

Proton and Ion Linear Accelerators

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Proton and Ion Linear Accelerators

13. RF accelerating structures, Lecture 3

Vyacheslav Yakovlev, Fermilab

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Education in Beam Physics and Accelerator Technology

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RF accelerating structures

Outline:

- Why SRF cavities?
- Multi-cell SRF cavities.
- Low- β cavities.

Chapter 5.

Why SRF cavities?

Why SRF?

The surface resistance

The radio-frequency surface resistance can be described in terms of three different contributions:

$$R_S(T, \omega, B, l) = R_{BCS}(T, \omega, l) + R_{fl}(B, l) + R_0$$

Where:

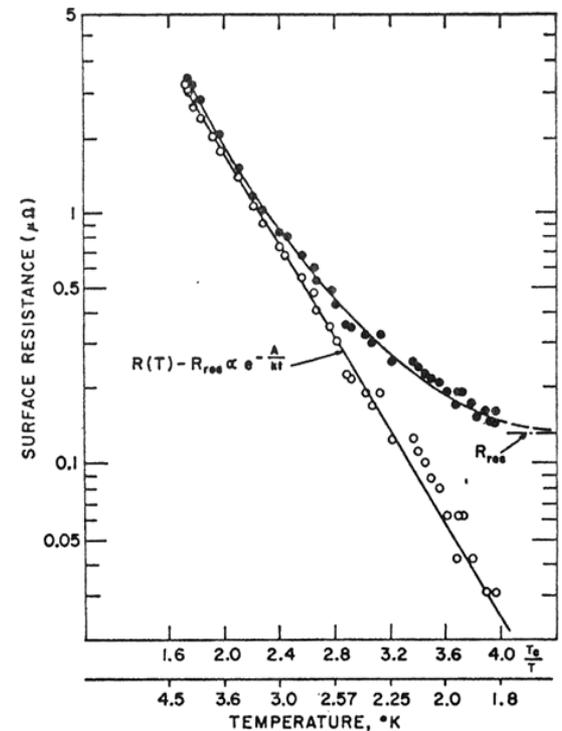
$$R_{BCS}(T, \omega, l) \cong \frac{A(l)\omega^2}{T} e^{-\frac{\Delta}{\kappa_B T}}$$

BCS resistance is caused by electron inertia;

$R_{fl}(B, l) \Rightarrow$ trapped flux surface resistance

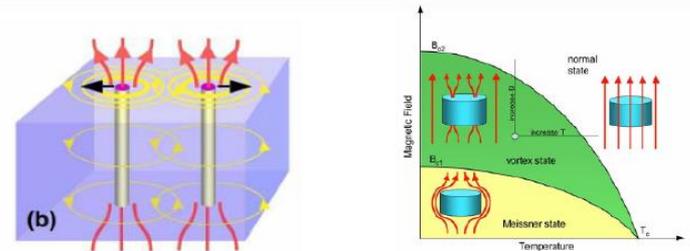
$R_0 \Rightarrow$ intrinsic residual resistance, due to:

- i. Sub-gap states
- ii. Niobium hydrides
- iii. Damaged layer
- iv. ...



J. R. Delaven. SRF1987

Type-II superconductors



Main thermodynamic parameters of type-II superconductors:

1. Critical temperature, T_c
2. Lower critical field H_{c1}
3. Upper critical field H_{c2}

Why SRF?

- For copper cavity at RT ($\sigma = 5.96e7$ S/m) for $f=1.3$ GHz one has $R_s = 9.5$ mOhm.
- For SRF Nb cavity at 2K one has $R_s = 8.5$ nOhm (ILC –type cavity, electropolishing),

It is 1.e6 times less!

Therefore, CW and high Duty Factor are possible at high gradient, even taking into account “conversion factor” for heat removal at 2K (~1000-1200W/W)

Why SRF?

Refrigeration efficiency ($W_{\text{grid}}/W_{\text{cryo}}$):

- Refrigerator's Coefficients of Performance (COP):

$$\text{COP}_{\text{real}} = 1 / (K * \eta_{\text{CARNOT}})$$

$$\eta_{\text{CARNOT}} = T / (300 - T)$$

- Refrigerator's Coefficients of Performance (COP) for different temperatures:

| Refrigeration Temperature | Carnot $1/\eta$ IDEAL WORLD | XFEL-Spec REAL WORLD | % Carnot |
|---------------------------|-----------------------------------|----------------------------|----------|
| 2 K | 149 | 870 | 17 |
| 5 K | 79 | 220 | 36 |
| 40 K | 7 | 20 | 33 |

$$P_{AC} = \sum_T \text{COP}_T \times (P_{\text{dynamic}} + P_{\text{static}})_T$$

In many cases SRF is more efficient than normal conducting RF!

- Low and medium beam loading**
- CW and long-pulse operation**

Why SRF?

Thus, SC provides the following benefits for ion and proton linacs:

1. Power consumption is much less

- operating cost savings, better conversion of AC power to beam power
- less RF power sources

2. CW operation at higher gradient possible

- shorter building, capital cost saving
- need fewer cavities for high DF or CW operation
- less beam disruption

3. Freedom to adapt better design for specific accelerator requirements

- large cavity aperture size
- less beam loss, therefore less activation
- HOMs are removed more easily, therefore better beam quality

Why SRF?

“Practical” gradient limitations for SC cavities:

- Surface magnetic field ~ 200 mT (absolute limit?) – “hard” limit
- Field emission, X-ray, starts at ~ 40 MeV/m surface field – “soft” limit
- Thermal breakdown (limits max surface field for $f > 2$ GHz for typical thickness of material, can be relaxed for thinner niobium) - “hard” limit

SRF allows significantly higher acceleration gradient than RT at high Duty Factor and CW!

Why SRF?

