Proton and Ion Linear Accelerators

Yuri Batygin,[1] Sergey Kurennoy,[1] Dmitry Gorelov,[1] Vyacheslav Yakovlev [2], Tyler Fronk[3]

[1]Los Alamos National Laboratory
[2]Fermi National Accelerator Laboratory
[3]Sandia National Laboratories

U.S. Particle Accelerator School

June 7 – July 2, 2021

Proton and Ion Linear Accelerators

13. RF accelerating structures, Lecture 2

Vyacheslav Yakovlev, Fermilab

U.S. Particle Accelerator School (USPAS)

Education in Beam Physics and Accelerator Technology

June 22, 2021

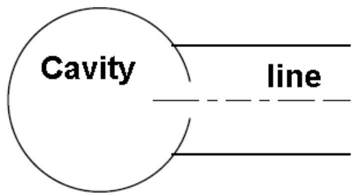
Chapter 3 (cont.).

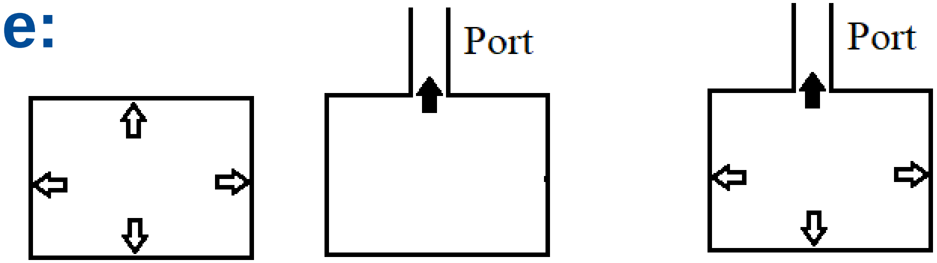
RF Cavities for Accelerators.

a. Cavity excitation by the input port;
b. Cavity excitation by the beam;
c. High-Order Modes (HOMs);
d. Types of the cavities and their application
e. Tools for cavity simulations

The cavity coupler to the line:

Let's consider the cavity coupled to the feeding line.





Wall loss: $Q_0 = \omega_0 U / P_0$

Port radiation: $Q_{ext} = \omega_0 U / P_{ext}$

Wall loss and port radiation: $Q_L = \omega_0 U / (P_0 + P_{ext})$

If the incident wave is zero (i.e., if the RF source is off), the loss in the cavity is a sum of the wall P_0 loss and the loss coasted by the radiation to the line P_{ext}:

$$P_{tot} = P_0 + P_{ext}$$

$$P_0 = \frac{V^2}{R/Q \cdot Q_0}, \quad P_{ext} = \frac{V^2}{R/Q \cdot Q_{ext}}$$

where we have defined an external quality factor associated with an input coupler. Such *Q* factors can be identified with all external ports on the cavity: input coupler, RF probe, HOM couplers, beam pipes, etc. The total power loss can be associated with the loaded *Q* factor, which is

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext1}} + \frac{1}{Q_{ext2}} + \ldots$$

because $P_{tot} = P_0 + P_{ext1} + P_{ext2} \ldots = \frac{V^2}{R/Q \cdot Q_L}$

*Details are in Appendix 7

Fermilab

Coupling parameter:

For each port a coupling parameter β can be defined as

$\beta \equiv \frac{Q_0}{Q_{ext}}$ and, therefore, $\frac{1}{Q_L} = \frac{1+\beta}{Q_0}$

It tells us how strongly the couplers interact with the cavity. Large implies that the power leaking out of the coupler is large compared to the power dissipated in the cavity walls:

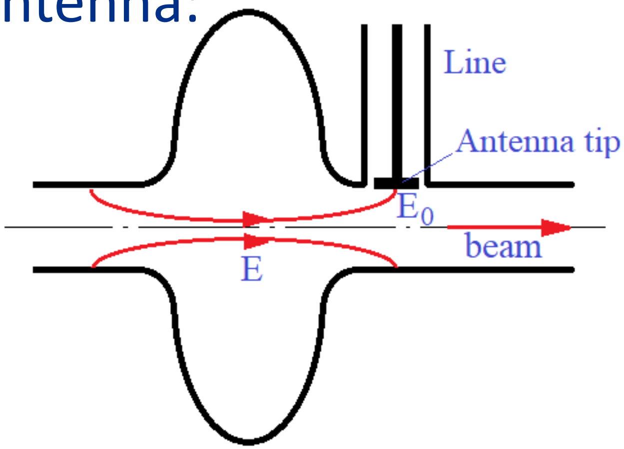
$$P_{ext} = \frac{V^2}{R/Q \cdot Q_{ext}} = \frac{V^2}{R/Q \cdot Q_0} \cdot \beta = \beta P_0$$

In order to maintain the cavity voltage, the RF source should compensate both wall loss and radiation to the line. Therefore, the RF source should deliver the power to the cavity which is

$$P_{tot} = P_{forw} + P_0 = (\beta + 1) P_0$$

The cavity coupler to the line

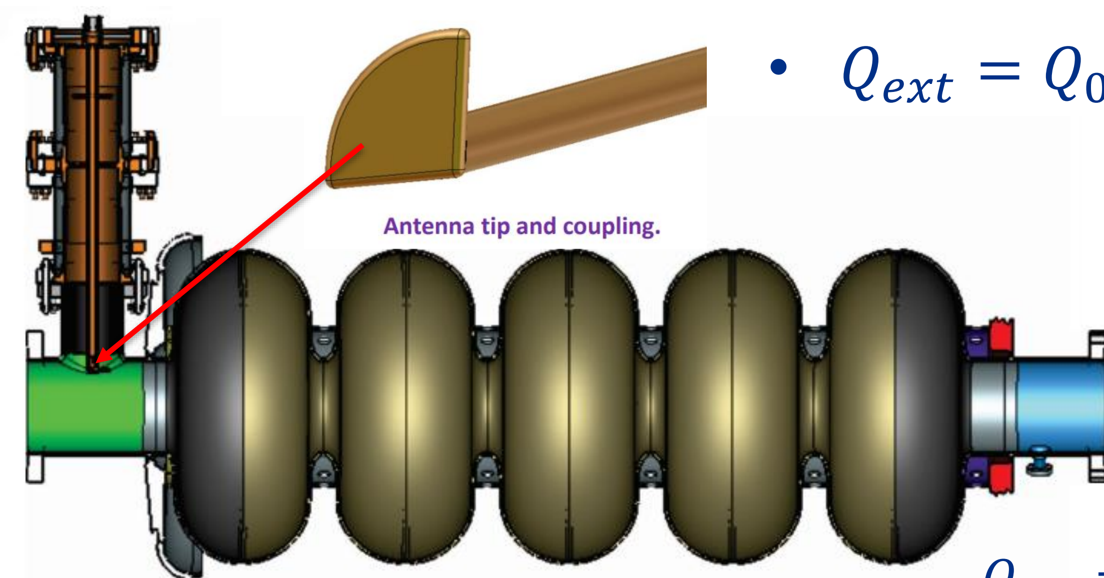
Antenna:



- Antenna tip square is S;
- The line has impedance Z;
- Electric field on the tip is E_0
- Antenna tip has a charge q:

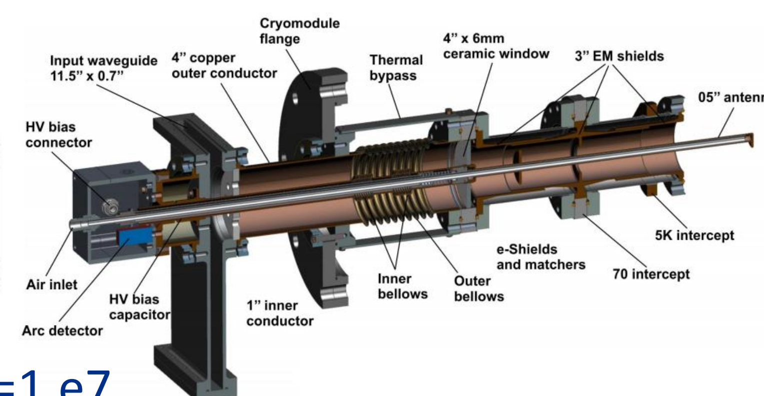
$$q = E_0 \varepsilon_0 S \rightarrow I = \omega q = \omega E_0 \varepsilon_0 S = kSE_0/Z_0;$$

- Radiated power $P_{ext} = \frac{1}{2} Z I^2$;
- Loss in the cavity $P_0 = \frac{V^2}{R/Q \cdot Q_0}$
- $Q_{ext} = Q_0 \frac{P_0}{P_{ext}} = 2 \frac{Z_0^2}{Z \cdot R/Q} \cdot \left(\frac{V}{kSE_0}\right)^2$



Antenna tip and coupling.

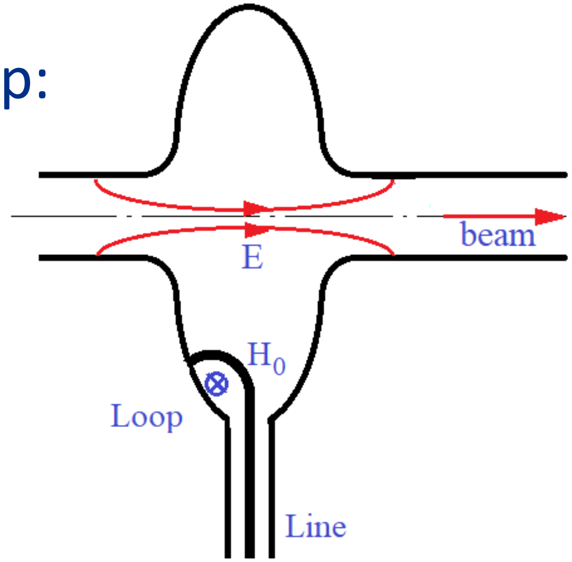
Structure of 650 MHz coupler, new design



$Q_{ext} = 1.e7$

The cavity coupler to the line

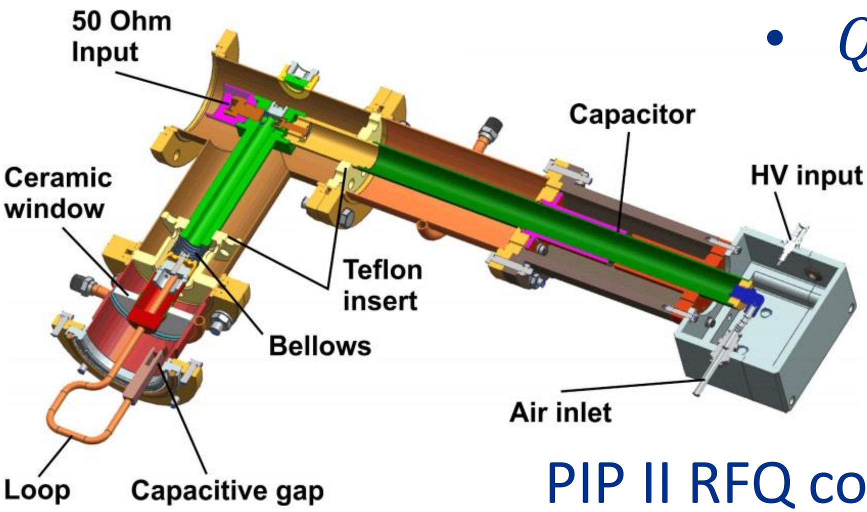
Loop:



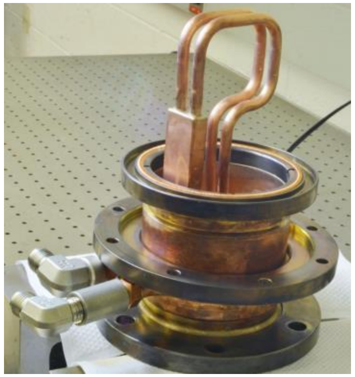
- Loop square is S;
- The line has impedance Z;
- Magnetic field on the loop is H_0
- Voltage induced on the loop U:

$U = \omega H_0 \mu_0 S;$ ⬅ $\mathrm{rot}\,\vec{E} = -i\omega\mu\vec{H}$

- Radiated power $P_{ext} = \frac{U^2}{2Z}$;
- Loss in the cavity $P_0 = \frac{V^2}{R/Q \cdot Q_0}$
- $Q_{ext} = Q_0 \frac{P_0}{P_{ext}} = 2\frac{Z}{R/Q} \cdot \left(\frac{V}{kSH_0Z_0}\right)^2$

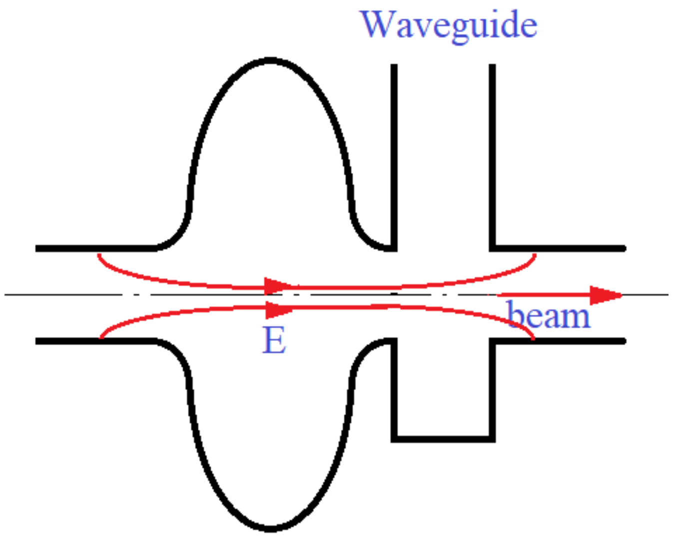


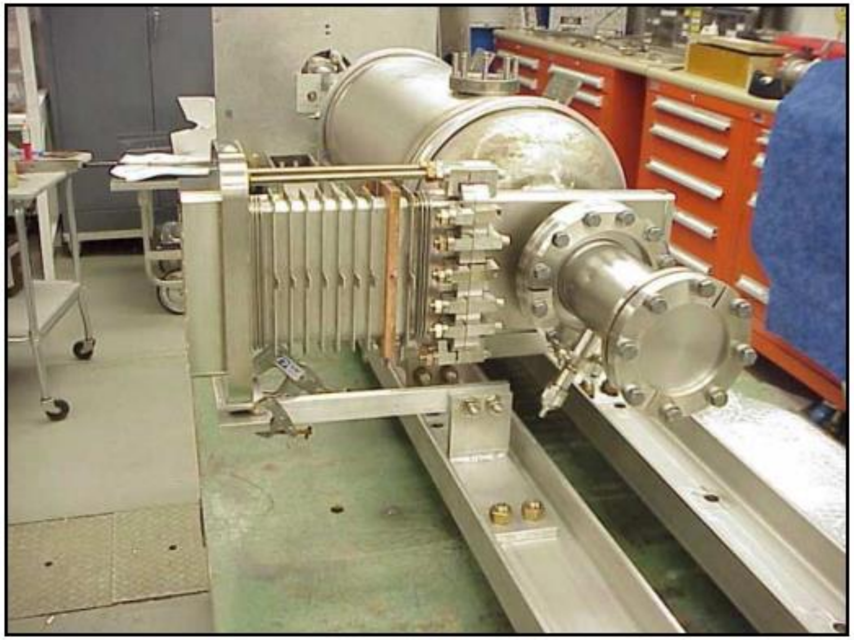
PIP II RFQ coupler



The cavity coupler to the line

Waveguide:





Waveguide on Cavity String

CEBAF couplers

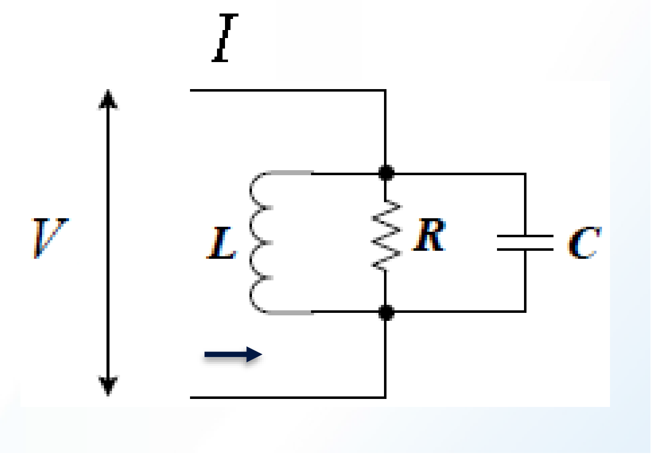
Cavity excited by the beam (Appendix 6):

- If the cavity is excited by the beam with the *average* current I having the bunches separated by the length equal to integer number of RF periods, i.e., in resonance, the excited cavity voltage provides maximal deceleration. The beam power loss is equal to the cavity loss, i.e., radiation and wall loss:

$$-VI = \frac{V^2}{\left(\frac{R}{Q}\right)Q_L} \qquad (1)$$

or

$$V = -I\left(\frac{R}{Q}\right)Q_L$$



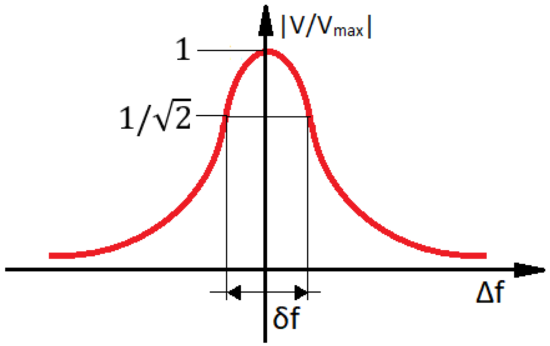
$L=(R/Q)/2\omega;$
$C=2/\omega(R/Q);$
$R=(R/Q)Q_L/2;$
$\omega=2\pi f$

- The cavity excited by the beam off the resonance, the voltage is $V \approx -\frac{I\left(\frac{R}{Q}\right)Q_L}{1+iQ_L\frac{2\Delta f}{f}}$

where Δf is the distance between the beam spectrum line and the cavity resonance frequency f.

