LECTURE 2 (#3) RADIATION FUNDAMENTALS

NOTE TAKER

1/16

GOALS: UNDERSTAND UNITS OF ACTIVITY, DOSE, SCALING etc

- NOTE: ALL ELEMENTS ABOVE 83 ARE UNSTABLE → THIS IS RADIOACTIVITY
- M. CURIE + P. CURIE ISOLATED RADIUM FROM PITCH BLEND.

THE QUANTITY OF 1 GRAM OF PURE RADIUM IS KNOWN AS THE CURIE

- RADIOACTIVE DECAY FOLLOWS A PREDICTABLE RANDOMNESS
- NATURE WANTS TO REDUCE TO LOWEST ENERGY STATE.
- THE INITIATION OF RADIOACTIVITY IS COMPLETELY RANDOM: IT IS A RESULT OF PROBABILITY + PARAMETERS SET FORTH BY QUANTUM MECHANICS. THE "REACTION" MUST BE ENERGETICALLY FAVORABLE, MEANING WHEN IT OCCURS ENERGY IS RELEASED IN THE PROCESS (ONE FORM BEING THE RADIATION WHICH WE ARE STUDYING)

THE PROBABILITY OF WITNESSING A SINGLE DECAY IS PROPORTIONAL TO THE AMOUNT OF THE RADIOACTIVE SUBSTANCE PRESENT.

UNIQUE DECAY TIMES FOR EACH ISOTOPE CAN BE EXPRESSED ~~AS~~ IN TERMS OF A HALF LIFE. THE AMOUNT OF TIME NEEDED TO REDUCE THE ISOTOPES ABUNDANCE BY 50%.

LET'S TAKE ^{226}Ra FOR EXAMPLE. $T_{1/2}$ 1700 yrs.

START WITH 1-MILLION IN yr 2000. IN YEAR 3700 500 THOUSAND WILL REMAIN

TIDBIT: SCOOP UP A CUP OF DIRT, TAKE ~ ACRE, 6" DEEP, ISOLATE ALL RADIUM ≈ 1 Ci.

TIDBIT: SCOOP UP SOME DIRT, WAIT FOR A Ra^{226} DECAY. THIS IS A MOMENTOUS EVENT. ALL RADIUM WAS CREATED WITH OUR SOLAR SYSTEM ~4.5 BILLION YEARS AGO. IT'S HELD ALL THAT ENERGY SINCE THEN [illegible]

IF AT ANY TIME WE KNOW THE QUANTITY OF A GIVEN RADIOACTIVE ELEMENT (SPECIFICALLY ISOTOPE) DENOTED N_0, AND ITS HALF-LIFE OR "DECAY CONSTANT" λ ONE CAN DETERMINE THE AMOUNT N FOR ALL REMAINING TIMES, t.