Different mechanisms limiting acceleration gradient:

Room Temperature:

- *Vacuum Breakdown;*
- *Metal fatigue caused by pulse heating;*
- *Cooling problems.*

Breakdown limit:

$$E_a \cdot t_p^{1/6} = \text{const}$$

$E_a \sim 20 \text{ MV/m}$ ($E_{pk} \sim 40 \text{ MV/m}$) @ 1ms or

$E_a \sim 7 \text{ MV/m}$ ($E_{pk} \sim 14 \text{ MV/m}$) @ 1sec (CW)

Superconducting:

- Breakdown usually is not considered for SC cavity;
- Thermal breakdown (quench) – for >2 GHz

Why SRF?

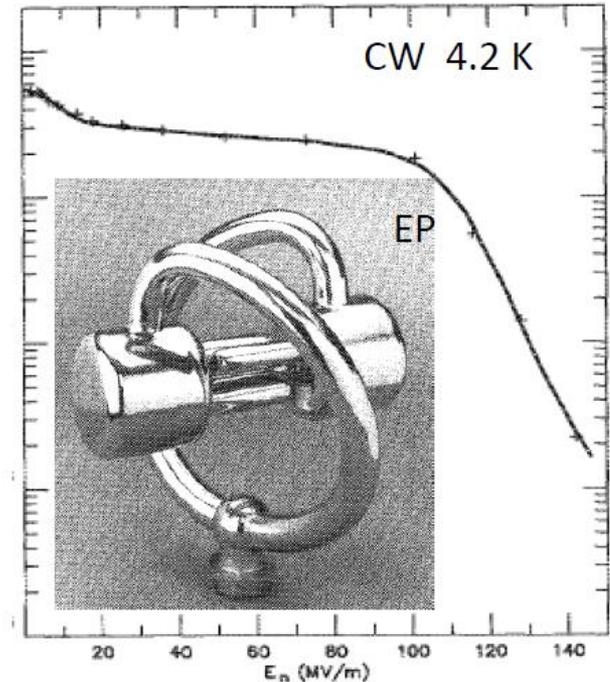
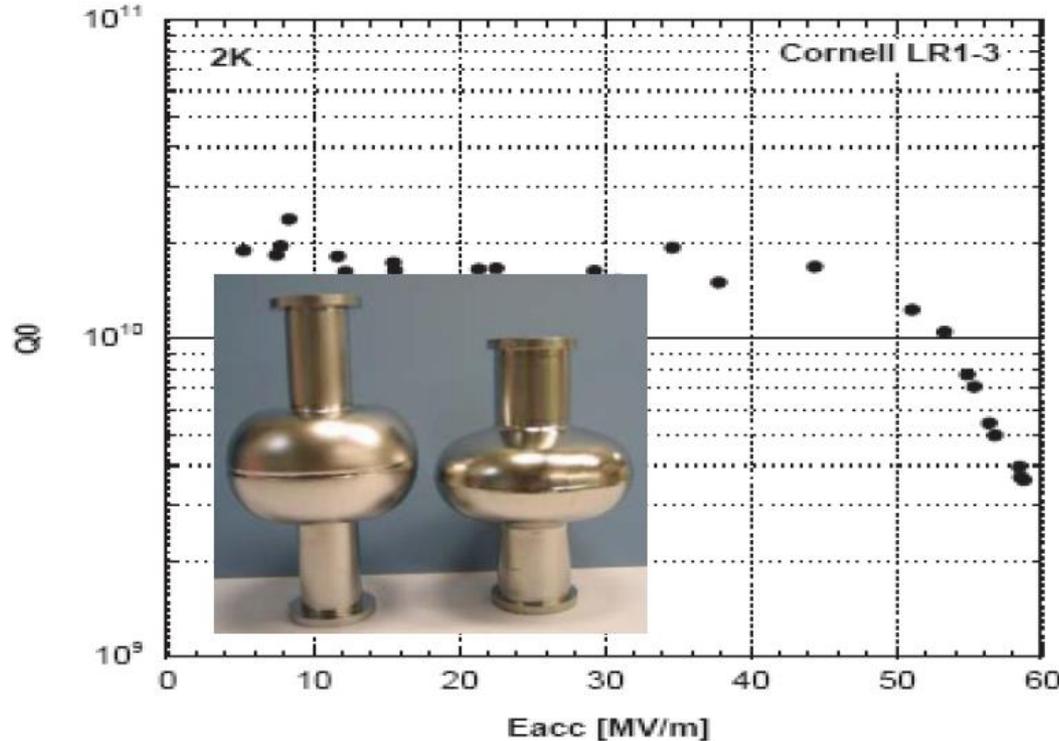
Achieved Limit of SRF electric field

- No known theoretical limit
- 1990: Peak surface field ~ 130 MV/m in CW and 210 MV/m in 1ms pulse.

J. Delayen, K. Shepard, "Test a SC rf quadrupole device", Appl. Phys. Lett, 57 (1990)

- 2007: Re-entrant cavity: $E_{\text{acc}} = 59$ MV/m ($E_{\text{pk}} = 125$ MV/m, $B_{\text{pk}} = 206.5$ mT).