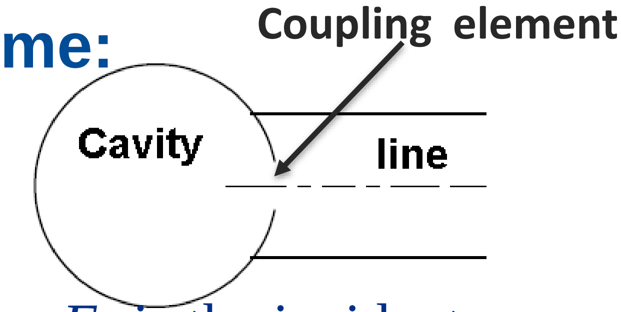
- Cavity bandwidth:

$\delta f = f/Q_L;$



Fermilab

Acceleration cavity operating in CW regime:



Energy conservation law:

$P_0 = P_{backward} + P_{diss} + P_{beam}$

$P_0 = E_0^2/(2Z)$, Z is the transmission line impedance; E_0 is the incident wave voltage in the transmission line.

- $P_{backward} = (E_0 - E_{rad})^2/(2Z)$, E_{rad} is the voltage of wave radiated from the cavity to the transmission line.

$\beta = P_{rad}/P_{diss}$, $P_{rad} = E_{rad}^2/(2Z) = \beta P_{diss} = \beta V^2/(Q_0 \cdot R/Q) = V^2/(Q_{ext} \cdot R/Q)$;

$P_{beam} = VI.$

- If the line is matched to the transmission line, $P_{backward} = 0$, $E_0 = E_{rad}$

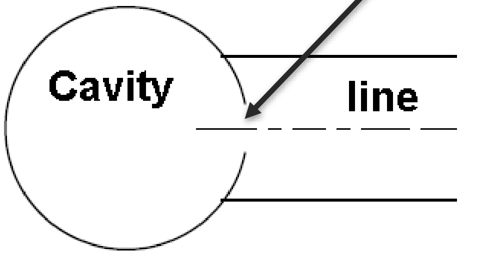
and therefore, $P_{rad} = P_0 = P_{diss} + P_{beam}$, or $\beta_{opt} V^2/(Q_0 \cdot R/Q) = V^2/(Q_0 \cdot R/Q) + VI$ and $\beta_{opt} = I \cdot Q_0 \cdot R/Q/V + 1$. For $\beta_{opt} >> 1$ $\beta_{opt} \approx I \cdot Q_0 \cdot R/Q/V$ and

$$Q_L = Q_0/(1+\beta_{opt}) \approx V/(R/Q \cdot I)$$

(see Slide 5)

Details are in Appendix 8

Cavity operating in pulse regime:



Energy conservation law:

$dW/dt = P_0 - P_{backward} - P_{diss} - P_{beam}$ (1)

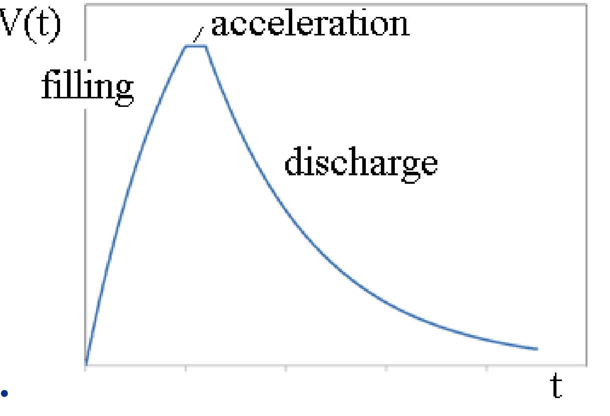
$P_0 = E_0^2/2Z$; $P_{backward} = (E_0 - E_{rad})^2/2Z$, $\beta = P_{rad}/P_{diss}$, $P_{rad} = E_{rad}^2/2Z$, ,
$P_{beam} = V(t)I$, $W = V(t)^2/(R/Q)\omega$; $\tau = 2Q_L/\omega$. (2)

V_0 –operating voltage.

If $\beta >> 1$ and $Q_L = V_0/(R/Q)I$ *(just substituting (2) to (1))*:

❑ RF on:

- $dV/dt = (2V_0 - V(t))/\tau$, filling, no beam, $V(t) = 2V_0(1 - exp(-t/\tau))$;
  If the filling time $t_f = \tau ln2$, $V(t_f) = V_0$.
- $dV/dt = (V_0 - V(t))/\tau$, acceleration, the beam is on,
  $V(t) = V_0$;



❑ RF is off:

$dV/dt = -V(t)/\tau$, the cavity discharge, $V(t) = V_0 \cdot exp(-t/\tau)$.

Example:

Let's consider a SC Nb pillbox cavity for high-energy electrons ($\beta \approx 1$), f=500 MHz, or wavelength $\lambda = c/f$=0.6 m. The mode is TM_{010}. The cavity voltage V is 3 MV. The beam current I is 1 A.

1. The cavity R/Q (Lecture 1, Slide 64):

$$R/Q = 0.98 Z_0 (d/b) T^2 = 196\ Ohm$$

2. Nb cavity unloaded quality factor Q_0 at 2 K (Lecture 1, Slides 64-65):

$$Q_0 = G/R_s = 9e10$$

3. The cavity loaded quality factor (Lecture 2, Slide 7):

$$Q_L \approx V/(R/Q)I = 1.5e3$$

3. The optimal coupling (Lecture 2, Slides 5,7):

$$\beta = Q_0/Q_L - 1 \approx Q_0/Q_L = 6e7$$

4. The power necessary for acceleration

(Lecture 1,Slide 66; Lecture 2, Slide 7):

$$P_0 = P_{diss} + P_{beam} \approx P_{beam} = VI = 3\ MW\ (compared\ to\ P_{diss} = 0.5\ W!)$$

High-Order Modes in cavities:

❑ Possible issues:

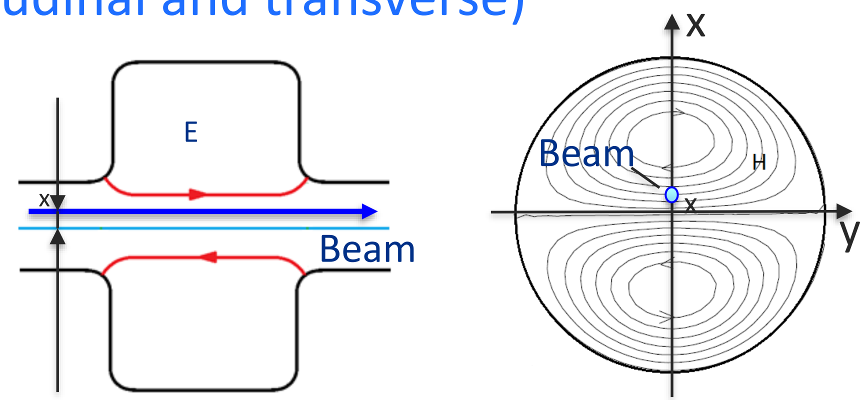
- Trapped modes;
- Resonance excitation of HOMs;
- Collective effects:
  - Transverse (BBU) and longitudinal (klystron-type instability) in linear accelerators;

- Additional losses;
- Emittance dilution (longitudinal and transverse)
- Beam current limitation.

❑ Longitudinal modes;

❑ Transverse modes.

❑ HOM dampers;



Dipole (transverse) mode

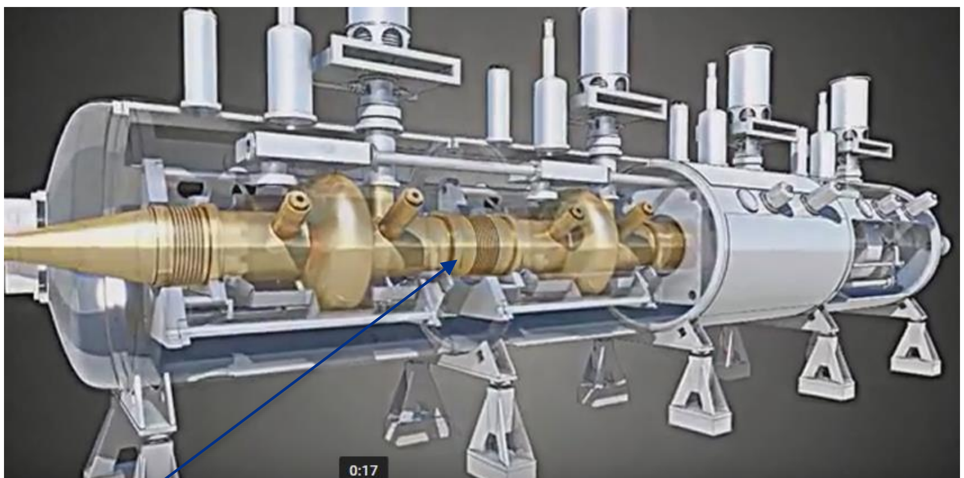
Near axis: E_z~x, E_x ~ const, H_y ~ const

High-Order Modes in cavities:

- Longitudinal modes:

$V_{HOM} = I_{beam} \cdot R_{HOM}$,   Longitudinal impedance: $R_{HOM} = (r_{||}/Q_{HOM}) \cdot Q_{load}$

- Design of the cavities with small R/Q (poor beam-cavity interaction)
- HOM dampers – special coupling elements connected to the load (low Q_{load}) .



LHC main cavity

LHC HOM coupler, Q_{ext} <200 for most "dangerous" modes

- Long wide waveguides between the cavity cells:
- HOMs propagate in the WGs and interact with the beam;
- No synchronism in the WGs (phase velocity >speed of light) → reduced R/Q_{HOM} for HOMs

High-Order Modes in cavities (Appendix 7):

- Transverse modes:

The beam interacts with the longitudinal component of the HOM electric field and provides transverse kick. For axisymmetric cavity for dipole TM-mode longitudinal field is proportional to the transverse coordinate next to the cavity axis.

Let's consider a cavity excited by a beam current I_0 having offset x_0. The kick caused by the dipole mode excited by the beam:

$$U_{kick} = i x_0 I_0 Q_{ext} \left(\frac{r_\perp}{Q}\right)$$ where $$\left(\frac{r_\perp}{Q}\right) \equiv \frac{\left|\int_{-\infty}^{\infty} \left(\frac{\partial E_z(x,0,z)}{\partial x}\right)_{x=x_0} e^{ikz} dz\right|^2}{kW\omega_0}$$

is transverse impedance, $k = \omega_0/c$ and $W = \frac{\varepsilon_0}{2}\int |\vec{E}|^2 dV$ - stored energy.

Compare to "longitudinal" (R/Q):

$$\left(\frac{R}{Q}\right) \equiv \frac{\left|\int_{-\infty}^{\infty} E_z(0,0,z)\, e^{ikz} dz\right|^2}{W\omega_0}$$

$\beta=1$ is considered.

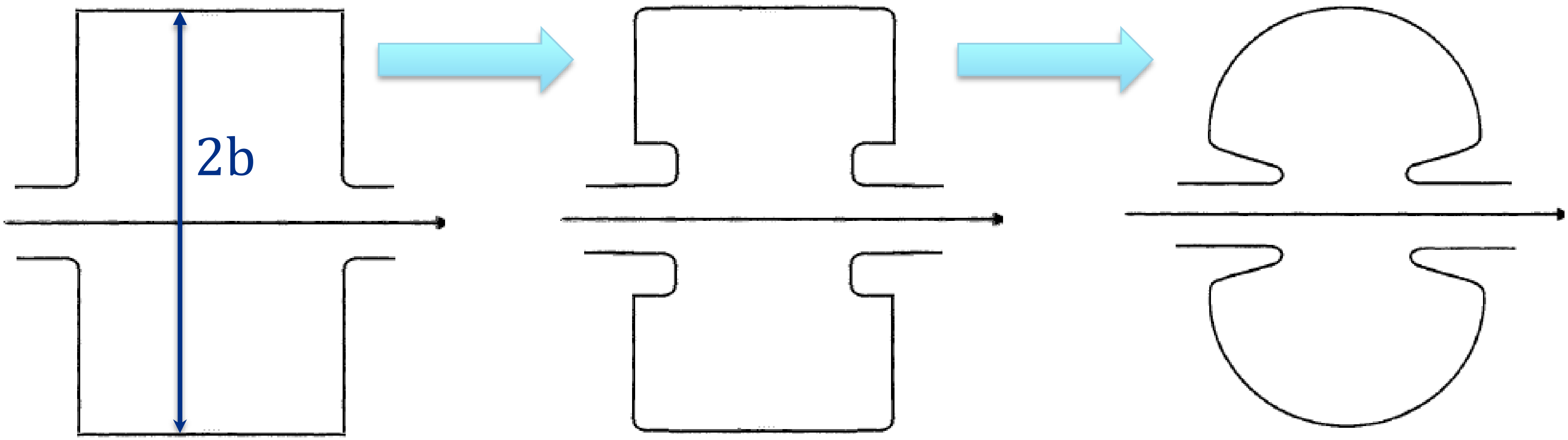
Note that $\left(\frac{r_\perp}{Q}\right)$ is measured in Ohm/m.