THIS IS MATHAMATICALLY EXPRESSED ~~AS~~ BY RELATING THE RATE OF DECAY

$$dN/dt$$

TO THE INITIAL QUANTITY

$$dN/dt = -\lambda N_0 = \underline{\text{ACTIVITY}}$$

THE SOLUTION TO THIS DIFFERENTIAL EQUATION IS THE DECAYING EXPONETIAL

$$N = N_0 e^{-\lambda t}$$

TIME

DECAY CONSTANT

QUANTITY AT $t=0$

QUANTITY @ TIME $= t$

WE CAN RELATE HALF-LIFE TO THE DECAY CONSTANT, λ, AS

$$T_{1/2} = \frac{\ln(2)}{\lambda} = \frac{0.693}{\lambda}$$

$\ln(2) \equiv$ NATURAL LOG OF 2 $= 0.693$

- ~~SEVERAL~~ USEFUL VARIATIONS OF

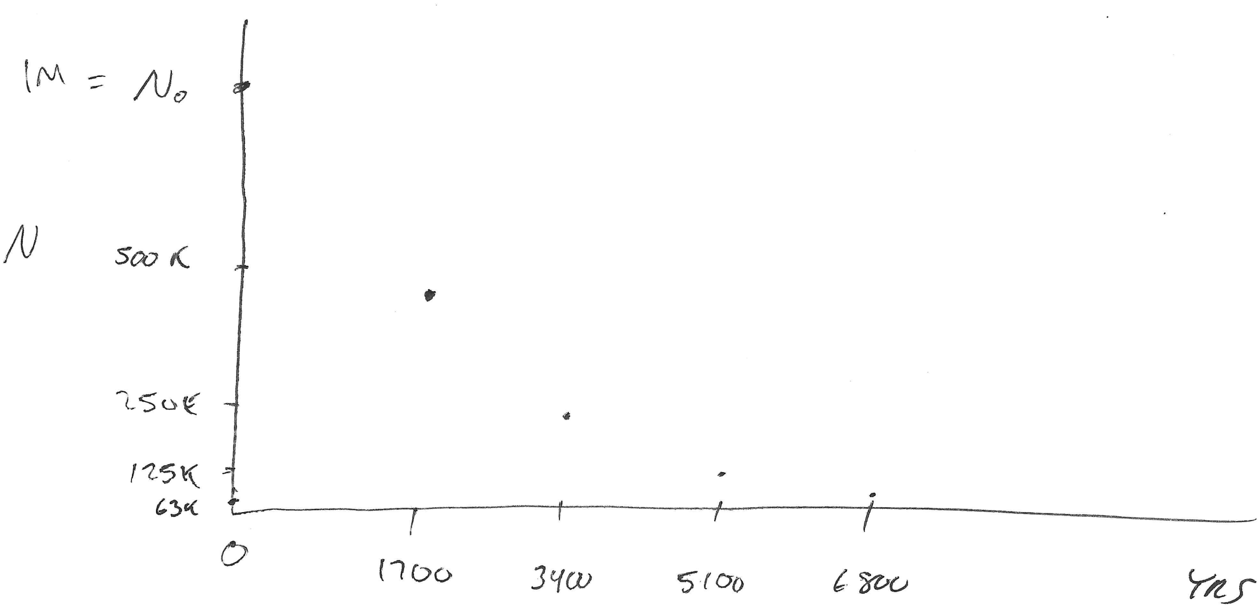
$$N(t) = N_0 e^{-0.693t/T_{1/2}}$$

ANOTHER APPROACH TO DETERMINING REMAINING QUANTITY is

$$N(t) = N_0 \left(\tfrac{1}{2}\right)^{t/T_{1/2}}$$

t ← elapsed time

$T_{1/2}$ ← half life



IN GENERAL TO INDICATE A SPECIFIC NUCLEAR SPECIES, OR NUCLIDE, (RADIO-NUCLIDE IF RADIOACTIVE) IN THE FORM

TOTAL # OF NUCLEONS, VARIES WITH ISOTOPE → A

$$ {}^{A}_{Z}X_{N} $$

X ← CHEMICAL SYMBOL

Z ← PROTON # (WHICH DEFINES CHEMICAL ELEMENT, SO IS REDUNDANT, TYPICALLY NOT WRITTEN)

N ← NEUTRON # $(A-Z)$ ← THUS IS REDUNDANT ALSO, VERY INFREQUENTLY WRITTEN

U, ${}^{238}_{92}U$ $\quad 238-92=146 \quad$ ${}^{238}_{92}U_{146}$

VERY USEFUL WHEN TRYING TO BALANCE A DECAY OR REACTION PROCESS

$$ {}^{60}_{27}Co \xrightarrow[5.27\,\text{years}]{\beta} {}^{60}_{28}Ni $$

~~Quick question, what does this tell us about ~~DECAY~~ products?~~

EXAMPLE:

$$ {}^{210}_{84}Po \xrightarrow[138.4\,d]{\alpha} {}^{206}_{82}Pb $$

$\alpha = He$ ${}^{4}_{2}He_{2}$

MORE COMPLICATED EXAMPLE OF

* — ^{60}Co — *

$$ {}^{60}_{27}Co \xrightarrow[5.27\,yr]{\beta^-} {}^{60}_{28}Ni $$

WHAT DOES THIS TELL US ABOUT THE DECAY?