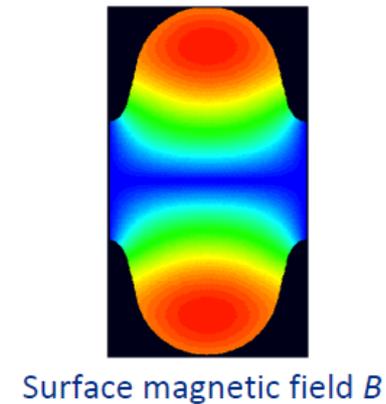
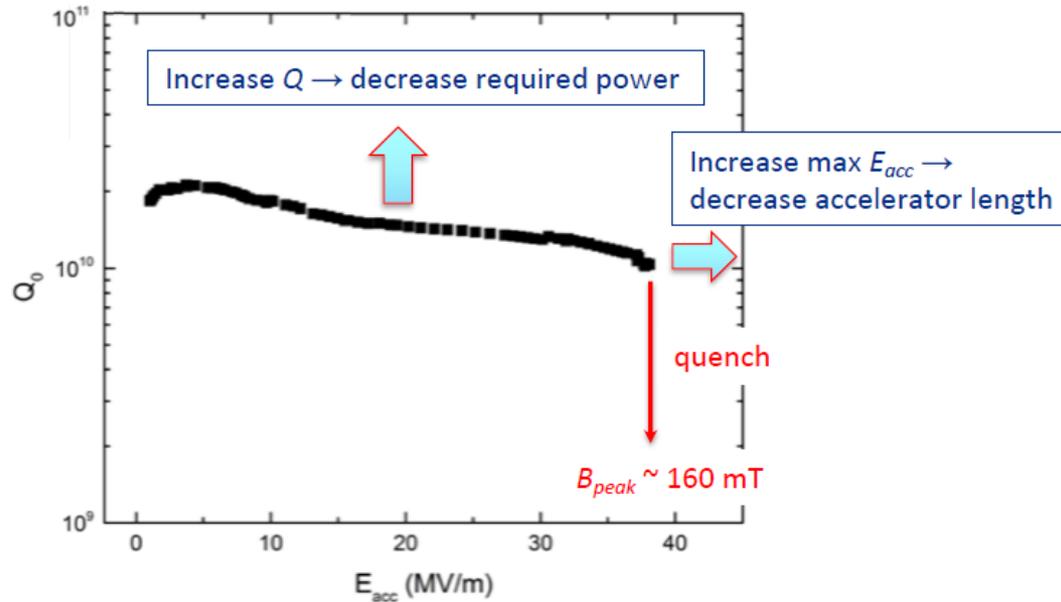
(R.L. Geng et. al., PAC07_WEPMS006) – World record in accelerating gradient



Why SRF?

Introducing Q_0 vs. E_{acc} plot:

Typical ILC-prepared TESLA cavity at $T = 2$ K (state of the art until recent breakthroughs)



- It is customary to represent performance of an SRF cavity using Q_0 vs. E_{acc} or $Q_0(E_{acc})$ plot.
- Peak surface electric and magnetic fields in the cavity are proportional to E_{acc} . Sometimes Q_0 is plotted vs. peak fields.

Why SRF?

SC cavity performance limitations

▪ **Ideal performance: Q_0 is constant until the maximal surface magnetic field is reached:**

→ fundamental limitation, limits accelerating gradient to ~ 60 MV/m for typical Nb elliptical cavity shapes.

▪ **Why is $Q_0(E_{acc})$ different in real life?**

Here are some limitations that historically plagued the SRF cavity performance:

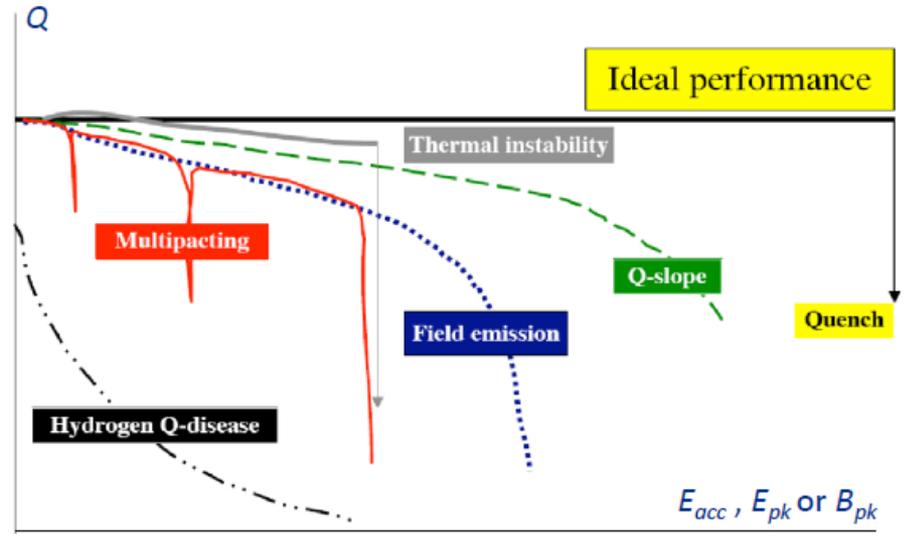
- High surface electric field → field emission

→ can be cured by applying proper preparation techniques: clean room (particulate-free) assembly, high-pressure DI water rinsing (HPR), mechanical polishing of the inner cavity surface.

- Thermal quench → use of high-purity material (RRR) to improve thermal conductivity, material quality control to avoid mechanically damaged surfaces, particulate free assembly.

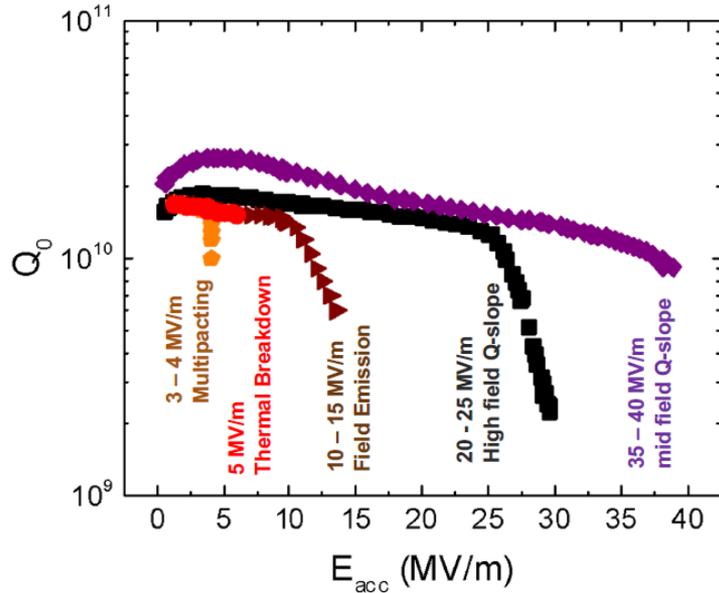
- Multipacting → use of elliptical cell shapes.