*Note that sometimes they use other transverse impedance, that is determined as:

$\left(\frac{r_\perp}{Q}\right)_1 = \frac{|U_{kick}|^2}{\omega_0 W_0} = \left(\frac{r_\perp}{Q}\right) \times \frac{1}{k}$. In this case, $U_{kick} = i(kx_0) I_0 Q_{ext} \left(\frac{r_\perp}{Q}\right)_1$, $\left(\frac{r_\perp}{Q}\right)_1$ **is measured in Ohm.**

RF cavity types

Pillbox RT cavities:



Pillbox with the beam pipes

Drift tubes
T improvement

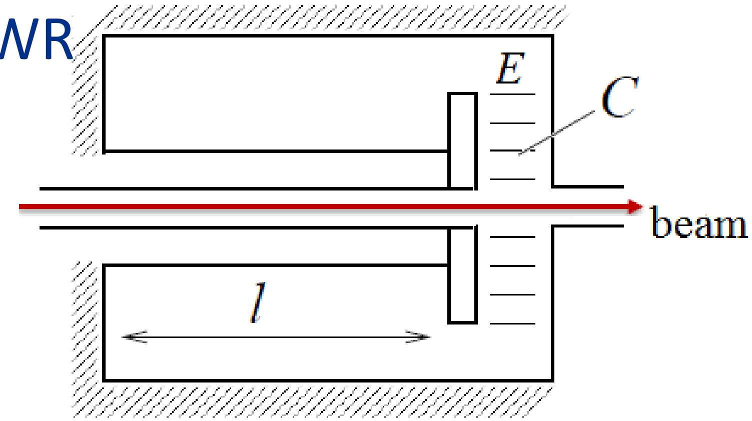
Rounding
R_{sh} improvement

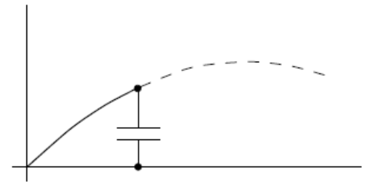
$$2b \lesssim 3/4\lambda$$

RF cavity types

Quarter-wave resonator (QWR) concept:

One-gap QWR





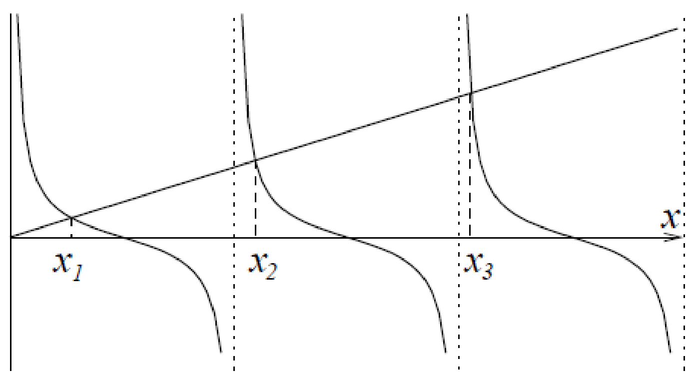
Resonance:

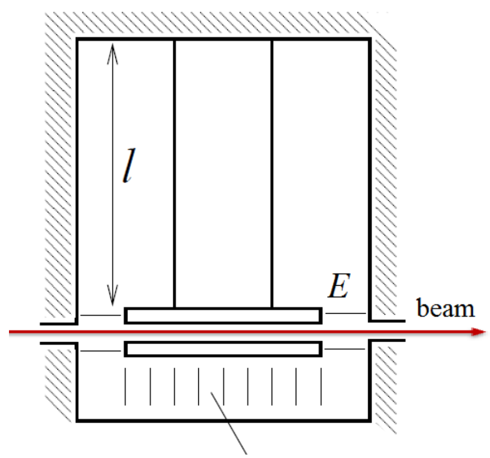
$$\frac{1}{j\omega C} + jZ_0 \tan\frac{\omega}{c}l = 0. \quad \text{or} \quad \cot\frac{\omega}{c}l = Z_0\omega C.$$

Z_0 here is the coaxial impedance

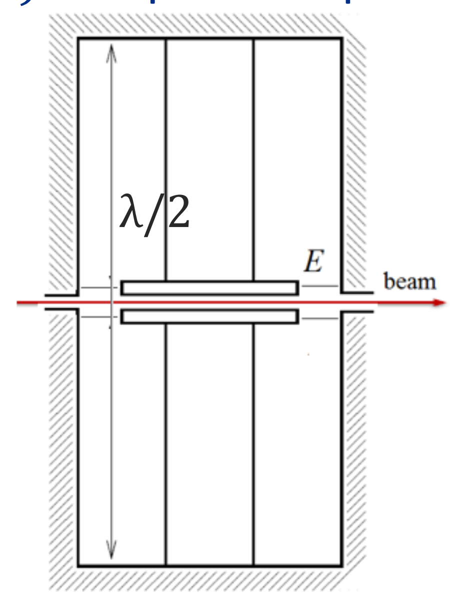
$$\frac{\omega}{c}l = x,\ A = \frac{Z_0 C\, c}{l} \longrightarrow \cot x = Ax.$$

Compact ($L \gtrsim \lambda/4$) compared to pillbox ($D \lesssim 3/4\lambda$).





Two-gap QWR



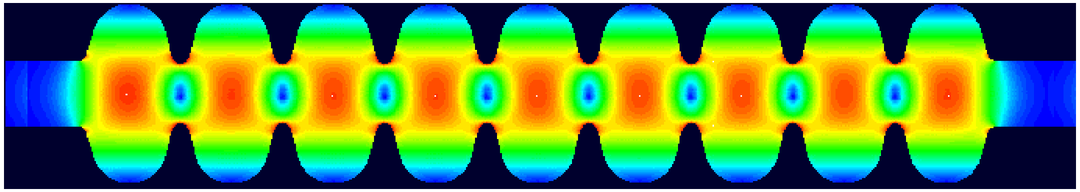
Half-wave resonator (HWR)

Tools for RF cavity simulations:

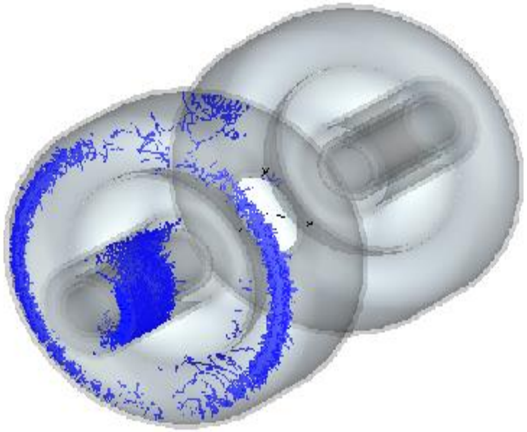
I. Field calculations:

-Spectrum, (r/Q), G, β (coupling)

-Field enhancement factors



- HFSS (3D);
- CST (3D);
- Omega-3P (3D);
- Analyst (3D);
- Superfish (2D)
- SLANS (2D, high precision of the field calculation).

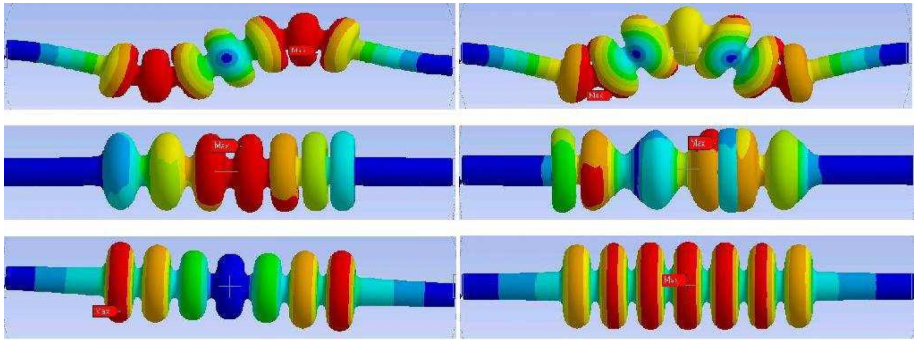


II. Multipactoring (2D, 3D)

- Analyst;
- CST (3D);
- Omega-3P

III. Wakefield simulations (2D, 3D):

- GdfidL;
- PBCI;
- ECHO.



IV. Mechanical simulations:

Lorenz force and Lorenz factor,

Vibrations,

Thermal deformations.

a. ANSYS

Fermilab

Summary (cont):

- The cavity coupled to the input port is characterized by the following parameters:
  - Coupling to the feeding line, β (do not mix with the relative particle velocity)
  - External Q, Q_{ext}
  - Loaded Q, Q_{load}
- The beam excites the cavity creating decelerating voltage, which is proportional at resonance to the shunt impedance and the beam current. This voltage should be compensated by the RF source to provide acceleration.
- High-Order Modes excited by the beam may influence the beam dynamics and lead to additional losses in the cavity.
  - Dipole modes are characterized by transverse impedance, $(r_{\perp}/Q)$, which relates transverse kick and stored energy.
  - Both monopole and dipole HOMs should be taken into account during the cavity design process, and damped if necessary.

Chapter 4.

Periodic acceleration structures.

a. Coupled cavities and periodic structure;
b. Travelling waves in a periodic structure;
c. Dispersion curve;
d. Phase and group velocities;
e. Parameters of the TW structures;
f. Equivalent circuit for a travelling – wave structure;
g. Losses in the TW structure;
h. Types of the TW structures;
i. Examples of modern TW structures.

Periodic acceleration structures:

- Single – cell cavities are not convenient to achieve high acceleration: a lot of couplers, tuners, etc.
- Especially it is important for electron acceleration:

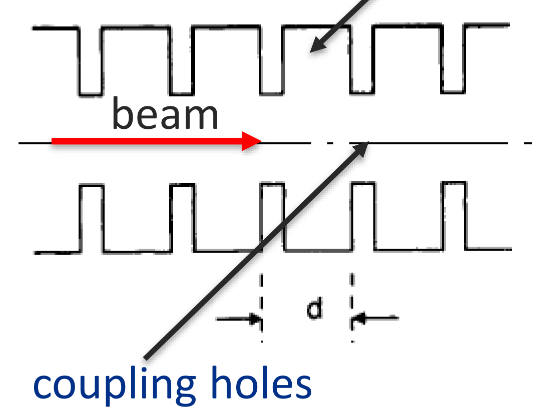
  $R_{sh} = R/Q \cdot Q_0 \sim \omega^{1/2}$, low Ohmic losses at high frequency;

  $v=c$, focusing is quadratic and does not depend on frequency.

high frequencies are preferable (typically up to few tens of GHz).

small cavity size, ~ 1 cm for RT, ~20 cm for SRF

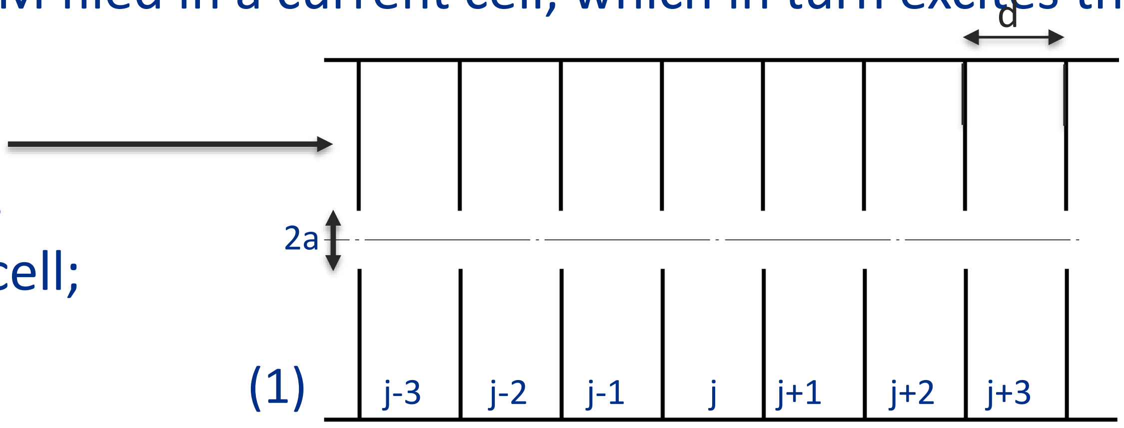
periodic structure of coupled cells.



- To provide synchronism with the accelerated particle, the particle velocity $v_p=\beta c=v_{ph}=\omega/k_z$ and the structure period $d = \varphi/k_z= \varphi\lambda/(2\pi\beta)$; φ is phase advance per period, $\varphi= k_z d$.

Periodic acceleration structures:

- Each previous cell excites EM filed in a current cell, which in turn excites the field in the next cell.



- Pillbox cells with thin walls
- $\vec{E}_j = X_j \vec{E}_0$ - field in the j^th cell;
- $\vec{E}_0$- eigen function

$$X_j\left[1-(1+K)\frac{\omega_0^2}{\omega^2}\right]+\frac{1}{2}K\frac{\omega_0^2}{\omega^2}\left[X_{j-1}+X_{j+1}\right]=0$$

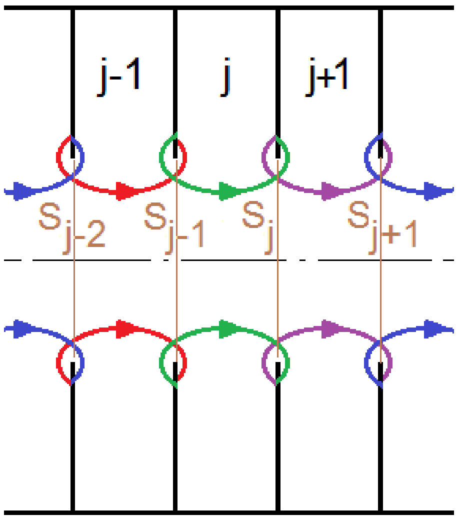
$$d=\frac{\beta\lambda\varphi}{2\pi}$$ - synchronism

where K is the coupling, dimensionless parameter:

$$K=\frac{2E_0^2a^3}{3Z_0W_0c}=\frac{2}{3}\cdot\frac{R/Q}{Z_0}\cdot\frac{k_0a^3}{d^2T^2} \qquad k_0=\frac{\omega_0}{c}$$

Fermilab

Periodic acceleration structures:



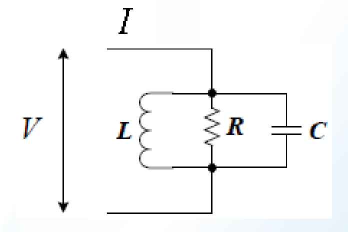
$$\vec{E}_j = X_j \vec{E}_0$$

$$X_j\left[1-(1+K)\frac{\omega_0^2}{\omega^2}\right]+\tfrac{1}{2}K\frac{\omega_0^2}{\omega^2}\left[X_{j-1}+X_{j+1}\right]=0$$

Why?

Excitation of a cavity by surface electric field:

$$X_j = \frac{\int_{S_{j-1}} \mathcal{K}_{\varphi j-1} H_{\varphi 0} dS}{W\left(1-\frac{\omega_0^2}{\omega^2}\right)} + \frac{\int_{S_j} \mathcal{K}_{\varphi j} H_{\varphi 0} dS}{W\left(1-\frac{\omega_0^2}{\omega^2}\right)}$$



Here $\boldsymbol{\mathcal{K}}$ is "magnetic" current* on the aperture, $\mathcal{K}_{\varphi j} = E_{rj}$.