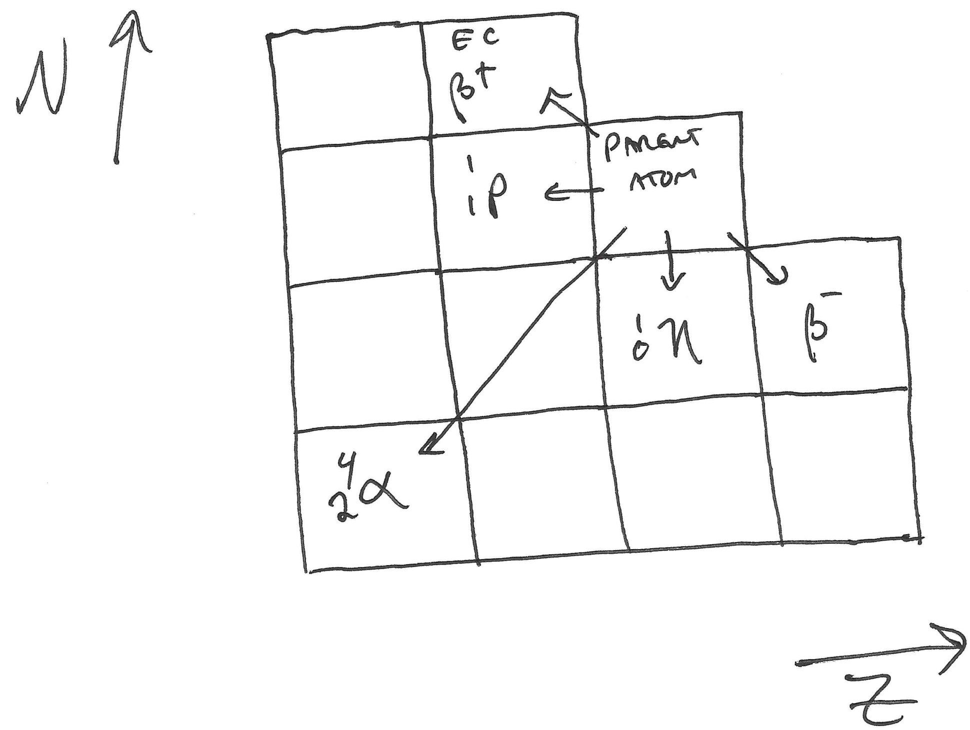
3.2.A

α

β

γ, X-RAY

n



- NUCLIDES WITH SAME Z, DIFFERENT N ARE CALLED ISOTOPES

LESS COMMON:

- A SEQUENCE OF NUCLIDES W/ SAME N BUT DIFFERENT Z ARE ISOTONES

- NUCLIDES W SAME <u>A</u> BUT DIFFERING N+Z ARE CALLED ISOBARS

$^{60}_{27}Co$ + $^{60}_{28}Ni$ ARE ISOBARS

DISCUSS QUANTITY.

HOW DO WE QUANTIFY HOW MUCH RADIOACTIVE "STUFF" WE HAVE? BY WEIGHT? BY NUMBER OF ATOMS?

SURE BUT,

STANDARD IS ~~TO~~ DESCRIBE AMOUT OF RADIOACTIVE MATERIAL BY ITS ACTIVITY, ~~BECAUSE~~ ~~of~~ RATE ~~PHYSICAL~~ DESCRIPTIONS OF MASS OR # OF ATOMS MIGHT BE MISLEADING.

BASIC UNIT OF QUANTITY IS "ACTIVITY", SIMPLY # OF UNSTABLE ATOMS DISINTIGRATING, DECAYING, TRANSFORMATIONS PER UNIT TIME. dN/dt.