Q-disease due to lossy niobium hydrides → perform acid etch at $T < 15^\circ\text{C}$, rapid cooldown, degassing at $600 - 800^\circ\text{C}$.

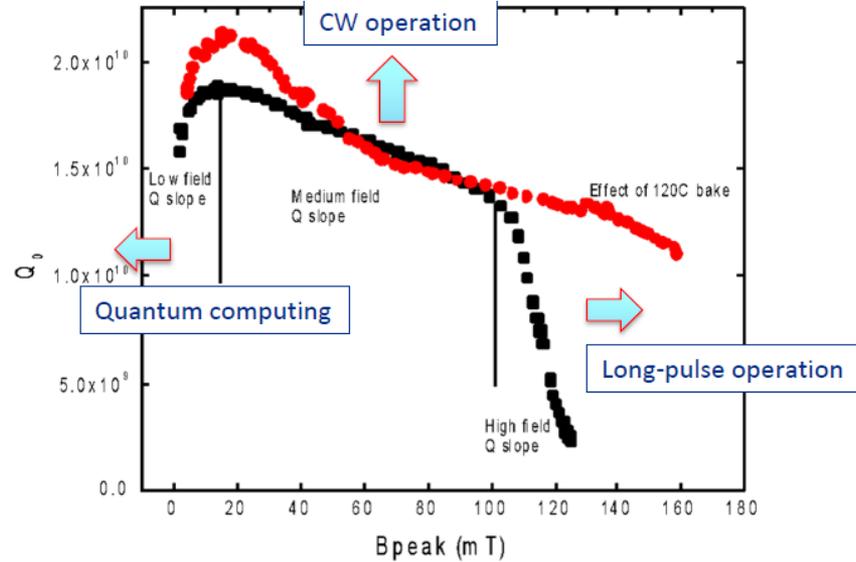


Why SRF?

$Q_0(E_{acc})$ with numbers



Q slopes



Three parts of the curve limiting performance of different applications:

1. Low field Q slope → SRF for quantum computing: need as high Q as possible to increase qubit coherence time;
2. Medium field Q slope → CW operation: cryogenics vs. linac cost optimization determines operating gradient (15-20 MV/m, LCLS-II);
3. High field Q slope → Long-pulse operation tends to favor the highest reliably achievable gradient (23.6 MV/m for XFEL, 31.5 MV/m for ILC)

Why SRF?

Standard SRF cavity surface treatments

Electron-Beam Welding - EBW

Buffered Chemical Polishing –BCP: $\text{HNO}_3 + \text{HF} + \text{H}_3\text{PO}_4$

- H_3PO_4 (phosphoric acid) is necessary to stabilize (buffer) the etching reaction between Nb and HNO_3 (nitric acid) + HF (hydrofluoric acid), which is exothermic and rapid.
- The mixture is used for Nb cavities contains HF(48%), HNO_3 (65%), H_3PO_4 (98%) in proportion 1:1:X, X=1-4.
- Still in use for low-frequency, medium gradient cavities;

Electro-Polishing –EP: $\text{H}_2\text{SO}_4 + \text{HF} + 10\text{-}12\text{V} \rightarrow$ smooth surface, lower surface fields, lower FE, higher E_{acc} and Q_0 .

- A cathode made of pure Al and a Nb cavity as an anode in mixture of sulfuric acid H_2SO_4 (93%) and hydrofluoric acid HF (50%) at 10:1 volume ratio.
- Nb is oxidized by sulfuric acid to niobium-pentoxide, which is dissolves simultaneously by hydrofluoric acid.
- Used for high-gradient cavities in pulsed regime and for medium-gradient cavities in CW.

High-Temperature Treatment

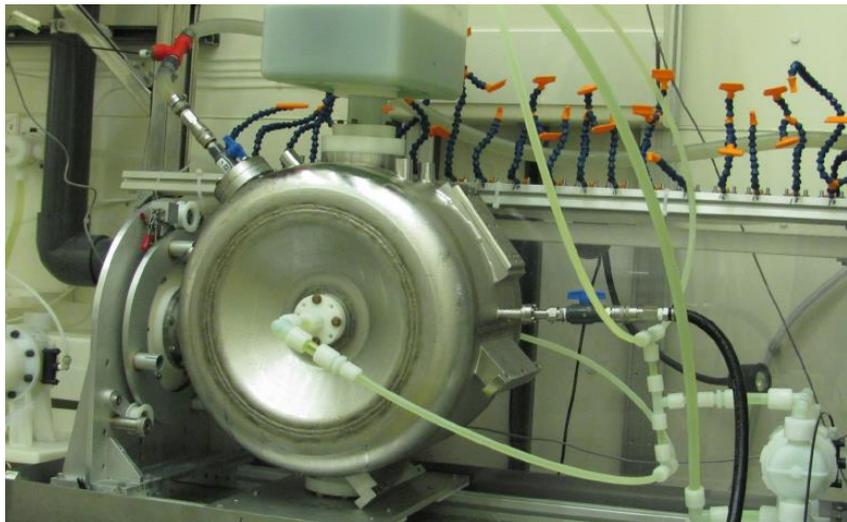
- 800C -900C backing in vacuum is used to relieve the stresses, remove defects and dislocations and degas of hydrogen.