$div\vec{E} = 0$ → in paraxial approximation $E_r = -\frac{r}{2}\frac{\partial E_z}{\partial z} \rightarrow \mathcal{K}_{\varphi j-1} \approx \frac{rE_{z0}}{4a}\left(X_j - X_{j-1}\right)$;

$\int_{S_{j-1}} \mathcal{K}_{\varphi j-1} H_{\varphi 0} dS = \frac{1}{2}K\frac{\omega_0^2}{\omega^2}\left(X_j - X_{j-1}\right)$, K depending on the aperture as a^3

(E_{z0} does not depend on r, $H_{\varphi 0} \sim r$, and $\int_{S_{j-1}} \mathcal{K}_{\varphi j-1} H_{\varphi 0} dS \sim a^3$). Therefore,

$$X_j\left(1-\frac{\omega_0^2}{\omega^2}\right)-\left(K\frac{\omega_0^2}{\omega^2}X_j-\frac{1}{2}K\frac{\omega_0^2}{\omega^2}X_{j-1}-\frac{1}{2}K\frac{\omega_0^2}{\omega^2}X_{j+1}\right)=0,$$ or

$$X_j\left(1-(1+K)\frac{\omega_0^2}{\omega^2}\right)+\frac{1}{2}K\frac{\omega_0^2}{\omega^2}\left(X_{j-1}+X_{j+1}\right)=0$$ (Exact derivation is in the Appendix 11)

*Surface electric current density is $\boldsymbol{\mathcal{J}} = \vec{H} \times \vec{n}$; surface magnetic current density is $\boldsymbol{\mathcal{K}} = \vec{E} \times \vec{n}$

Travelling–Wave acceleration structures:

In the infinite chain of cavities equation (1) has solution (travelling wave):

$$X_j = Xe^{ij\varphi}$$

and

$$\omega(\varphi) = \omega_0[1 + K(1 - cos\varphi)]^{1/2}$$

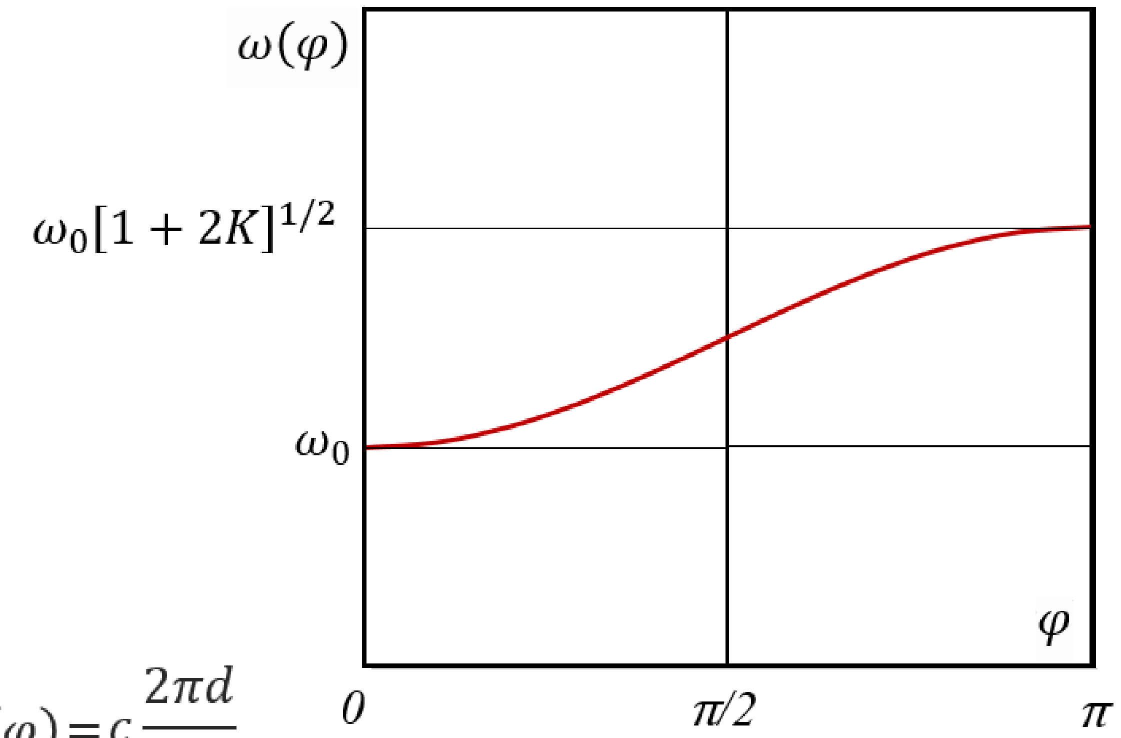
For small K we have:

$$\omega(\varphi) \approx \omega_0[1 + \tfrac{1}{2}K(1 - cos\varphi)]$$

One can see that

$$K = \frac{\omega(\pi) - \omega(0)}{\omega(0)}$$

$$v_{ph}(\varphi) = c\frac{2\pi d}{\varphi\lambda}$$



Travelling–Wave acceleration structures:

- In the arbitrary infinitely long periodic structure, or in the finite structure matched on the ends, there are travelling waves (TW) having arbitrary phase shift per cell φ. Longitudinal wavenumber, therefore, is $k_z = \varphi/d$. Dispersion equation is the same:

$$\omega(k_z) \approx \omega_{\pi/2}\left(1 - \frac{K}{2}cos(\varphi)\right) = \omega_{\pi/2}\left(1 - \frac{K}{2}cos(k_z d)\right)$$

  Therefore, the phase velocity $v(\varphi)$ is:

$$v_{ph}(\varphi) = \frac{\omega(k_z)}{k_z} = c\frac{2\pi d}{\varphi\lambda}$$

- For acceleration of the particle having velocity $v_p=\beta c$, the cavity cell length d should be equal to

$$d = \frac{\beta\lambda\varphi}{2\pi},$$

  because for synchronism we need $v_p = v_{ph}$
  For example, for $\varphi = \pi$ the cell should have the length of $\beta\lambda/2$.

Travelling–Wave acceleration structures:

- The group velocity $v_{gr}(\varphi)$ is

$$v_{gr}(\varphi) = \frac{d\omega}{dk_z} \approx c\frac{\pi K d}{\lambda} sin(\varphi)$$

For $\varphi=0$ and $\varphi=\pi$ group velocity is zero. For $\varphi=\pi/2$ group velocity is maximal:

$$v_{gr}(\pi/2) = c\frac{\pi K d}{\lambda}.$$

- For small K group velocity is small compared to the speed of light.
- In contrast to a waveguide, $v_{ph} \cdot v_{gr} \neq c^2$.

Travelling–Wave acceleration structures :

John Stewart Bell

- For TW in a periodic structure:

- Average stored energy per unit length for electric field w_E is equal to the average stored energy per unit length for magnetic field w_H (the 1st Bell Theorem*):

$$w_E = w_H = w/2$$

- The power P flow is a product of the average stored energy per unit length and the group velocity (the 2d Bell Theorem*):

$$P = v_{gr} w.$$

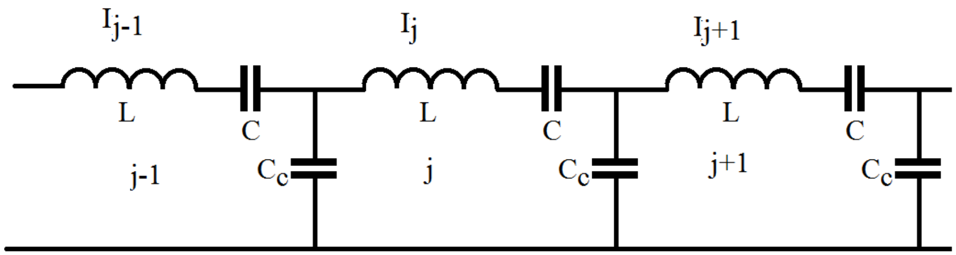
***J.S. Bell, "Group velocity and energy velocity in periodic waveguides," Harwell, AERE-T-R-858 (1952)**

Fermilab

Travelling–Wave acceleration structures

Equivalent circuit:

Note that the electrodynamics in the periodic structure is described by the equivalent circuit



For j[th] cell we have from Kirchhoff theorem:

$$\left(i\omega L + \frac{1}{i\omega C}\right) I_j + \frac{(I_j - I_{j-1})}{i\omega C_c} + \frac{(I_j - I_{j+1})}{i\omega C_c} = 0,$$

For the capacity voltage $X_j = \dfrac{I_j}{i\omega C}$ we have the same equation as for EM model:

$$X_j\left[1 - (1+K)\frac{\omega_0^2}{\omega^2}\right] + \tfrac{1}{2}K\frac{\omega_0^2}{\omega^2}\left[X_{j-1} + X_{j+1}\right] = 0$$

Here $\omega_0^2 = \dfrac{1}{LC}$, $K = \dfrac{2C}{C_c}$, $C = \dfrac{2}{\omega_0 R/Q}$, $L = \dfrac{R/Q}{2\omega_0}$.

Fermilab

Travelling–Wave acceleration structures

Loss in the cells:

Ohmic loss on the metallic surface:

$$\omega_0 \rightarrow \omega_0\left(1+\frac{i}{2Q_0}\right)$$

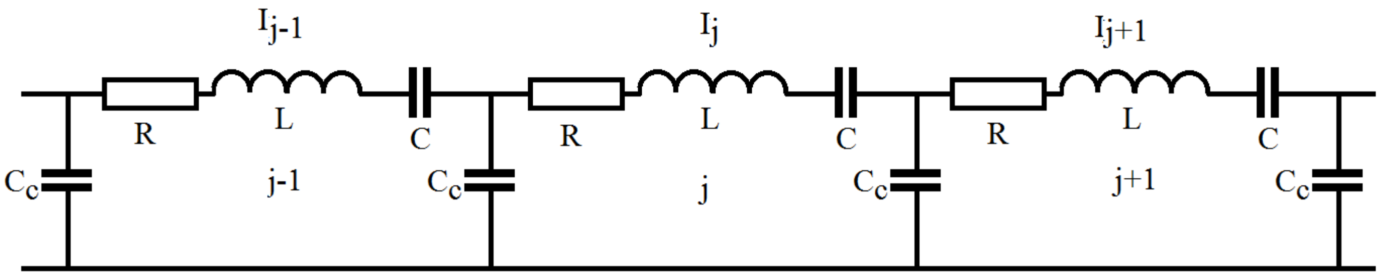
$$\vec{E},\vec{H} \sim e^{i\omega_0 t - t/\tau} = e^{i\omega_0 t\left(1+\frac{i}{2Q_0}\right)}$$

$$\tau = \frac{2Q_0}{\omega_0}$$

and

$$X_j\left[1-(1+K)\frac{\omega_0^2}{\omega^2}+i\frac{\omega_0^2}{Q_0\omega^2}\right]+\frac{1}{2}K\frac{\omega_0^2}{\omega^2}\left[X_{j-1}+X_{j+1}\right]=0$$

Equivalent circuit is the following:



where

$$R=\frac{R/Q}{2Q_0} \qquad \omega_0^2=\frac{1}{LC},\ \ K=\frac{2C}{C_c},\ C=\frac{2}{\omega_0 R/Q},\ L=\frac{R/Q}{2\omega_0}.$$

Fermilab

Travelling–Wave acceleration structures

However, in a long periodic TW structure Ohmic losses change acceleration field distribution along the structure.

Energy conservation law in the j[th] cell:

$$\frac{dW_{0,j}}{dt} = -P_j + P_{j-1} - \frac{\omega_0 W_{0,j}}{Q_0},$$

Taking into account that $w = \frac{W_0}{d}$ and $P = w \cdot v_{gr}$ we have

$$\frac{\partial w}{\partial t} = -\frac{(w \cdot v_{gr}|_j - w \cdot v_{gr}|_{j-1})}{d} - \frac{\omega_0 w}{Q_0} \approx -\frac{\partial (w v_{gr})}{\partial z} - \frac{\omega_0 w}{Q_0}$$

In steady-state case we have $\frac{dw}{dz} = -\frac{w}{v_{gr}}\left(\frac{dv_{gr}}{dz} + \frac{\omega_0}{Q_0}\right)$

- Constant impedance structure:

$$v_{gr} = const \rightarrow w(z) = w(0)e^{-\frac{z\omega_0}{v_{gr}Q_0}} \rightarrow E(z) = E(0)e^{-\frac{z}{v_{gr}\tau}} \qquad \tau = \frac{2Q_0}{\omega_0}$$

- Constant gradient structure:

$$v_{gr}(z) = v_{gr}(0) - z\frac{\omega_0}{Q_0} \rightarrow w(z) = w(0) \rightarrow E(z) = E(0) = const$$

Aperture a should decrease with z.

Travelling–Wave acceleration structures

Tolerances:

If the cell frequencies have resonant frequency deviation $\delta\omega_0$, it changes the longitudinal wave number k_z and violates synchronism.

$$\delta k_z = \frac{dk_z}{d\omega_0}\delta\omega_0 = \frac{1}{v_{gr}}\delta\omega_0$$

It means that it is necessary to operate in the middle of dispersion curve, when group velocity is maximal, $\varphi \sim \pi/2 - 2\pi/3$.

If φ close to π, the structure is unstable.

Travelling–Wave acceleration structures

TW structure parameters:
For TW stricture R *and* R/Q are calculated per unit length of the structure.

- Shunt impedance R is measured in MOhm/m. For geometrically similar cells R scales as $\omega_0^{1/2}$.
- R/Q is measured in Ohm/m. For geometrically similar cells R/Q scales as ω_0

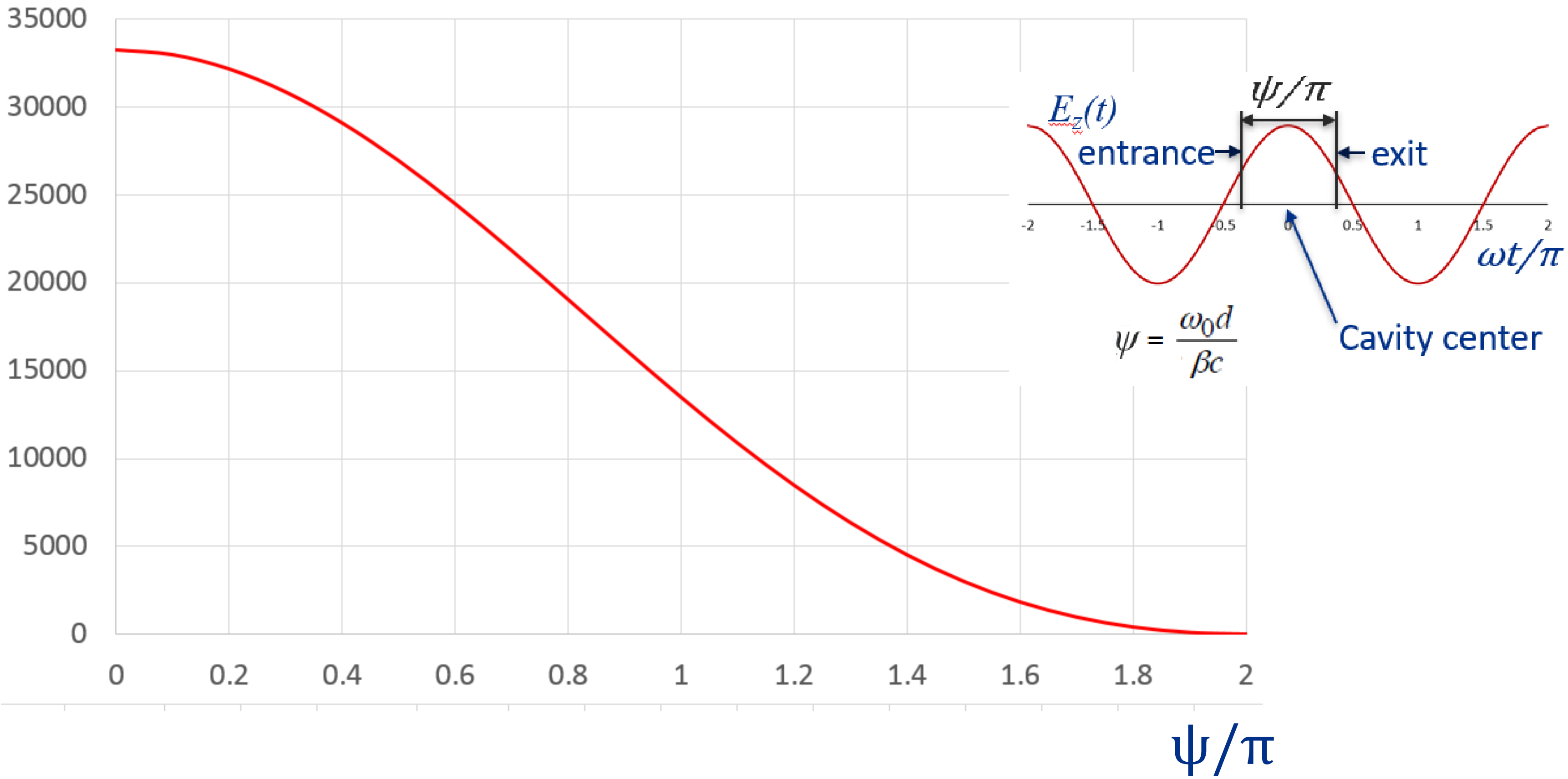
Travelling–Wave acceleration structures

TW structure parameters: *(R/Q)* for pillbox, f=10 GHz (here b is the cavity radius)