THE FIRST UNIT WAS THE CURIE, WHICH IS DEFINED AS THE NUMBER OF ~~226~~ RA ATOMS DECAYING PER SECOND IN 1-GRAM OF ^{226}Ra.

$$3.7 \times 10^{10} \text{ DIST/SECOND} \quad dps$$

SO IT SHOULD THEN MAKE SENSE THAT GEIGER COUNTERS MEASURE IN DIST. PER. MIN. <JOKINGLY>

3.7×10^{10} dps → 37 Billion dps OR

LESS COMMON 37 GHz 37,000 MHz etc.

~~ANOTHER~~ SENSE OF SCALE

- EXEMPT QUANITY BUTTON SOURCES ARE ~ 1 μCi

  SMOKE DETECTORS CONTAIN ABOUT ~~[illegible]~~ 1 μCi OF ^{241}Am

  ↳ ~ μR/hr

- SOURCE I FOUND AT 13 WAS 100 mCi 50R/hr

- ^{60}Co IRR 130,000 Ci 10 MR/hr

ANOTHER UNIT OF ACTIVITY IS THE BECQUEREL Bq

$$1 \text{ Bq} = 1 \text{ dps.}$$

MUCH MORE REASONABLE FOR LOW ACTIVITY, ENVIRONMENTAL SAMPLES.

$$1 \mu Ci = 3.7 \times 10^4 \text{ dps} = 37 \text{ kBq}$$

^{60}Co DECAY EXAMPLE

Q: HOW LONG DOES IT TAKE FOR AN ISOTOPE NOT TO BE RADIOACTIVE?

A: ∞, BUT THIS IS UNSATISFACTORY TO REGULATORS, SO A NUMBER OF ABOUT 10 HALF LIVES IS TYPICALLY QUOTED.

$$A(t) = A_0 \left(\tfrac{1}{2}\right)^{10} = A_0 (\tfrac{1}{2})(\tfrac{1}{2})(\tfrac{1}{2})(\tfrac{1}{2})(\tfrac{1}{2}) \cdots (\tfrac{1}{2}) = N_0 \frac{1}{1024}$$

$5.27 \times 10\ T_{1/2}$'s $= 527$ years $\qquad \approx$ ~~1%~~ $0.01\%\ N_0$

IN 527 years

$1\ \mu Ci \longrightarrow \sim 100\ pCi$ OR $\sim$ ~~4~~ dis per second

pretty dead.

OK HOW ABOUT OUR 100 kCi ^{60}Co IRRADIATOR?

100,000 Ci IN 527 years $\sim$ 100 Ci

— STILL EXTREMELY ~~RAD~~ ACTIVE —

SPECIFIC ACTIVITY

PROB OF RADIOACTIVE DECAY FOR A GIVEN RADIONUCLIDE IS A UNIQUE PHYSICAL PROPERTY. THE NUMBER OF DECAYS THAT OCCUR IN A GIVEN TIME FOR A SPECIFIC NUMBER OF ATOMS OF THAT RADIONUCLIDE IS ALSO A FIXED QUANTITY.

SPECIFIC ACTIVITY IS DEFINED AS THE ACTIVITY PER QUANTITY OF ATOMS:

$$Bq/gram \quad \text{or} \quad Ci/Gram$$

$$-\frac{dN}{dt} = \lambda N$$

MASS OF RADIONUCLIDE:

$$\frac{N}{N_A}[mol] \times m[g/mole]$$

N_A ← AVOGADRO'S NUMBER

SPECIFIC ACTIVITY DEFINED AS

$$a[Bq/g] = \frac{\lambda N}{mN/N_A} = \frac{\lambda N_A}{m}$$

OR

$$a = \frac{N_A \ln(2)}{T_{1/2}\, m}$$

~~LETS TAKE RADIUM 226 AS AN EXAMPLE~~

IN GENERAL GENERIC FORMULA: $a[Bq/g] = \dfrac{4.17 \times 10^{23} [mol^{-1}]}{T_{1/2} \times 365 \times 24 \times 60 \times 60 [s/yr] \times m} \simeq \dfrac{1.32 \times 10^{16} \, mol^{-1} s^{-1} yr}{T_{1/2}[yr] \cdot m[g \cdot mol^{-1}]}$