High-Pressure Rinsing (HPR)

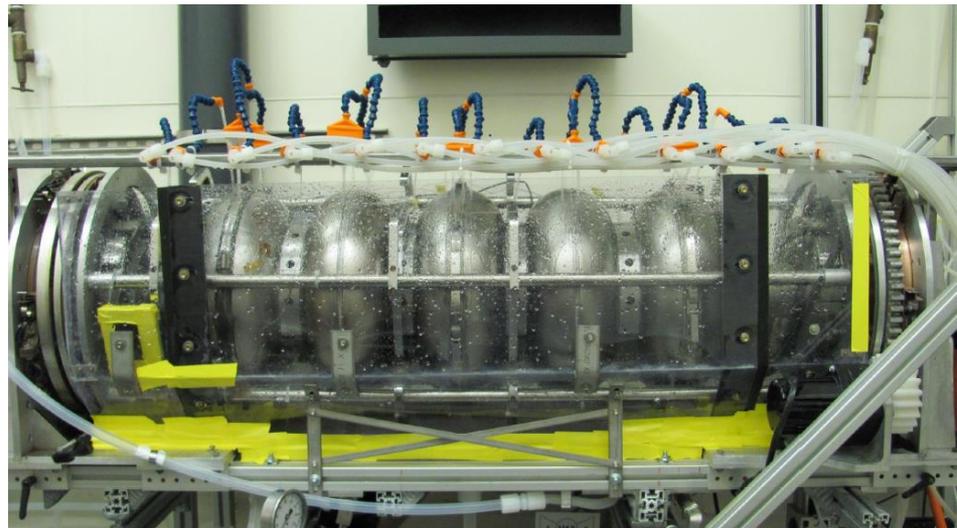
- 100 bar rinsing before assembly in a clean room

Why SRF?

BCP processing for a 325 MHz spoke cavity.



EP processing of 650 MHz elliptical cavity



Why SRF?

- Q_0 Improvement:
 - Improvement of cavity processing recipes;
 - High Q_0 preservation in CM.
- The goal is to achieve $Q_0 > 2.5e10 - 4e10$ in CM

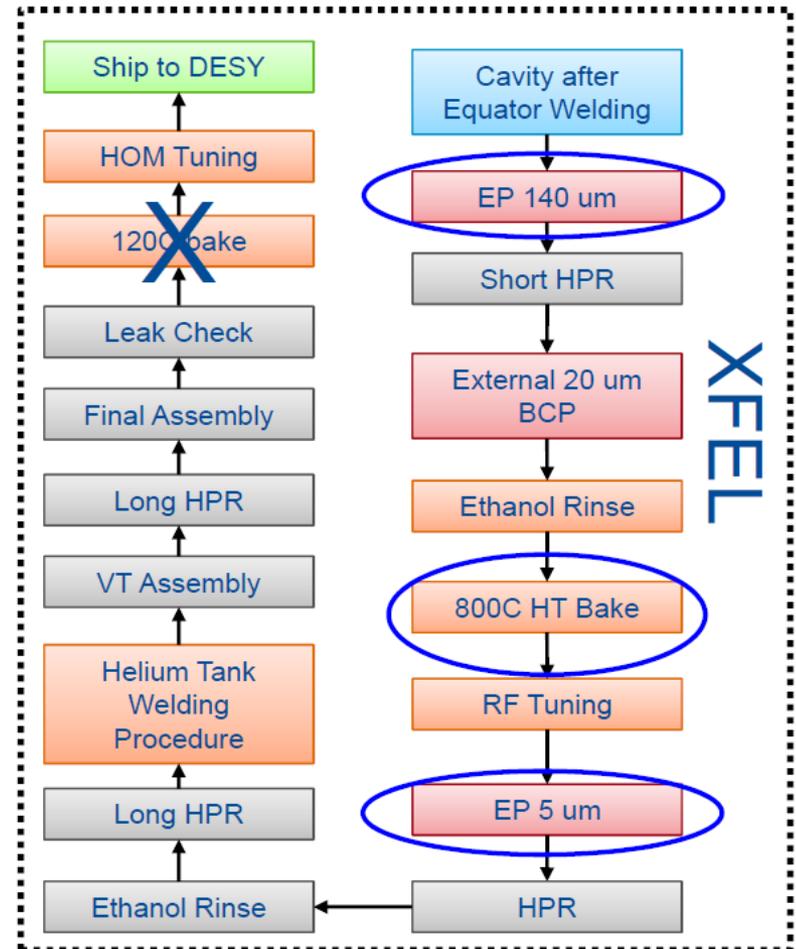
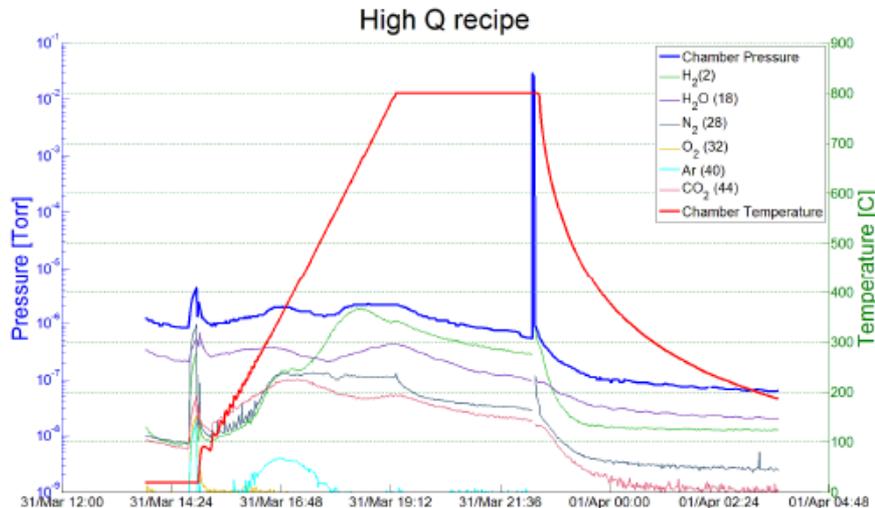


Recent breakthrough in Q_0 increase: N-doping.

- “Standard” XFEL technology provides $\sim 1.4e10@2K$, 20-23 MeV/m (CM);
- N-doping: discovered in the frame of R&D on the Project-X SC CW linac (A. Grassellino).