R/Q, Ohm/m

$$\frac{R}{Q} = \frac{0.98 Z_0 T(\psi)^2}{b}, \qquad b = \frac{2.405c}{2\pi f}$$

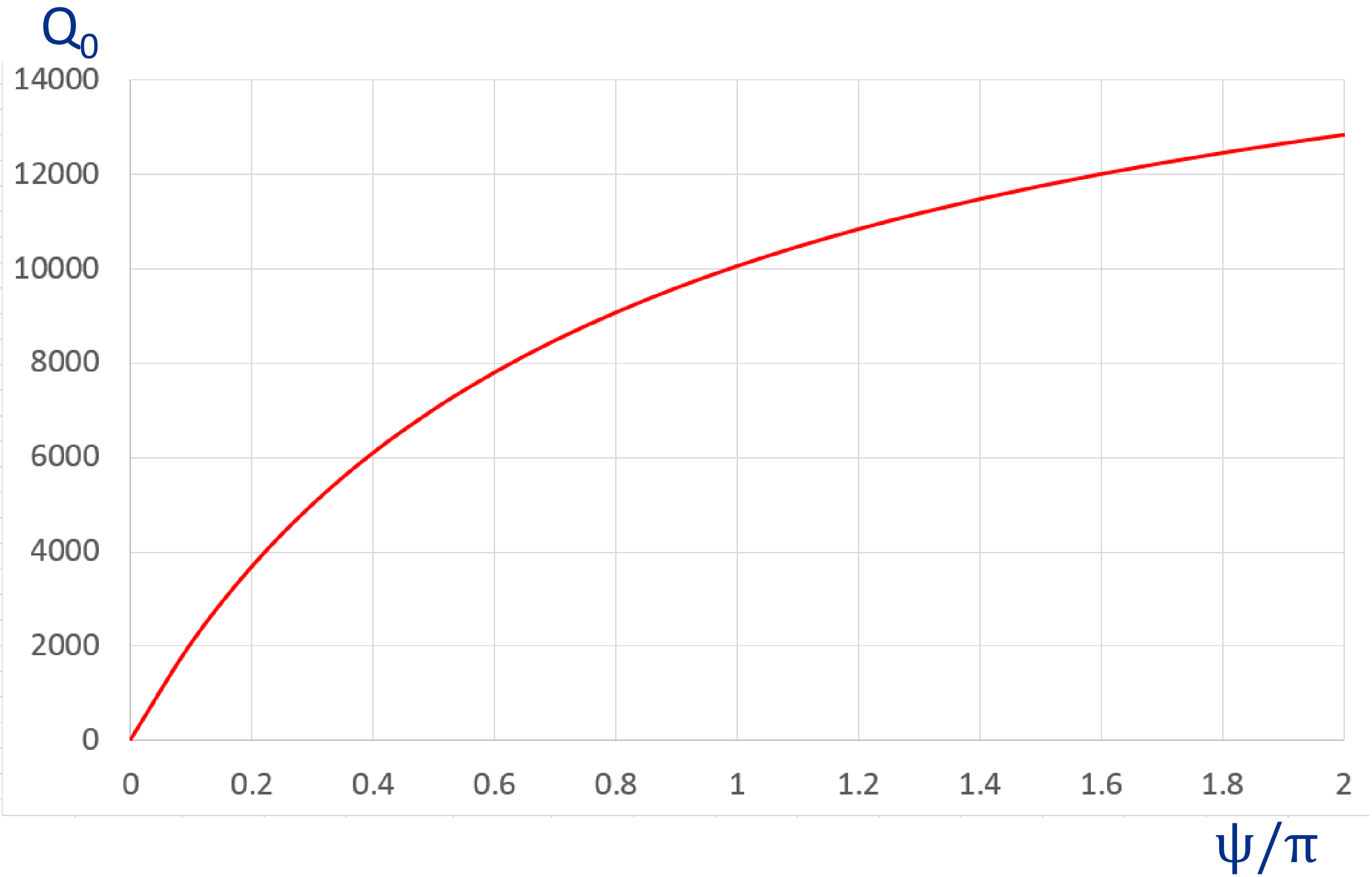
(See Lecture 1, slide 51)



Fermilab

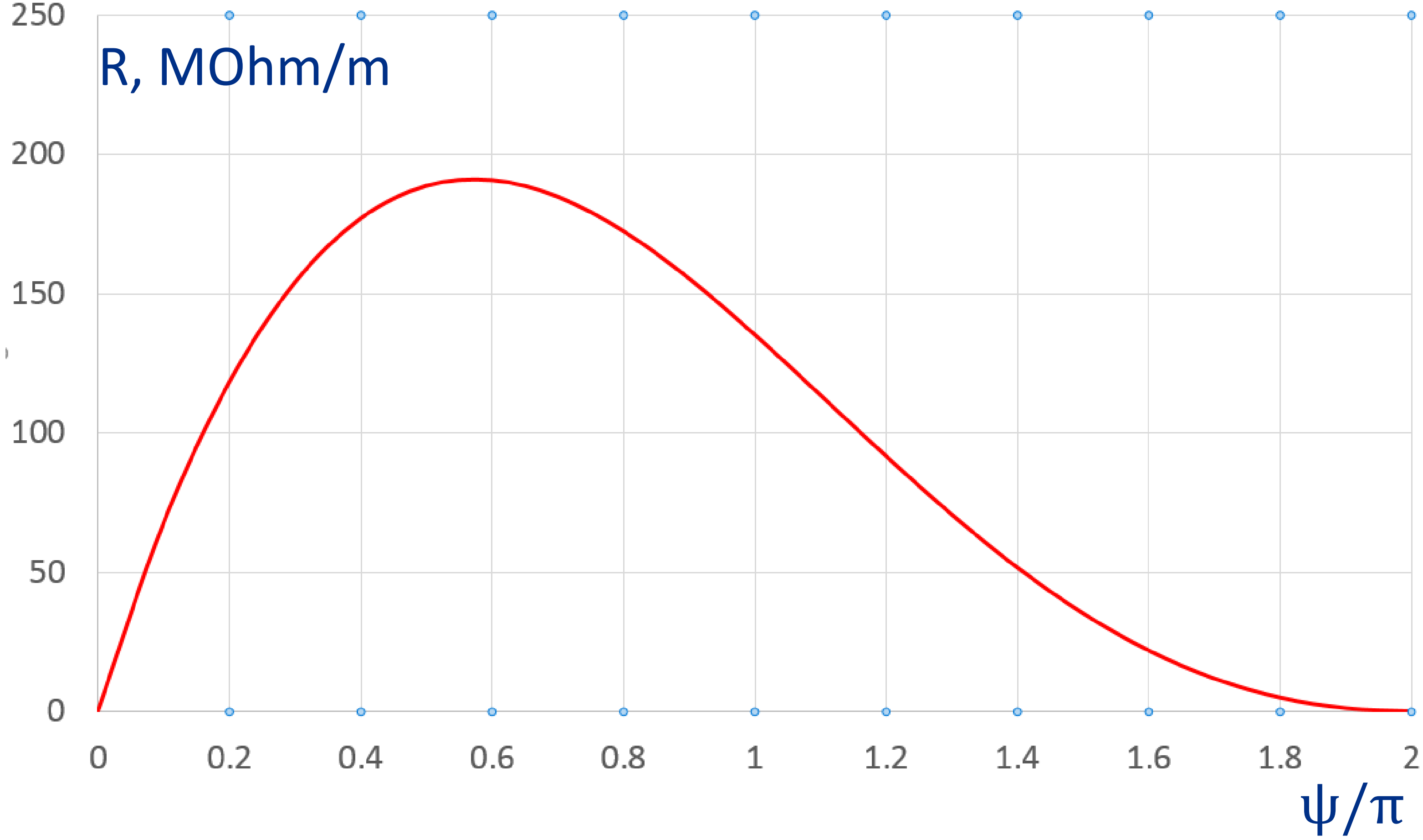
Travelling–Wave acceleration structures

TW structure parameters: Q_0 for pillbox at 10 GHz (see Lecture 1, slide 54)



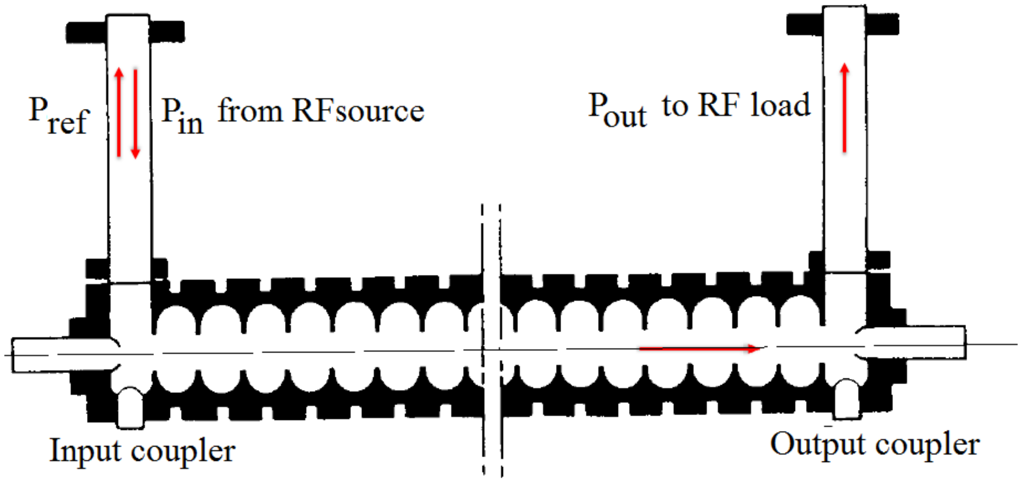
Travelling–Wave acceleration structures

TW structure parameters: Shunt impedance $R=(R/Q)\cdot Q_0$ for pillbox at 10 GHz (see Lecture 1, slide 58)



R is maximal at $\psi \sim 0.6\pi$. Typically, they use $\psi = 2\pi/3$.

Travelling–Wave acceleration structures



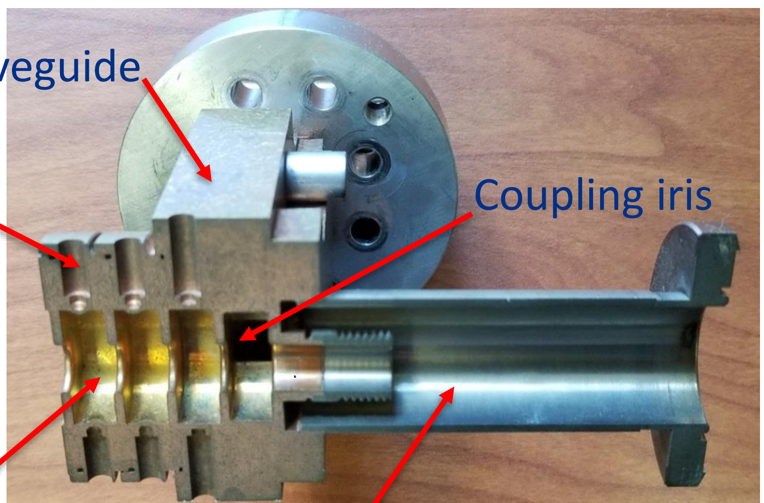
$$P_{in} = P_{ref} + P_{out} + P_{loss} + P_{beam}$$
$$P_{in} \gg P_{out}$$

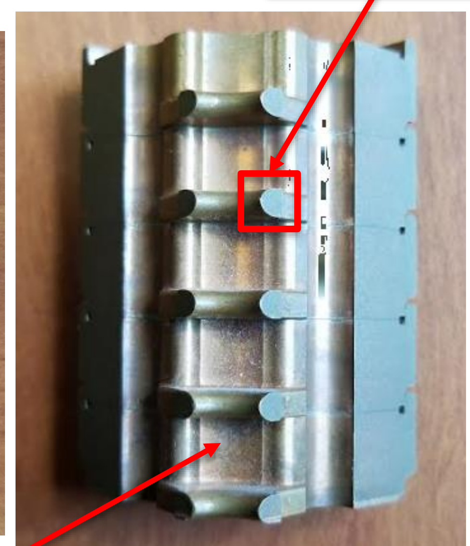
Aperture has elliptical shape to minimize surface electric field

Input waveguide

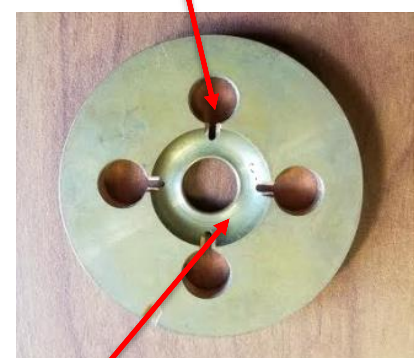
Tuning channels

Coupling iris





HOM dampers



Acceleration cells

Beam pipe

Acceleration cells

Acceleration cell

NLC structure with HOM damping

Travelling–Wave acceleration structures

TW structures for acceleration of electrons are widely used is different fields.

❖ **High – energy physics:**

- SLAC (1968): 3 km, 47 GeV (max), 2π/3 2.856 GHz (S-band) , 3 m structures.
- SLC (1987) – first e^+e^- linear collider based on the SLAC linac.
- CLIC collider (R&D): up to 50 km, up to 3 TeV c.m., 2π/3 12 GHz

❖ **FELS:**

- SwissFEL (PSI) 5.7 GHz linac (2017), 0.74 km, 5.8 GeV, 2π/3 6 GHz

❖ **Industrial and medical accelerators**

- Varian S-band (2.856 GHz) and X-band (11.424 GHz) linacs for medical applications
- Industrial linacs