226 RA EXAMPLE

$$a[Bq/g] = \frac{1.32 \times 10^{16}}{1600 \, yr \cdot 226} \simeq 3.7 \times 10^{10} \, Bq/g \quad \text{OR } 1 \, Ci$$

NOTE: ^{226}Ra is no longer used, processed or available — YOU CANNOT GET A LICENSE PERMIT (OUTSIDE OF ANTIQUITIES)

SPECIFIC ACTIVITY VALUES FOR SELECTED ISOTOPES.

^{226}Ra 1

^{3}H Tritium 9.7×10^{3} [Ci/g]

^{222}Rn 1.5×10^{5}

Ar^{41} 4.3×10^{7} — a gas.

^{60}Co 1.1×10^{3}

U natural ~~5~~ 7×10^{-7} by weight

~~^{235}U (0.7% NAT ABUND.) (7.1×10^{-7})~~

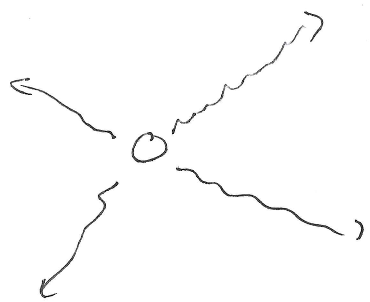
~~[illegible]~~

DU 5.0×10^{-7} Ci/g NAT U or DU LEGAL TO OWN UP TO 15 lbs

LETS DISCUSS INVERSE SQUARE LAW

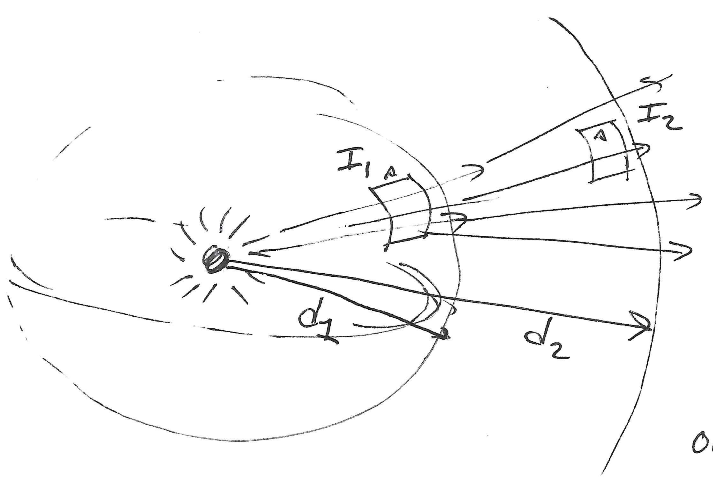
Q: WHERE DOES THE EMITTER RADIATION GO?

A: EVERYWHERE & ANY WHERE. ALL ~~THE~~ LAWS OF PHYSICS ARE UP HELD,



THE AGGREGATE OF RADIATION (UNSHIELDED) IS NOMINALLY ISOTROPIC.

THE FLUX, FLUENCE, RATE / UNIT AREA ("INTENSITY") FOLLOWS THE $1/r^2$ OR INVERSE SQUARE LAW.



$$\text{INTENSITY} \propto \frac{1}{\text{DISTANCE}^2}$$

$$I \propto \frac{1}{d^2}$$

$$\frac{I_1}{I_2} = \frac{d_2^2}{d_1^2}$$

OR

$$I_1 d_1^2 = I_2 d_2^2$$

^{60}Co DECAY EXAMPLE + GEOMETRY

$t_0 =$ DEC 7, 2011 ACTIVITY $1 \mu Ci$

$\Delta t =$ DEC 7, 2011 $\rightarrow$ AUG 30, 2017

5 years, 8 month, 30 days 5 yrs

OR

2,100 days (181,440,000 seconds)