Cavity Treatment:

- Bulk EP
- 800 C anneal for 3 hours in vacuum
- 2 minutes @ 800C nitrogen diffusion
- 800 C for 6 minutes in vacuum
- Vacuum cooling
- 5 microns EP

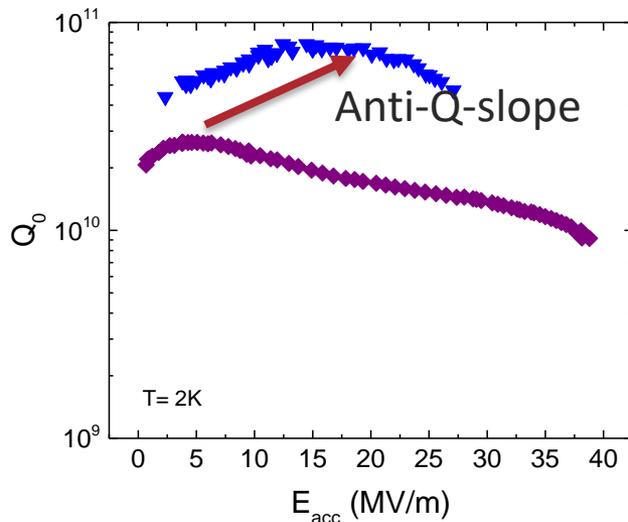


A. Grassellino, N-doping: progress in development and understanding, SRF15

N-doping

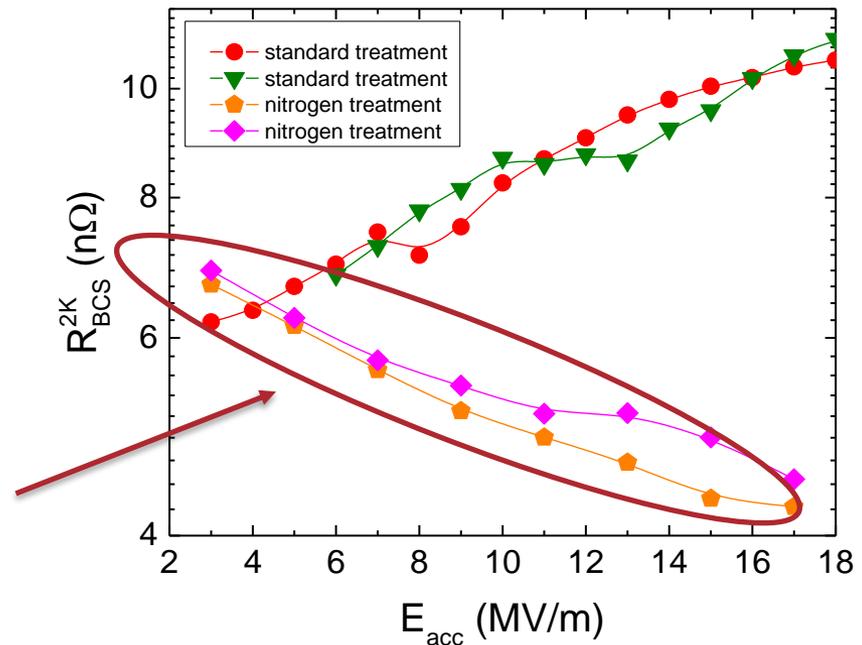
Origin of the anti-Q-slope for N-doping

$$R_S(2 K) = \boxed{R_{BCS}(2 K)} + R_0 + R_{fl}$$



Anti-Q-slope emerges from the BCS surface resistance decreasing with field

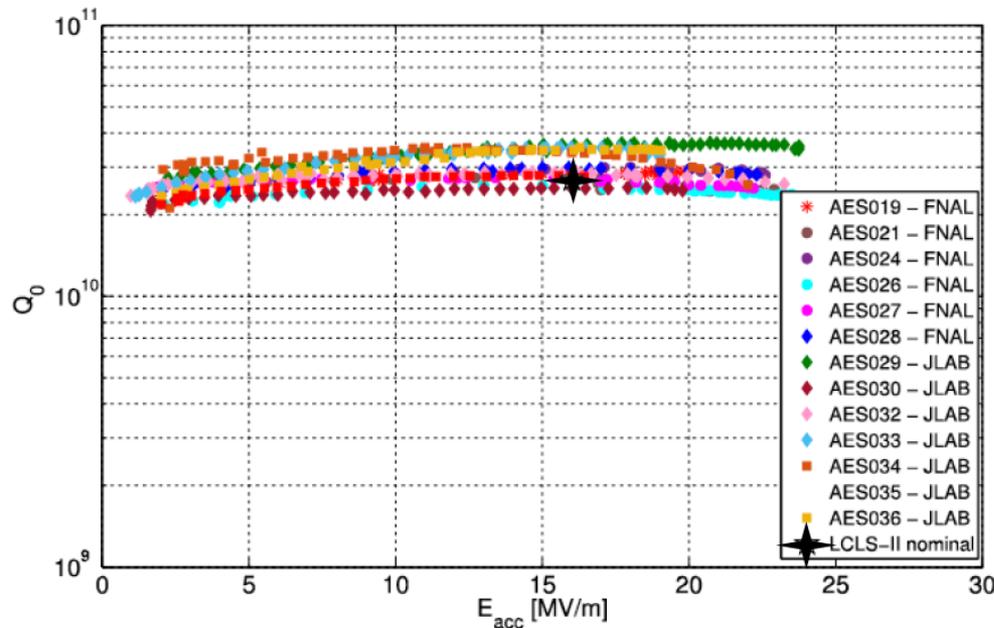
A. Grassellino et al, Supercond. Sci. Technol. **26** 102001 (2013) - Rapid Communications
A. Romanenko and A. Grassellino, Appl. Phys. Lett. **102**, 252603 (2013)



M. Martinello, M. Checchin

N-doping:

- Provides Q_0 2.5-3 times higher than “standard” processing.
- Trade-off:
 - Lower acceleration gradient, 20-22 MeV/m – not an issue for ion and proton linacs;
 - Higher sensitivity to the residual magnetic field.
- Remedy:
 - Magnetic hygiene and shielding improvement
 - Fast cooldown



VTS test results of dressed prototype cavities

Fast cooldown

- $Q_0 = G/R_s$; $R_s = 10$ nOhm for $Q_0 = 2.7e10$

$$R_s = R_0 + R_{BCS} + R_{TF}$$

$R_{TF} = s * \eta * B_{res}$, s is sensitivity to residual magnetic field B_{res} , η is flux expulsion efficiency.