Travelling–Wave acceleration structures

Modern TW structures: 12 GHz CLIC structure*

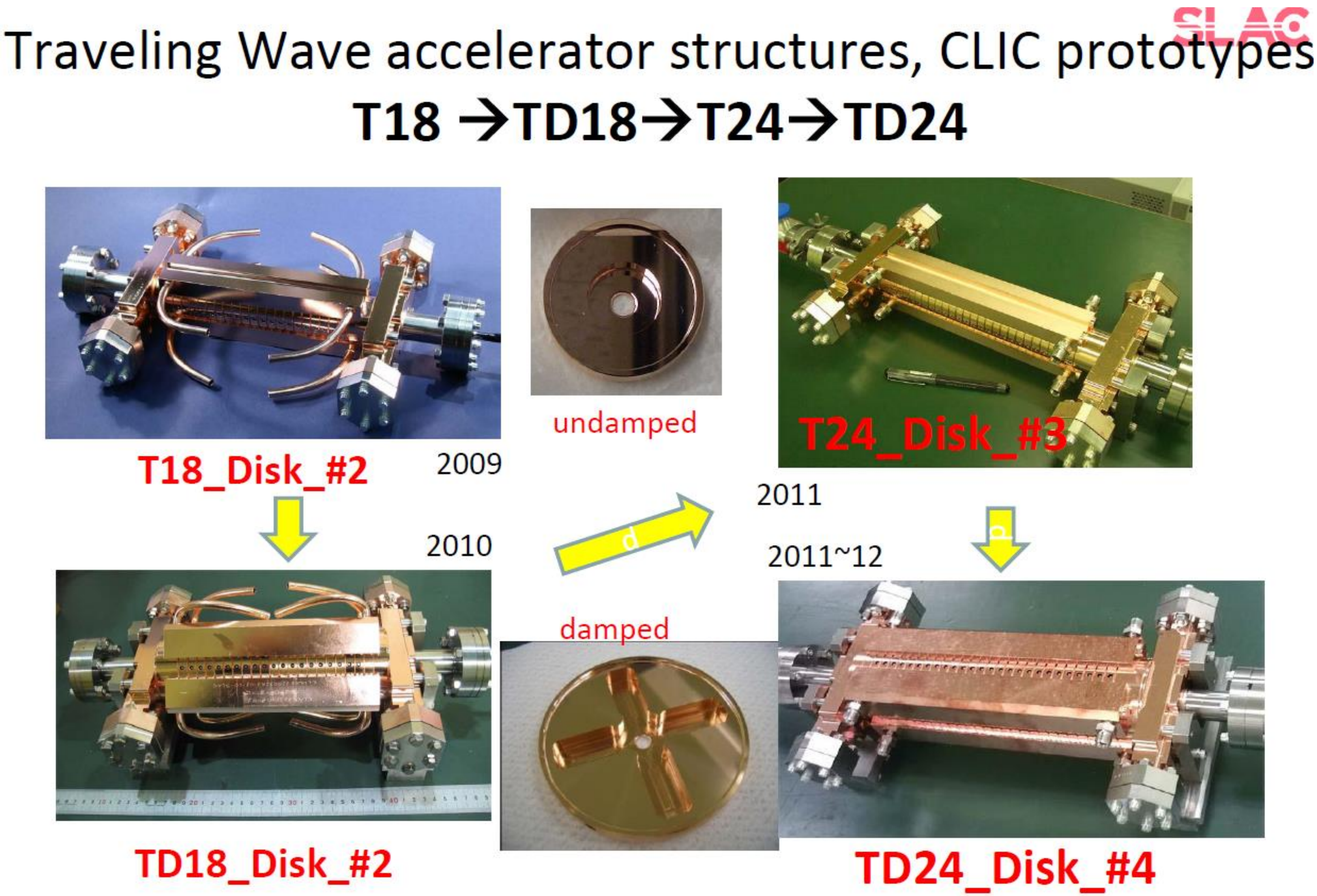
Accelerating structure parameters

Loaded gradient* [MV/m]	100
Working frequency [GHz]	11.994
Phase advance per cell	2π/3
Active structure length [mm]	217
Input/output radii [mm]	3.15/2.35
Input/output iris thickness [mm]	1.67/1.00
Q factor [Cu]	7112/7445
Group velocity [%c]	1.99/1.06
Shunt impendence [MΩ/m]	107/137
Peak input power [MW]	60.9
Filling time [ns]	49.5
Maximum E-field [MV/m]	313
Maximum modified Poynting vector[MW/mm^2]	7.09
Maximum pluse heating temperature rise [K]	35

*V. Dolgashev, SLAC, EAAC 2015

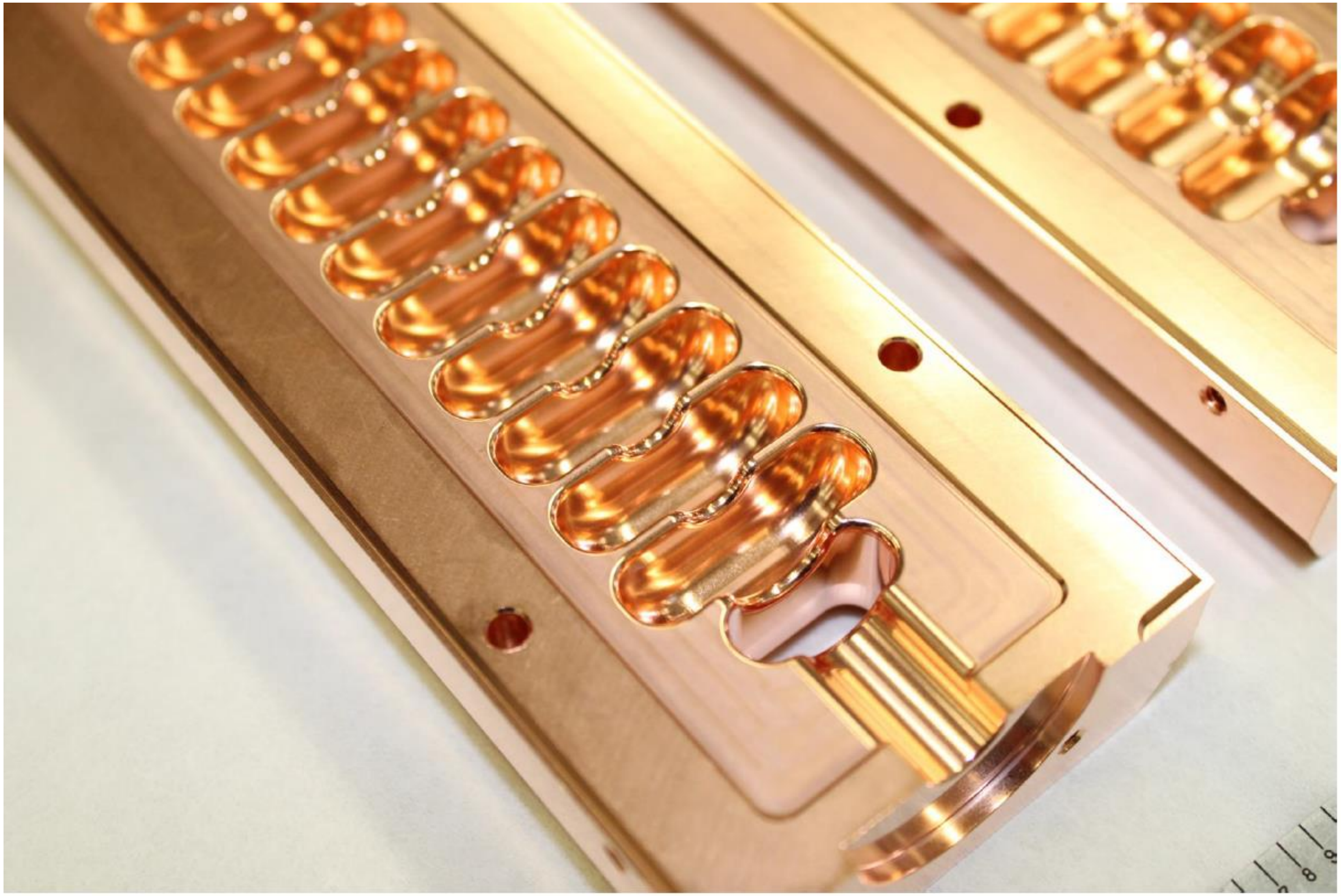
Travelling–Wave acceleration structures

Modern TW structures: 12 GHz CLIC structure*



Travelling–Wave acceleration structures

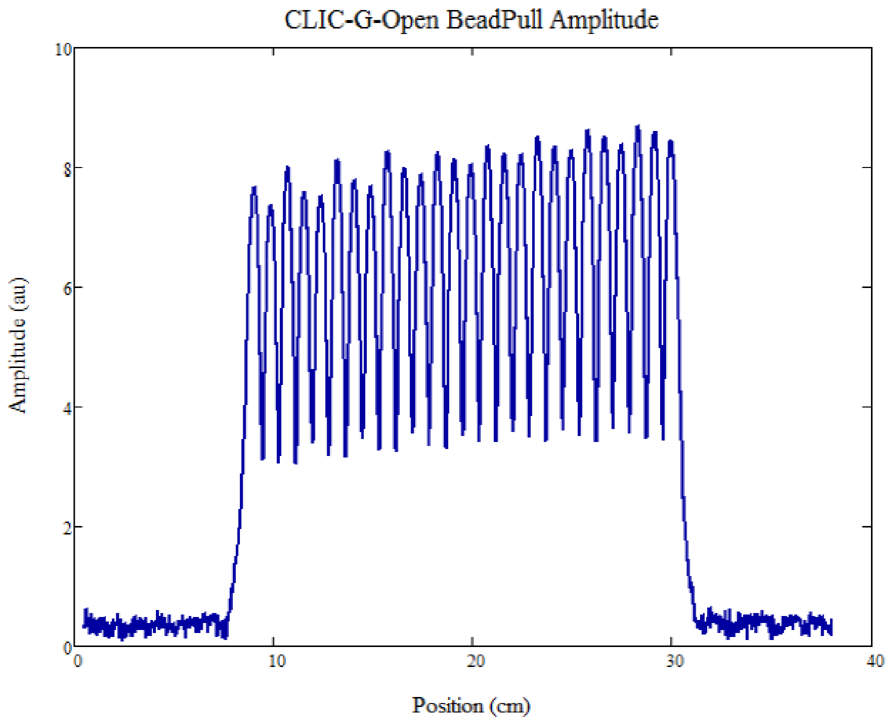
Modern TW structures: 12 GHz CLIC structure*



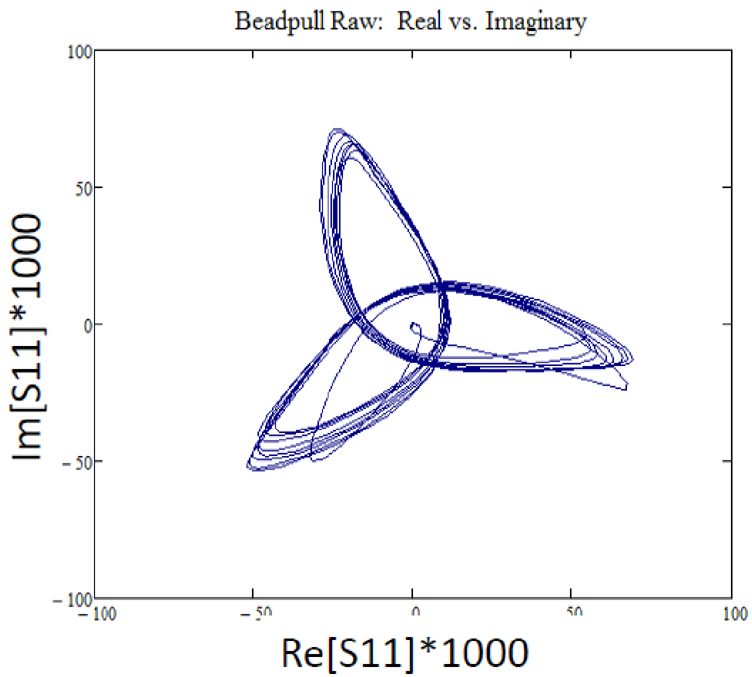
SLAC-CERN

Travelling–Wave acceleration structures

Final beadpull of tuned CLIC-G-OPEN



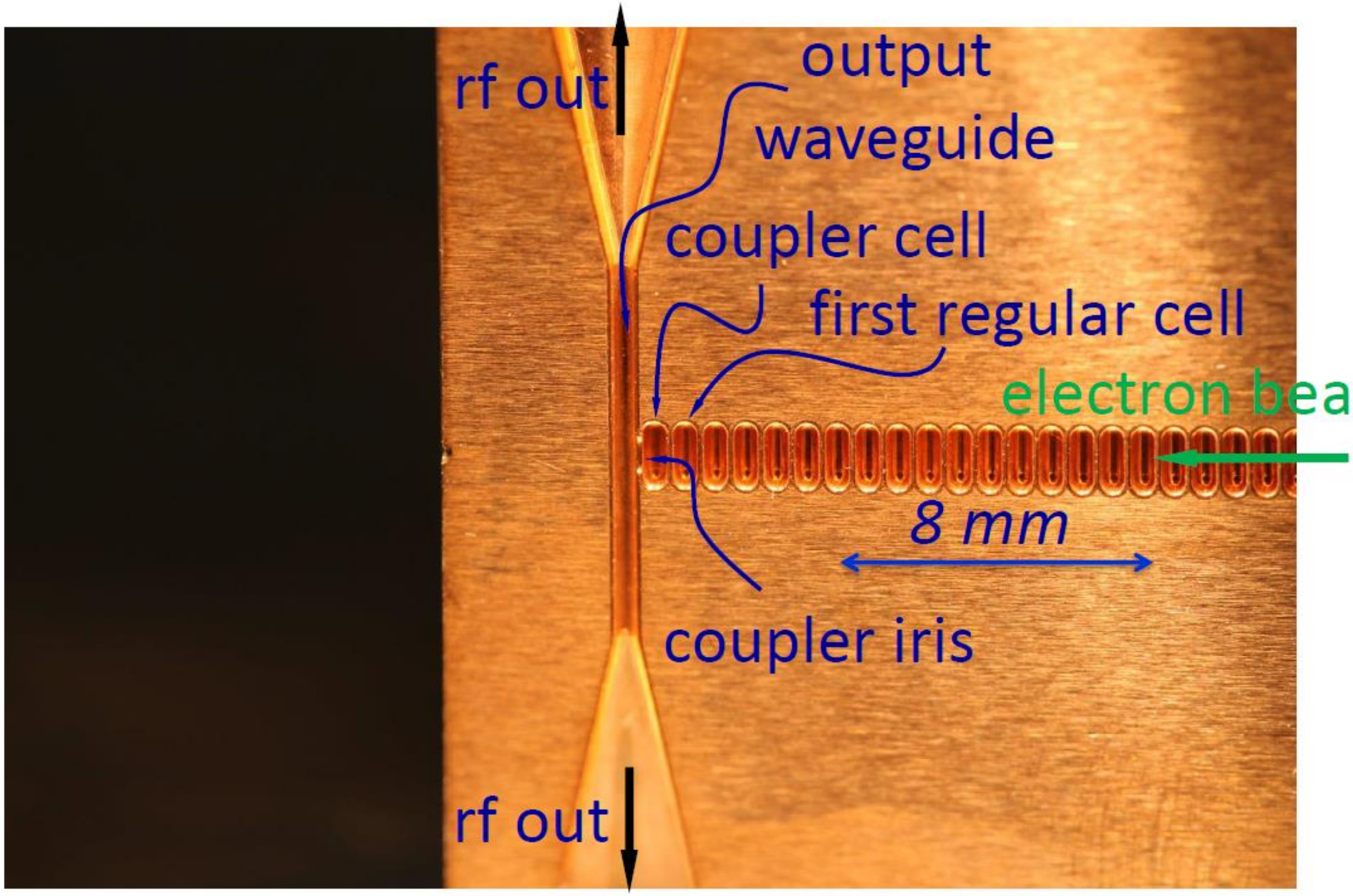
On-axis field amplitude.



Polar plot of beadpull data.

Travelling–Wave acceleration structures

Output Part of the Open 100 GHz Copper Traveling Wave Accelerating Structure



SLAC-INFN

Summary:

- Single – cell cavities are not convenient in order to achieve high acceleration: a lot of couplers, tuners, etc. Especially it is important for acceleration of electrons.
- Periodic structures are used for acceleration, where travelling wave is excited.
- Phase advance per cell, and therefore, phase velocity depend on the phase advance per cell. The accelerating wave has the same phase velocity as the accelerated particles.
- Average energy of magnetic field is equal to average energy of electric field (the 1st Bell theorem); Power flaw is equal to the product of the group velocity to the average stored energy per unit length (the 2d Bell theorem).
- The passband depends on the value of coupling between the cells *K* ; it depends on the coupling hole radius a as ~ a^3-a^4 ;
- Group velocity is maximal if phase advance per cell is ~$\pi/2$;
- Maximal shunt impedance per unit length is at the phase shift of ~$2\pi/3$;
- Losses may change the field distribution. To achieve field flatness along the structure, group velocity (coupling) should decrease from the structure beginning to the end.

Chapter 5.

Standing –Wave acceleration structures.

a. Standing - wave structures;
b. Equivalent circuit for a SW structure;
c. Dispersion curve;
d. Normal modes;
e. Perturbation theory for SW structures;
e. Parameters of SW structures;
f. Bi-periodic SW structures;
g. Inductive coupling;
h. Types of the SW structures;

Standing–Wave acceleration structures

❖ **TW structures work very good for RT electron accelerators:**

- High frequency → lower power (R~ $f^{1/2}$);
- A lot of cells (many tens) → high efficiency (all the power is consumed in the structure, and small fraction is radiated through the output port).