^{60}Co $T_{1/2} = 5.27$ years $\Rightarrow$ 1,924 days

$$A = A_0 e^{-\lambda t}$$

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1924 \text{ days}} = 3.6 \times 10^{-4} \text{ day}^{-1}$$

3.6×10^{-4} day^{-1}

$$A = 1 \mu Ci \; e^{-\lambda t} = 1 \mu Ci \; e^{-(3.6 \times 10^{-4} \text{ day}^{-1})(2{,}100 \text{ days})}$$

$$= 1 \mu Ci \; e^{-0.756}$$

$$= (1 \mu Ci)(0.47) =$$

$$= 470 \text{ nCi}$$

TODAY'S ACTIVITY 470 nCi

10/10

OK, WHAT IS THAT IN TERMS OF DIST. PER SEC?

$$\frac{470\times10^{-9}\,Ci}{1\,Ci}\left|\,\frac{3.7\times10^{10}\,disint}{sec}\right. \approx 17{,}390\ dis/sec$$

$$\boxed{\approx 17.4\ kBq\ \cancel{/sec}}$$

CAN WE VERIFY THIS WITH A COUNTER?

2π ~~Sensitivity~~ Geometry Capture

EXPECT ~(8.7×10^3) 8,700 EVENTS GOING INTO DETECTOR PER SECOND. OR 520,000 PER MINUTE

BUT COUNTER READS IN COUNTS PER MINUTE.

WITH SOURCE UP AGAINST ~~DETECTOR~~ IT READS 10K CPM ! WHY THE DISCREPENCY?

$\frac{10}{520} = 0.02$ 2% EFFICIENCY

SERIAL RADIOACTIVE DECAY

TWO ~~ISOTOP~~ RADIONUCLIDES N_1 + N_2, $N_1 \rightarrow N_2$, $T_{1/2}$ T_1 + T_2 ACTIVITY A_1 + A_2

1. SECULAR EQUILIBRIUM $(T_1 >> T_2)$

~~RELATIVELY~~ LONG LIVED PARENT DECAYS INTO RELATIVELY SHORT LIVED DAUGHTER

- ASSUME PERIODS OF TIME SHORTER THAN T_1 SO ~~E~~ N_1 DOESNT APPRECIABLY CHANGE.
- SO A_1 CAN BE CONSIDERED CONSTANT.
- TOTAL ACTIVITY IS $A_1 + A_2$
- RATE OF CHANGE ~~of ACTIVITY CHANGE~~, OF DAUGHTER ATOMS N_2 PER UNIT TIME $\frac{dN_2}{dt}$ IS EQUAL TO THE RATE AT WHICH THEY ARE PRODUCED, A_1, MINUS THEIR RATE OF DECAY $\lambda_2 N_2$

$$\frac{dN_2}{dt} = A_1 - \lambda_2 N_2$$

SOLVE FOR N_2.

WE FIRST SEPARATE VARIABLES BY WRITING

$$\frac{dN_2}{A_1 - \lambda_2 N_2} = dt$$

WHERE A_1 CAN BE CONSIDERED A CONSTANT.

INTRODUCE VARIABLE $u = A_1 - \lambda_2 N_2$

WE HAVE $du = -\lambda_2 \, dN_2$

RE-WRITE

$$\frac{du}{u} = -\lambda_2 \, dt$$

INTEGRATING GIVES

$$\ln(A_1 - \lambda_2 N_2) = -\lambda_2 t + C$$

C IS A CONSTANT WE CAN DETERMINE

IF $N_{2,0}$ REPRESENTS # ATOMS N_2 @ $t=0$

$$C = \ln(A_1 - \lambda_2 N_{2,0})$$

$$\ln \frac{A_1 - \lambda_2 N_2}{A_1 - \lambda_2 N_{2,0}} = -\lambda_2 t \quad \text{OR}$$