η is material-dependent!

- For pCM Nb (Wah Chang):

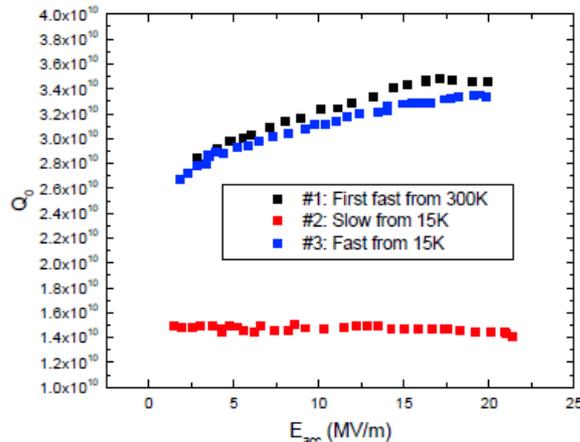
$$R_{BCS} = 4.5 \text{ nOhm}, R_0 = 1-2 \text{ nOhm}, R_{TF} \approx 1 \text{ Ohm for } 5\text{mG} \rightarrow Q_0 = 3.5e10$$

- For production material:

Change heat treatment temperature from 800 C to 900 C+ deeper EP (S. Posen):

$$R_{BCS} = 4.5 \text{ nOhm}, R_0 \approx 2 \text{ nOhm}, R_{TF} \approx 2 \text{ Ohm for } B_{res} \approx 5\text{mG} \rightarrow Q_0 > 3e10$$

Dressed N₂ doped 9 cell Sensitivity Test at 2K

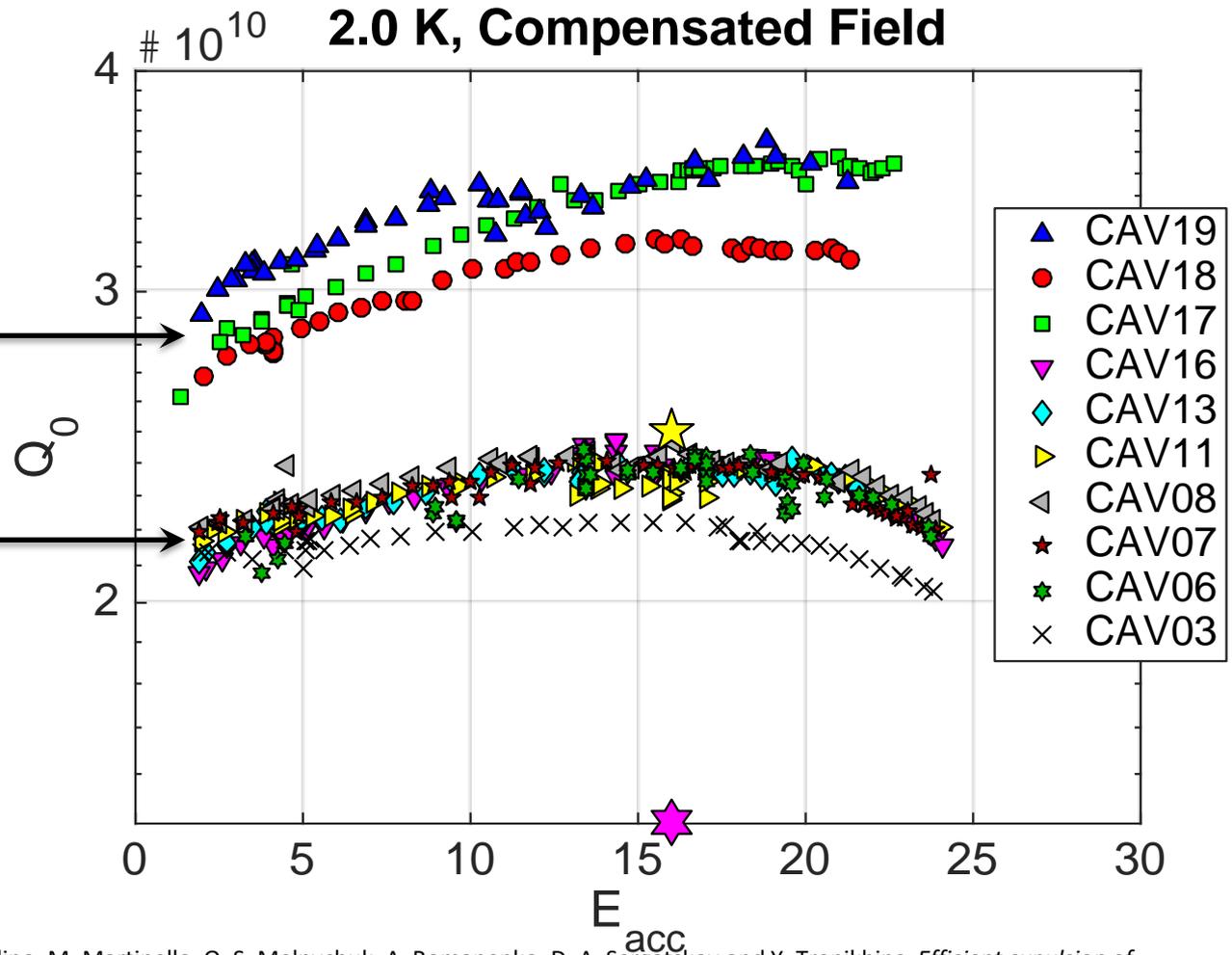


“Fast”: 2 – 3 K/minute, “slow”: < 0.5 K/minute

Impact of Modified LCLS-II Recipe on Q_0

Cavities 17, 18, 19:
modified recipe - 900
C degas, $\sim 200 \mu\text{m}$ EP,
2min/6min N doping
at 800 C

Cavities 03...16: First
production tests at
Fermilab, baseline
LCLS-II recipe - 800 C
degas, $\sim 130 \mu\text{m}$ EP,
2min/6min N doping
at 800 C



Studies leading to modified recipe:

S. Posen, M. Checchin, A. C. Crawford, A. Grassellino, M. Martinello, O. S. Melnychuk, A. Romanenko, D. A. Sergatskov and Y. Trenikhina, *Efficient expulsion of magnetic flux in superconducting radiofrequency cavities for high Q_0 applications*, J. Appl. Phys. **119**, 213903 (2016), [dx.doi.org/10.1063/1.4953087](https://doi.org/10.1063/1.4953087).