❖ **TW structures are not good for RT proton accelerators:**

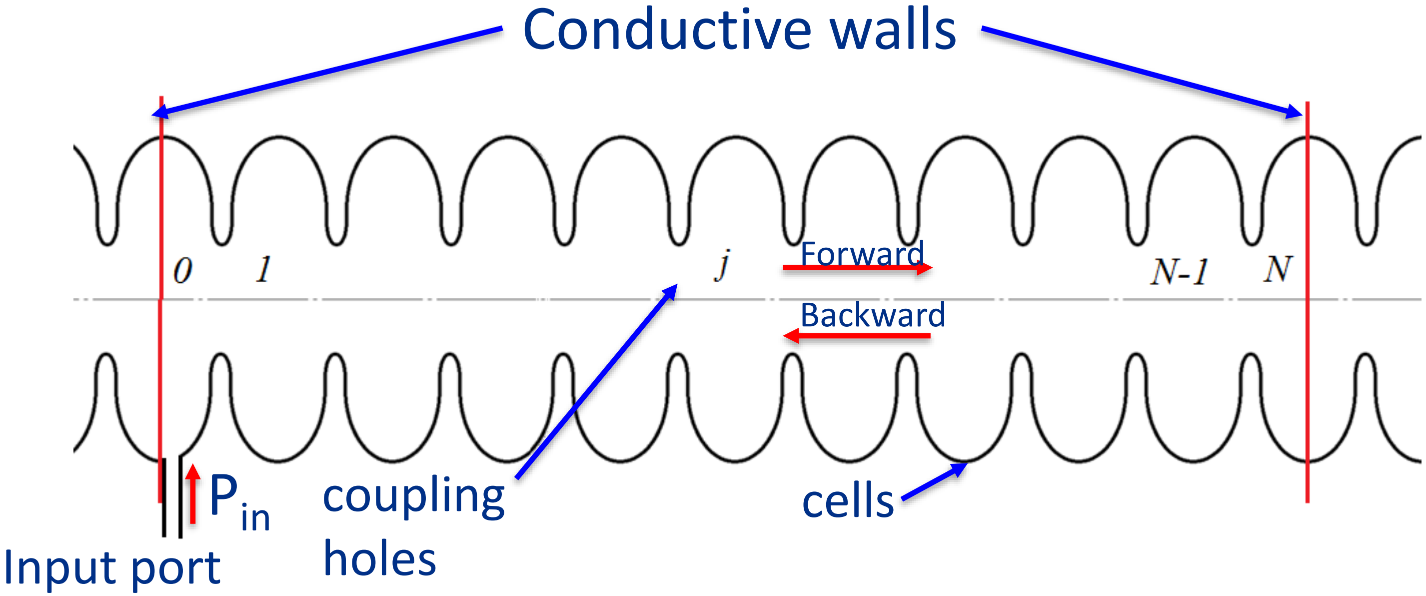
- High frequency is not practical (defocusing is proportional to *f*)
- Low beam loading → large number of cells (impractical from the point of view of focusing and manufacturing, especially if the cell diameter is large because of low frequency);

❖ **TW structures are not good for SRF accelerators:**

- High frequency is not practical (BCS surface resistance is proportional to f^2)
- Small decay in the cavities
  - Very large number of cells + large cell size (impractical from the point of view of manufacturing and processing);
  - Feedback waveguide - still under R&D

Standing–Wave acceleration structures

Standing Wave structures:



Putting reflective conductive walls in the middle of the end cells, we do not violate boundary conditions for EM field for TM_{010}-like modes.

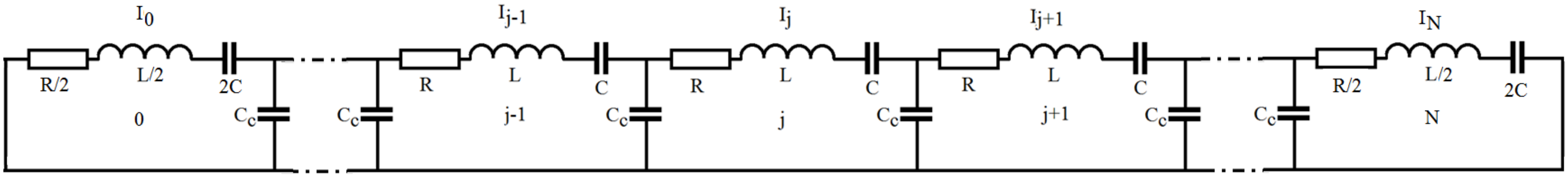
Forward and backward travelling waves form standing wave.

- N may be small, even N=2;
- Frequency may be small, up to hundreds of MHz→ proton acceleration
- Suitable for SRF
- $P_{in} << P_{forward} \approx P_{backward}$

Fermilab

Standing–Wave acceleration structures

Equivalent circuit of the SW structure containing half-cells on the ends:



$$X_0\left[1-\frac{\omega_0^2}{\omega^2}+i\frac{\omega_0^2}{Q_0\omega^2}\right]+K\frac{\omega_0^2}{\omega^2}X_1=0$$

$$X_j\left[1-\frac{\omega_0^2}{\omega^2}+i\frac{\omega_0^2}{Q_0\omega^2}\right]+\frac{1}{2}K\frac{\omega_0^2}{\omega^2}\left[X_{j-1}+X_{j+1}\right]=0 \qquad (1)$$

$$X_N\left[1-\frac{\omega_0^2}{\omega^2}+i\frac{\omega_0^2}{Q_0\omega^2}\right]+K\frac{\omega_0^2}{\omega^2}X_{N-1}=0$$

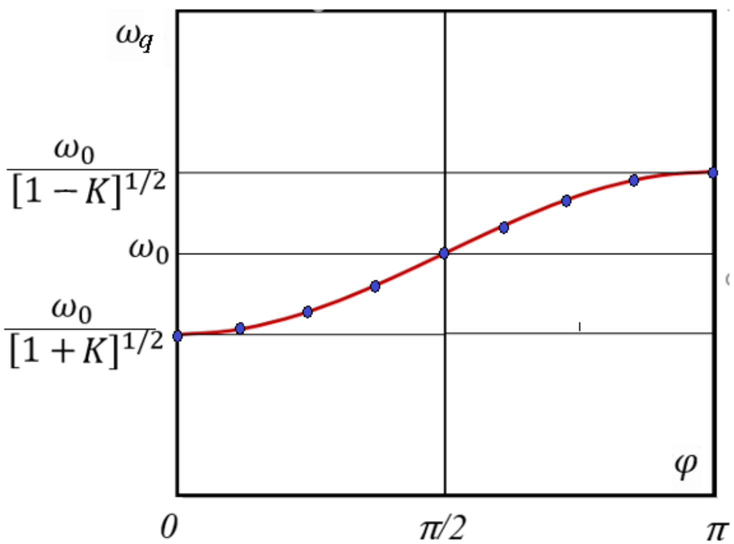
Here ω_0 corresponds to the center of dispersion curve.

Standing –Wave acceleration structures

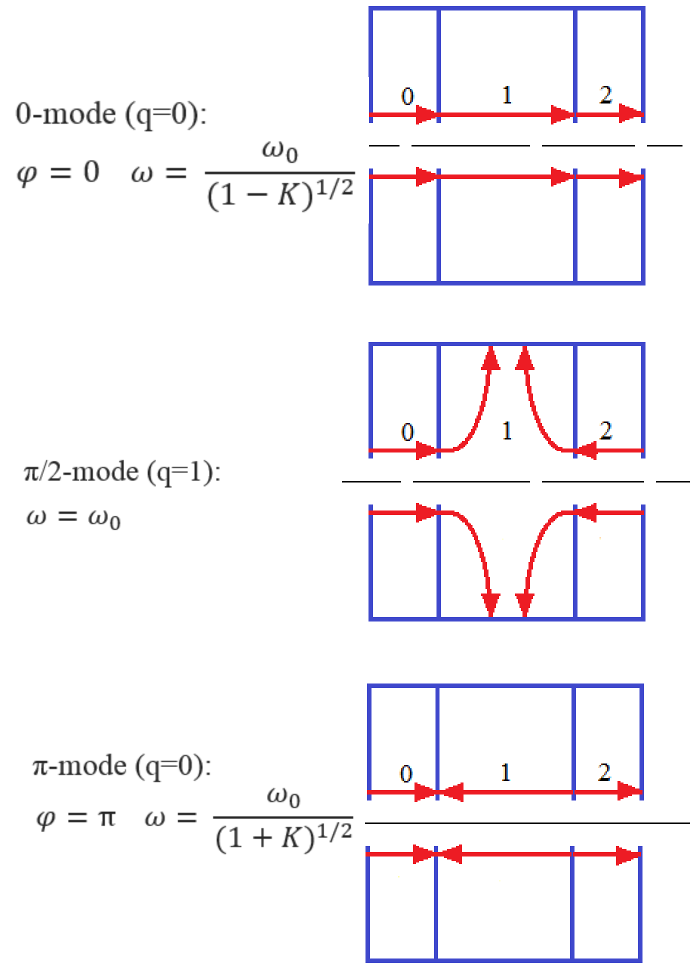
Eigenvectors and eigenvalues:

$$\hat{X}_j^q = cos\frac{\pi qj}{N};\ \omega_q^2 = \frac{\omega_0^2}{1 + Kcos\frac{\pi q}{N}}, q = 0,1, \dots N$$

Phase advance per cell: $\varphi = \frac{\pi q}{N}, q = 0,1, \dots N$



3-cell cavity (N=2)



Orthogonality:

$$\hat{X}^q \cdot \hat{X}^r \equiv \sum_{j=0}^{N} W(j)\, \hat{X}_j^q \hat{X}_j^r = \frac{N\delta_{qr}}{2W(q)},\quad \delta_{qq} = 1, and\ \delta_{qr} = 0, if\ q \neq r$$

Fermilab

Standing –Wave acceleration structures

- Perturbation of the cell resonance frequencies causes perturbation of the mode resonance frequencies $\delta\omega_q$;
- the field distribution $\delta\hat{X}_q$.

$$\omega_{0j}^{2\prime} = \omega_0^2 + \delta\omega_{0j}^2 \quad \rightarrow \hat{X}^{q\prime} = \hat{X}^q + \delta\hat{X}^q, \quad \hat{X}^q \cdot \delta\hat{X}^q$$

Variation of the equation (1) in matrix form, see

Appendix 12 $M\delta\hat{X}^q = \frac{\omega_0^2}{\omega_q^2}\left[\delta\hat{X}^q + \Omega\hat{X}^q - \frac{\delta\omega_q^2}{\omega_q^2}\hat{X}^q\right]$, gives

(here $\Omega = \begin{bmatrix} \frac{\delta\omega_{01}^2}{\omega_0^2} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{\delta\omega_{0N}^2}{\omega_0^2} \end{bmatrix}$)

$$\frac{\delta\omega_q^2}{\omega_q^2} = [2W(q)/N] \cdot \hat{X}^q\Omega\hat{X}^q;$$

$$\delta\hat{X}^q = \sum_{q'\neq q} \frac{2W(q')\hat{X}^q\Omega\hat{X}^q}{N\left(\frac{\omega_q^2}{\omega_{q'}^2} - 1\right)}\hat{X}^{q'}$$

$$\left|\delta\hat{X}^q\right| \sim \frac{\left|\delta\omega_{0j}\right|_{av}}{\left|\omega_q - \omega_{q\pm 1}\right|}$$

Fermilab

Standing –Wave acceleration structures

π/2-mode ($q=N/2$): N-even, N is the number of cells in the cavity

$$|\delta\hat{X}^{N/2}| \sim \frac{|\delta\omega_{0j}|_{av}}{|\omega_{N/2}-\omega_{N/2-1}|} \sim N\frac{|\delta\omega_{0j}|_{av}/\omega_0}{K}$$

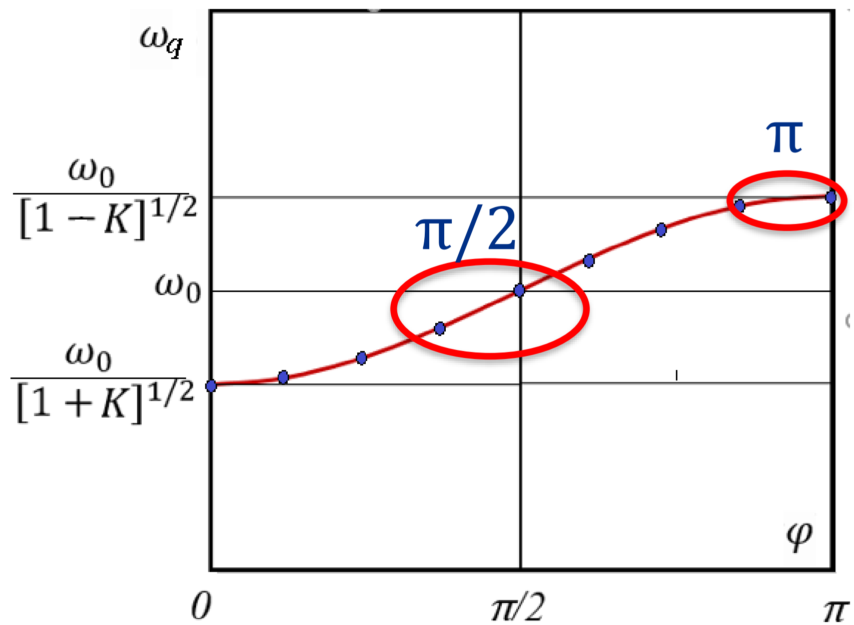
π-mode ($q=N$):

$$|\delta\hat{X}^{N}| \sim \frac{|\delta\omega_{0j}|_{av}}{|\omega_{N}-\omega_{N-1}|} \sim N^2\frac{|\delta\omega_{0j}|_{av}/\omega_0}{K}$$

SW π-mode is much less stable than π/2-mode !

For π-mode problems with

- Tuning
- Temperature stability at RT



$$\omega_q^2 = \frac{\omega_0^2}{1+K\cos\frac{\pi q}{N}}, q = 0,1,\ldots N$$

Fermilab

Standing –Wave acceleration structures

Solutions:

- ❖ Operate at $\pi/2$ mode;
- ❖ Operate at π mode:
  - Small number of cells N;
  - Increase K.

1. Operating at $\pi/2$ mode:

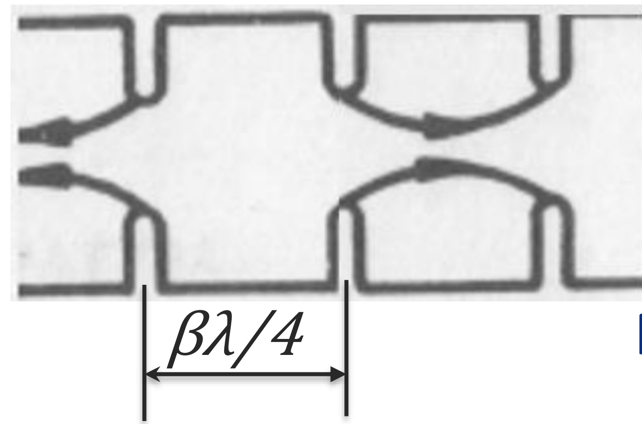
$$\hat{X}_j = cos\frac{\pi j}{2}$$

Even cells are empty!