$$A_1 - \lambda_2 N_2 = (A_1 - \lambda_2 N_{2,0}) e^{-\lambda_2 t}$$

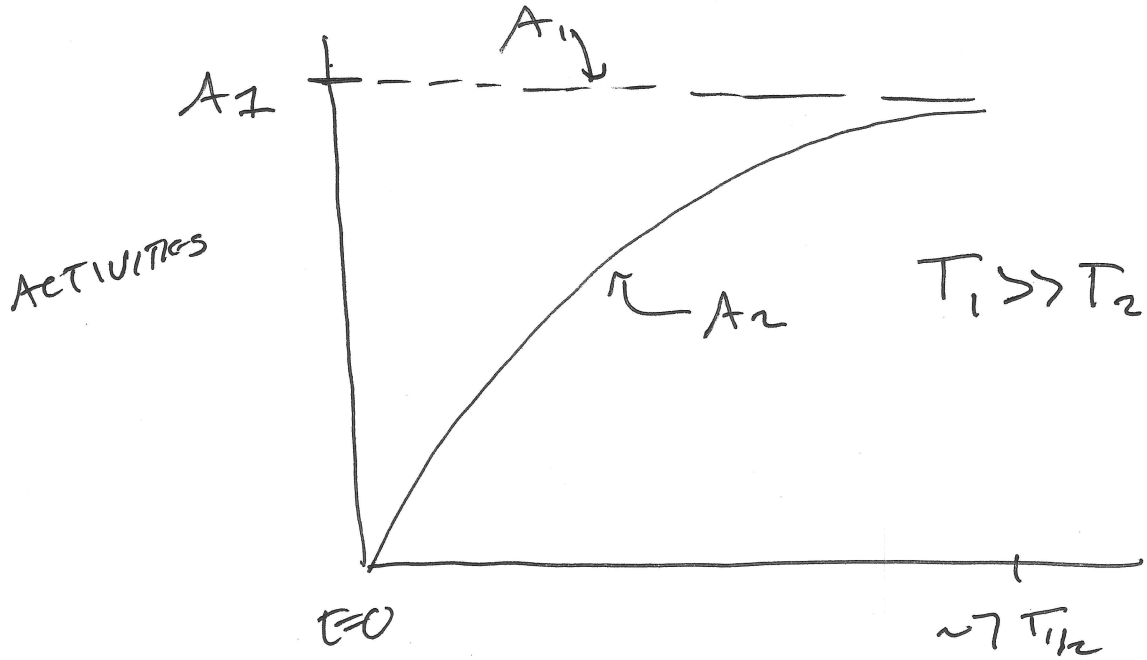
SINCE $\lambda_2 N_2 = A_2$ "ACTIVITY OF NUCLIDE N_2"

$\lambda_2 N_{2,0} = A_{2,0}$ IS INITIAL ACTIVITY

$$A_2 = A_1 (1 - e^{-\lambda_2 t}) + A_{2,0} e^{-\lambda_2 t}$$

ASSUME @ $t=0$ PURE SAMPLE OF PARENT NUCLIDE, i.e. $t=0$ $A_{2,0}=0$

THEN A_2 BUILDS UP



$\sim 7\,T_{1/2}$ $\quad t \gtrsim 7\,T_{1/2,2}$ $\quad e^{-\lambda_2 t} \ll 1$

AND $\quad A_2 = A_1\left(1-\underbrace{\cancel{e^{-\lambda_2 t}}}_{\sim 0}\right) + \underbrace{\cancel{A_{2,0}}}_{0}\, e^{-\lambda_2 t}$

