438 \frac{D^2}{K} \frac{\Delta P}{K}$$
For example from previous list

\[ \Delta P = 0.00000280 \frac{KW^2V}{d^4} \]

Where \( \Delta P \) is pressure drop in psi, \( V \) is the specific volume \((\text{in}^3/\text{lbm})\), \( K \) is the total resistance coefficient = \( fL/d \) so is dimensionless, \( W \) is the mass flow rate \((\text{lbm}/\text{hr})\), and \( d \) is the pipe inner diameter (in).

Compare to

\[ \Delta P = (0.811)\frac{m^2}{\rho D^5}L \times 4 \times f \]

from slide 34 -- no unit conversions, and a different definition of friction factor. Note! Some sources define \( f \) based on hydraulic radius and some on diameter, a factor 4 difference for pipes!
Example pressure drop analyses

- See Excel file C5_39_relief_calcs-TJP.xls
  - Illustrates relief venting calculation with stepwise reassessment of Re, friction factor, and fittings losses for constant mass flow

- See Excel file PressureDropLongPipeDec2008.xls
  - Pipe divided into sections for reassessment of properties
Heat capacity discussion

• The following plots illustrate the fact that the heat capacity of metals becomes vanishingly small at liquid helium temperatures
  – Cool-down to ~80 K is dominated by removal of heat from the solid materials
  – Cool-down below ~20 K is dominated by removal of heat from the helium
Heat capacity

Heat capacity per unit mass

Temperature (K)

Heat capacity (J/gK)

Stainless Steel

Helium
Heat capacity per unit volume

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<thead>
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<th>Temperature (K)</th>
<th>Stainless Steel</th>
<th>Helium</th>
</tr>
</thead>
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<td>0.0100</td>
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</tr>
<tr>
<td>1000</td>
<td>1.0000</td>
<td>10.0000</td>
</tr>
</tbody>
</table>

Heat capacity (J/cc-K)
Provisions for cool-down and warm-up

• Cool-down
  – Return vapor may block liquid supply flow in the same channel; a simple fill from the top or one end might not work. A cool-down vent and/or a bottom-fill port may be required.

• Warm-up
  – Flow will stratify. Local electric heat, a bottom vent port, or other feature to force heat down to the lower parts of a cold mass may be required.

  • The small “capillary” tubes connected to a manifold and providing helium to the bottoms of helium vessels in TESLA-style cryomodules were included primarily with warm-up in mind.
Subcooling in a liquid bath

Saturated liquid (in equilibrium with vapor at its surface) has a higher pressure below the surface by virtue of the weight of the liquid. This pressure provides the possibility for a slightly elevated temperature below the surface without boiling and/or some subcooling below the vapor pressure at that higher pressure. The pressure under a head of liquid is \( P = \rho gD \).
Pressure head of helium column

Thus, for helium at 4.5 K,

\[
\frac{P}{D} = \rho g = 0.125 \text{g/cm}^3 \times 980 \text{cm/sec}^2 = \\
122 \text{g/cm}^2 \text{sec}^2 = 0.12 \text{mbar/cm} = 12 \text{mbar/meter}
\]

Now, it would be good to understand the relationship of this elevated pressure below the surface to a new saturation temperature at that pressure. This new temperature, higher than the saturation temperature at the surface, tells us how much delta-T is available for heat transfer without the onset of boiling.
Clapeyron Equation

The Clapeyron equation, \( \frac{dP}{dT} = \frac{\Delta h_{12}}{T \Delta v_{12}} \), relates the rate of change of pressure with respect to temperature for any two phases of a pure substance in equilibrium, where \( \Delta h_{12} \) and \( \Delta v_{12} \) are enthalpy change and specific volume change during the phase change at temperature \( T \).
Basis for Clapeyron Equation

- The Clapeyron Equation comes from two substances in equilibrium over a phase transition satisfying $\Delta G = 0$
  - Where $G = \text{Gibbs free energy}$ is defined as
    \[ G = H - TS \]
Delta-T available under a head of liquid helium at 4.5 K

Returning again to helium, inserting values for $\Delta h_{12}$ and $\Delta v_{12}$ for helium at 4.5 K, we find $\frac{dP}{dT} = 1.11 \text{ mbar/mK}$. Thus, the $\Delta T$ available for subcooling or convection below the surface of a saturated bath of 4.5 K liquid helium is

$$\frac{12 \text{ mbar/meter}}{1.11 \text{ mbar/mK}} = 10.8 \text{ mK/meter}.$$

Note that 10.8 mK/meter, although a small number, implies a significant saturation temperature increase at, for example, 10 meters depth, for example down to an accelerator tunnel or experimental hall.
Delta-T available under a head of liquid helium at 2.0 K

If we follow a similar analysis for helium at 2.0 K, we find from the Clapeyron equation, \( \frac{dP}{dT} = 0.093 \text{ mbar/mK} \).

The somewhat denser liquid at 2.0 K results in \( \frac{P}{D} = 0.14 \text{ mbar/cm} \), resulting in the \( \Delta T \) available for subcooling or heat transport below the surface of a saturated bath of 2.0 K of 1.5 mK/cm or 150 mK/meter.

Note that 0.14 mbar/cm or 14 mbar/meter is a significant delta-P relative to the total pressure of 30 mbar at 2.0 K.
Saturated bath of liquid Argon

- Pure liquid argon is a common component of high energy physics detectors
  - Liquid argon calorimetry
  - Time Projection Chamber (TPC)
- In both cases, especially in the latter, purity and lack of bubbles are important for minimal noise and good signal
  - TPC depends on electron drift to a charged plate, and electron lifetime is critical
Looking at maximum heat flux via free convection (no boiling) for D0 liquid argon calorimeter back in 1988.

Analytical process:
Calculate available $\Delta T$ as a function of depth.
Given $\Delta T$, calculate free convective heat flux for various geometries and orientations.
Liquid Argon TPC

• Heat input from supports and other heat sources to the liquid argon bath should not produce bubbles
  – Heat transport by free convection without nucleation of bubbles
  – Free convection driven by liquid density differences, due to temperature differences
  – Temperature differences limited by saturation temperature at depth
Total pressure vs vapor pressure

• Consider a glass of water open to normal air and at the same temperature as the air
  – 100% humidity in air ➔ water vapor in air in equilibrium with liquid in the glass
  – Now warm the water slightly ➔ saturation pressure of liquid higher than vapor pressure of water in air
  – Why does it not boil?

• Answer: bubbles must overcome total pressure, not just vapor pressure of the water in air

• Typically our cryogenic systems involve pure helium and nitrogen, so vapor pressure = total P
Inhibit boiling with total pressure

• One could accomplish the same sort of thing for a liquid argon bath
• Pressurize the system with helium gas
• Total pressure much higher than vapor pressure of LAr
• (Problem for detectors: argon purity with some helium dissolving in LAr)
References

• Ovid Baker, "Design of Pipelines for the Simultaneous Flow of Oil and Gas," Oil and Gas Journal (July 26, 1954) p. 185-195
• Crane Technical Paper #410 “Flow of Fluids through Valves, Fittings, and Pipes”