Solution – biperiodic structures:

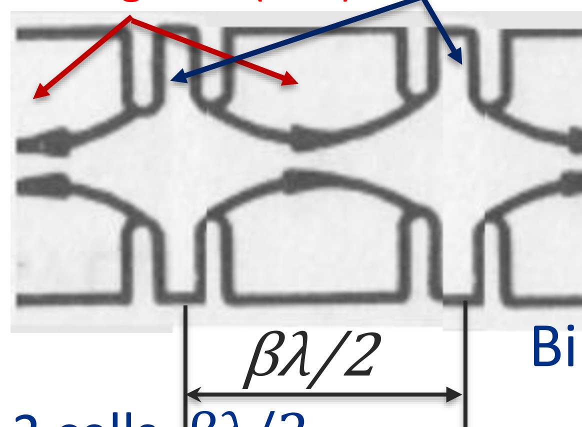
- Narrow even cells (coupling cells)
- Long odd cells (acceleration cells)
- Same length of the period containing 2 cells, βλ/2
- The structure is “π/2 for RF” and “π for the beam”

odd even



Periodic

Accelerating cells (odd) Coupling cells (even)



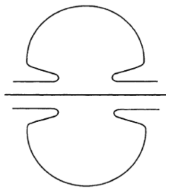
Biperiodic

Fermilab

Standing –Wave acceleration structures

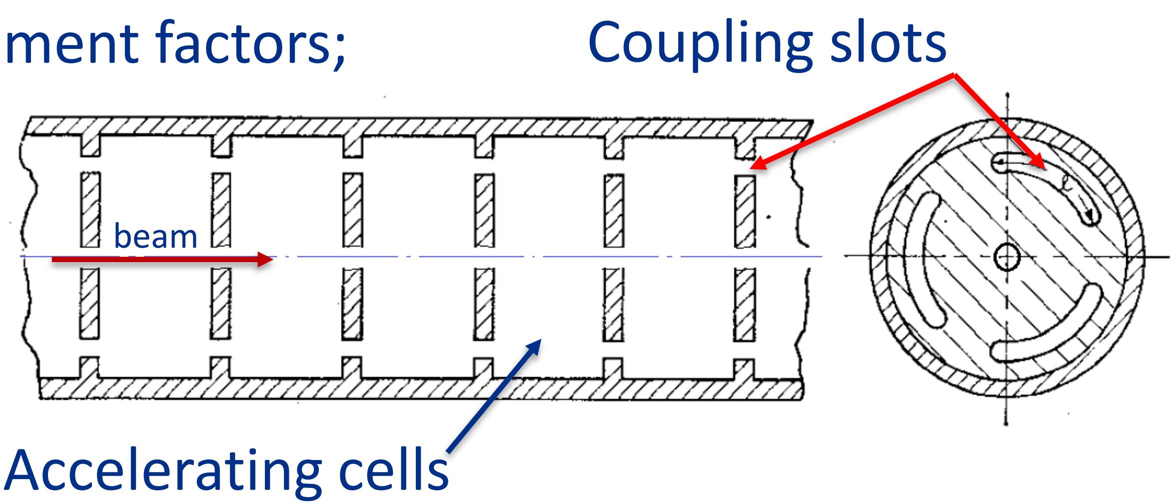
2. Increase K:

- Coupling through the aperture holes does not provide high K;
  - Aperture is limited by surface elecrtic field
  - At β<c acceleration gain on the axis drops as $\sim exp\ (ka/\beta)$*
- In this case, R_{sh} is modest (the drift tubes cannot be used) →



Solution: inductive coupling through the side slots.
Aperture may be small in this case, which provides

- Small field enhancement factors;
- High R/Q and R_{sh}.

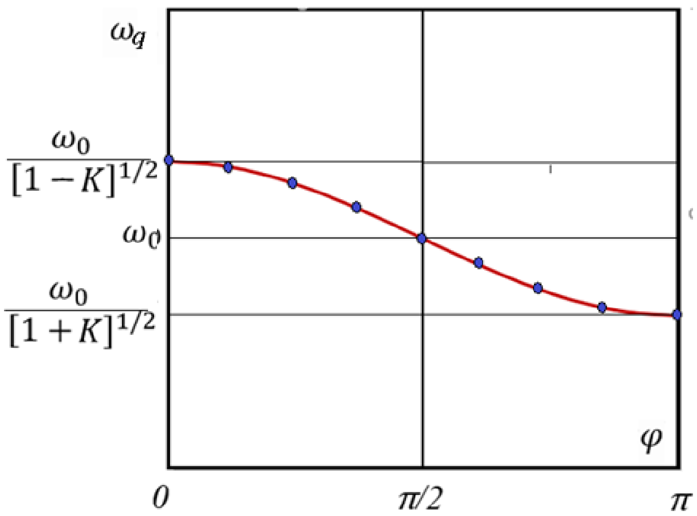


*See Lecture 1, slide 29

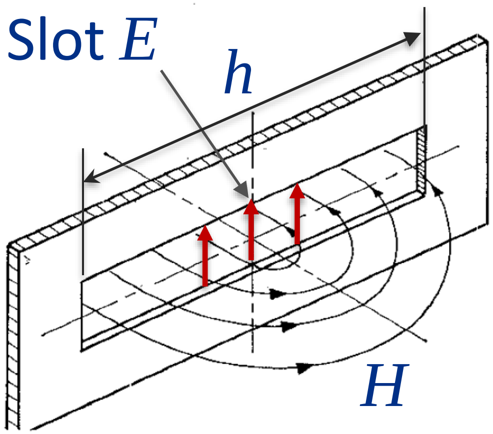
Standing–Wave acceleration structures

TEM wave in the slot → high electric field → high coupling

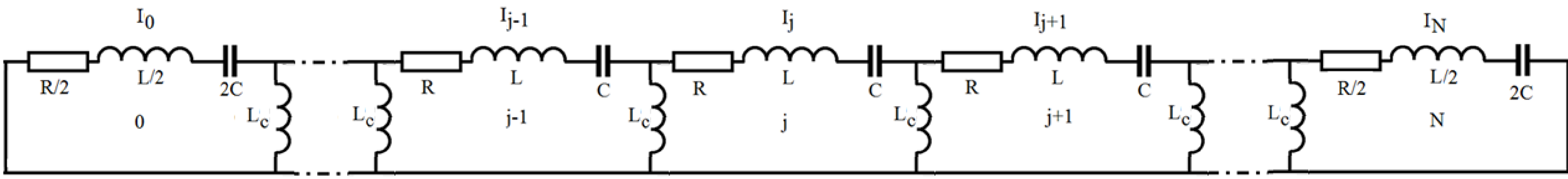
Induction coupling gives negative K



$$K = -\frac{2L_c}{L_c}$$



Slot resonance: $h = \lambda/2$. Typically, $h < \lambda/2$



Equivalent circuit below the slot resonance

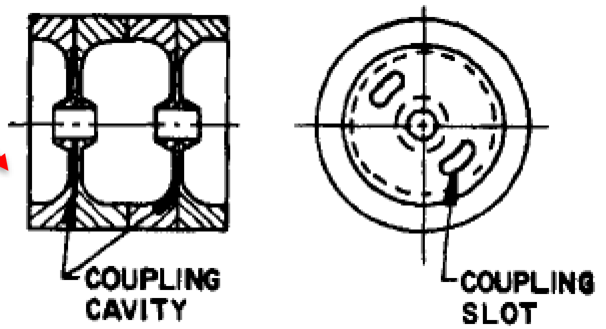
Standing–Wave acceleration structures

Combination:

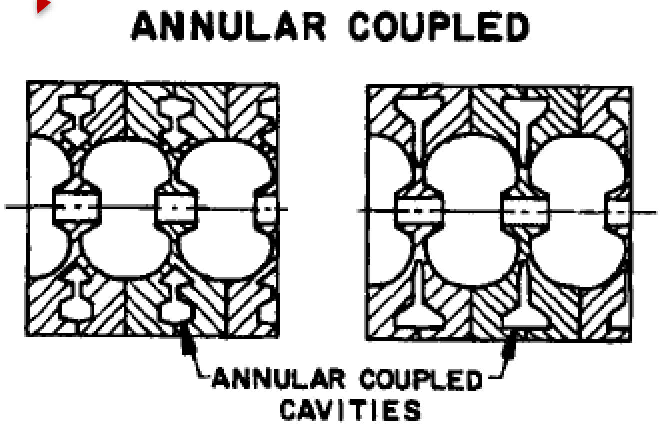
- Inductive coupling
- Biperiodic structure

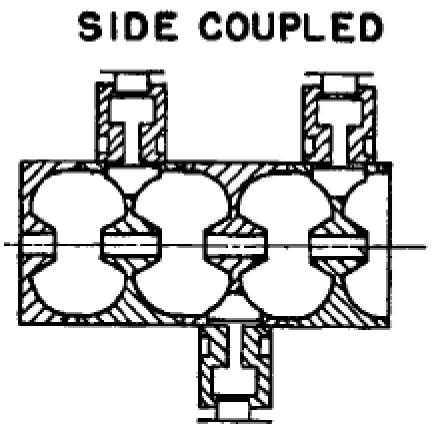
Biperiodic structures with induction coupling

- Coupling cells between accelerating cells



- Side coupling cells

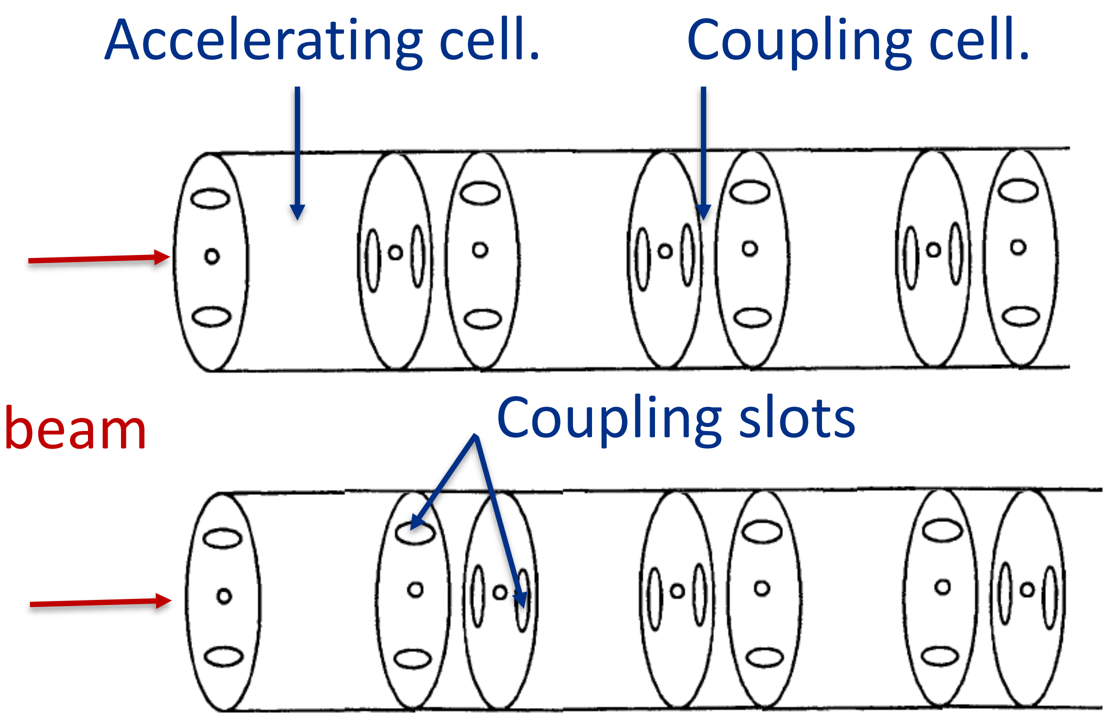




Standing–Wave acceleration structures

Inductive coupling slots cause multipole perturbation of the acceleration field, which may influence the beam dynamics:

$$x'_f = \frac{\Delta p_\perp}{p_{||}} \approx \frac{m}{ka}\left(\frac{V_{max}(a)}{\gamma m_0 c^2}\right)\left(\frac{x_i}{a}\right)^{m-1}$$



Coupling slot orientation:

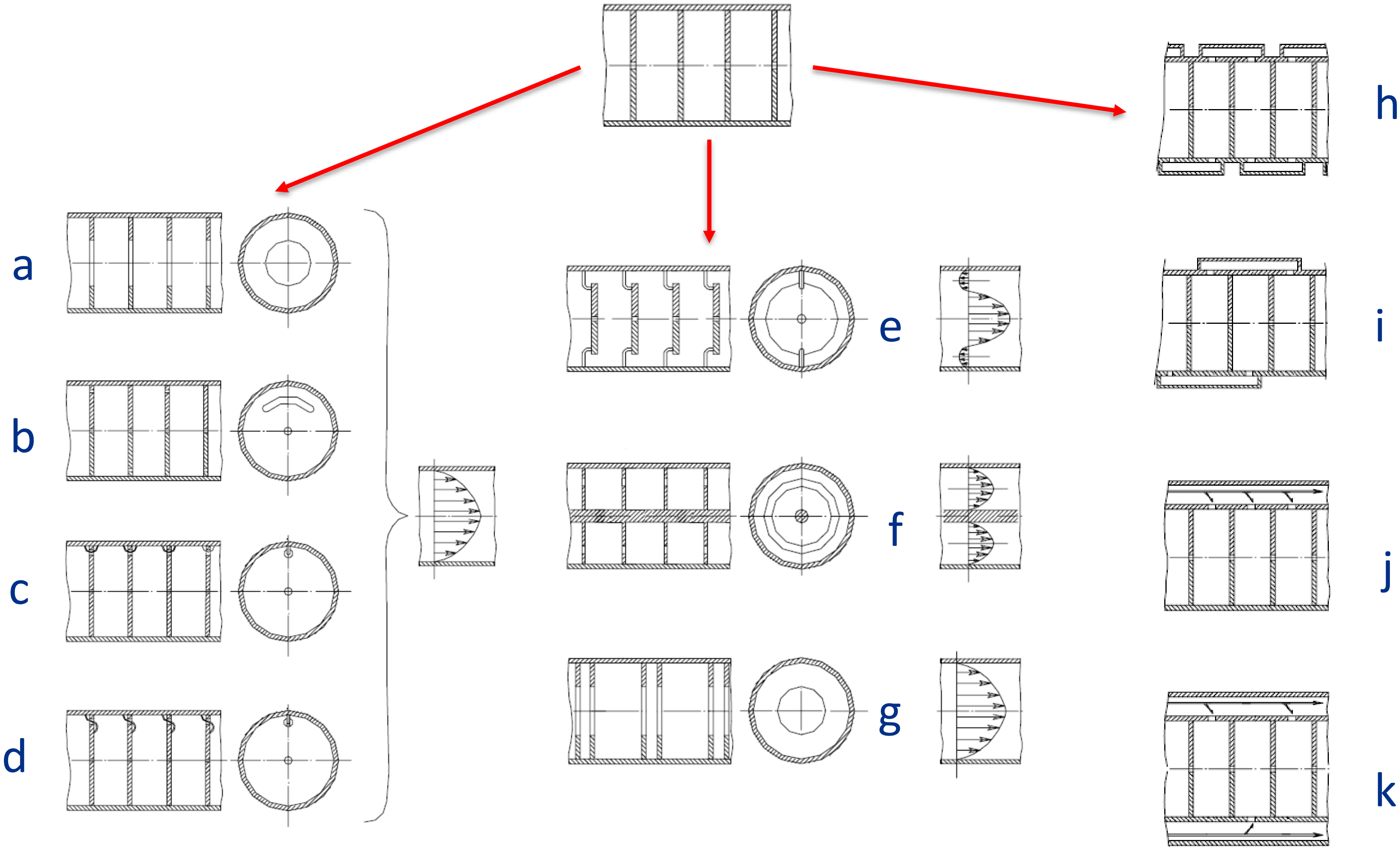
Wrong! Strong quadrupole defocusing in one of transverse directions.

Right! Strong quadrupole focusing in both directions.

Fermilab

Standing–Wave acceleration structures

Different types of the RT SW acceleration structures:



Summary:

- TW structures are not practical for RT proton accelerators (low beam loading.
- TW structures are not practical for SRF accelerators, proton and electron.
- The cure is a standing – wave structure.
- In the SW structure the operating mode is split, the number of resulting modes is equal to the number of cells.
- π/2 - mode is the most stable versus cell frequency perturbation, field distribution perturbation is proportional to the number of cells.
- 0- mode and π- mode are less stable versus cell frequency perturbation, field distribution perturbation is proportional to the number of cells squared, wich does not allow large number of cells.
- Remedy:
  - biperiodic structures;
  - inductive coupling.