$$A_2 = A_1$$

SECULAR EQUILIBRIUM

TOTAL ACTIVITY OF SAMPLE IS $2A_1$ OR $2A_2$

IN TERMS OF # ATOMS

$$\lambda_1 N_1 = \lambda_2 N_2$$

GENERAL CASE, NO RESTRICTIONS ON RELATIVE MAGNATUDES OF T_1 & T_2

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

WITH INITIAL CONDITION OF $N_{2,0} = 0$, SOLUTION IS

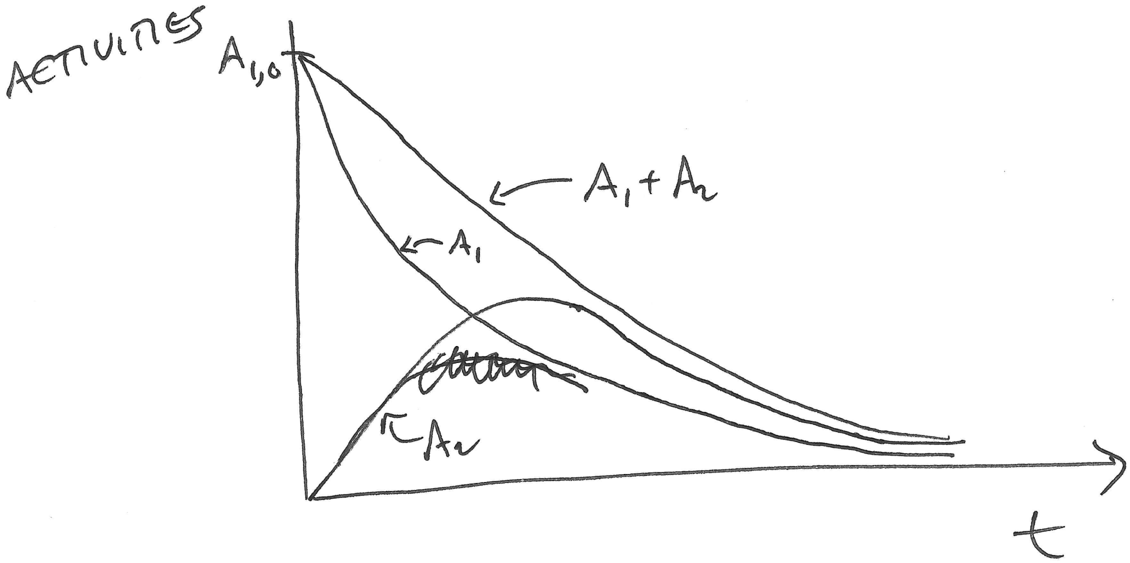
$$N_2 = \frac{\lambda_1 N_{1,0}}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

~~TRANSIENT EQUILIBRIUM,~~ CONSIDER ~~$N_{2,0} = 0$ @ $t=0$~~

~~AND~~

NO EQUILIBRIUM $T_1 < T_2$

5/5



EXAMPLE

10^{10} Bq OF PURE ^{90}Sr @ $t=0$

HOW LONG WILL IT TAKE FOR ^{90}Sr + ^{90}Y TO BUILD UP TO 1.5×10^{10} Bq

SERIES OF DECAYS, MOST GENERAL FORM

$$dN_i = \lambda_{i-1} N_{i-1}\, dt - \lambda_i N_i\, dt$$

STARTING FROM PURE SAMPLE OF 1 PARENT WITH N_0, ACTIVITY OF n^{TH} MEMBER OF THE DECAY CHAIN IS GIVEN BY

$$A_n = N_0 \sum_{i=1}^{n} c_i e^{-\lambda_i t} = N_0 \left(c_1 e^{-\lambda_1 t} + c_2 e^{-\lambda_2 t} + \cdots + c_n e^{-\lambda_n t} \right)$$

WHERE $$c_m = \prod_{i=1}^{n} \lambda_i \Big/ \prod_{i=1}^{n}{}' (\lambda_i - \lambda_m) = \frac{\lambda_1 \lambda_2 \lambda_3 \cdots \lambda_n}{(\lambda_1 - \lambda_m)(\lambda_2 - \lambda_m)\cdots(\lambda_n - \lambda_m)}$$

IN SPECIAL CASE OF SECULAR EQUILIBRIUM

$$\lambda_1 N_1 = \lambda_2 N_2 = \cdots = \lambda_n N_n$$