A. Romanenko, A. Grassellino, A. C. Crawford, D. A. Sergatskov and O. Melnychuk, *Ultra-high quality factors in superconducting niobium cavities in ambient magnetic fields up to 190 mG*, Appl. Phys. Lett. **105**, 234103 (2014); [http://dx.doi.org/10.1063/1.4903808](https://doi.org/10.1063/1.4903808).

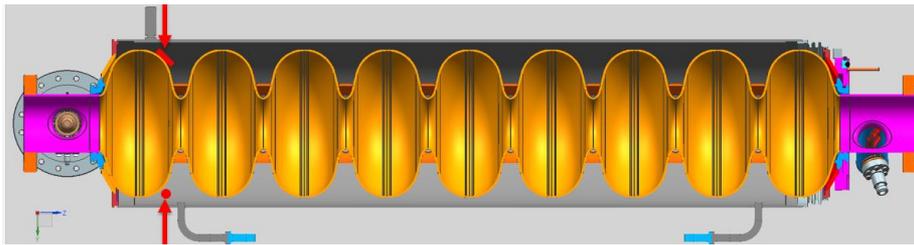
A. Grassellino, A. Romanenko, S Posen, Y. Trenikhina, O. Melnychuk, D.A. Sergatskov, M. Merio, N-doping: progress in development and understanding, Proceedings of SRF15, <http://srf2015proc.triumf.ca/prepress/papers/moba06.pdf>.

Ambient Magnetic Field Management Methods

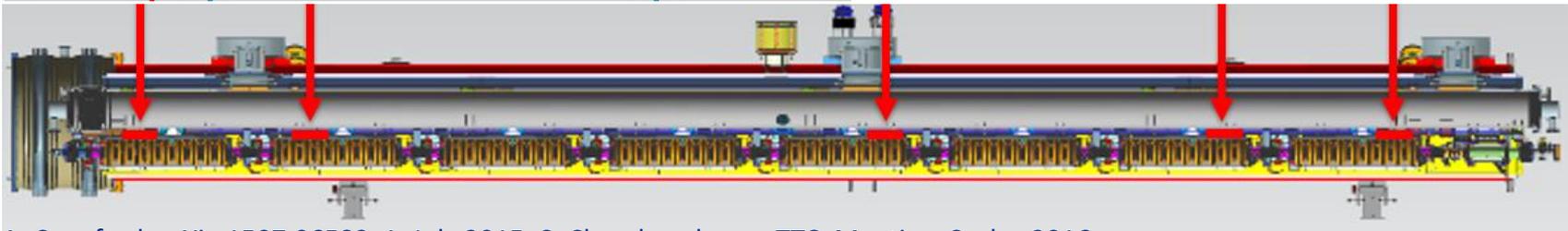
- 2-layer passive magnetic shielding
 - Manufactured from Cryoperm 10
- Strict magnetic hygiene program
 - Material choices
 - Inspection & demagnetization of components near cavities
 - Demagnetization of vacuum vessel
 - Demagnetization of assembled cryomodule / vessel
- Active longitudinal magnetic field cancellation

Magnetic field diagnostics:

- 4 cavities instrumented with fluxgates inside helium vessel (2 fluxgates/cavity)
- 5 fluxgates outside the cavities mounted between the two layers of magnetic shields



Fluxgates monitored during cryomodule assembly

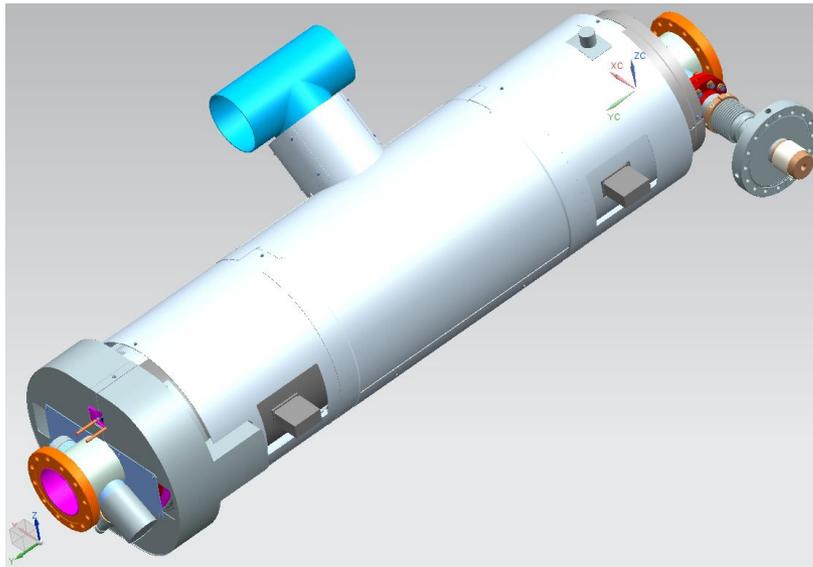


A. Crawford, arXiv:1507.06582v1, July 2015; S. Chandrasekaran, TTC Meeting, Saclay 2016

Ambient Magnetic Field Management Methods

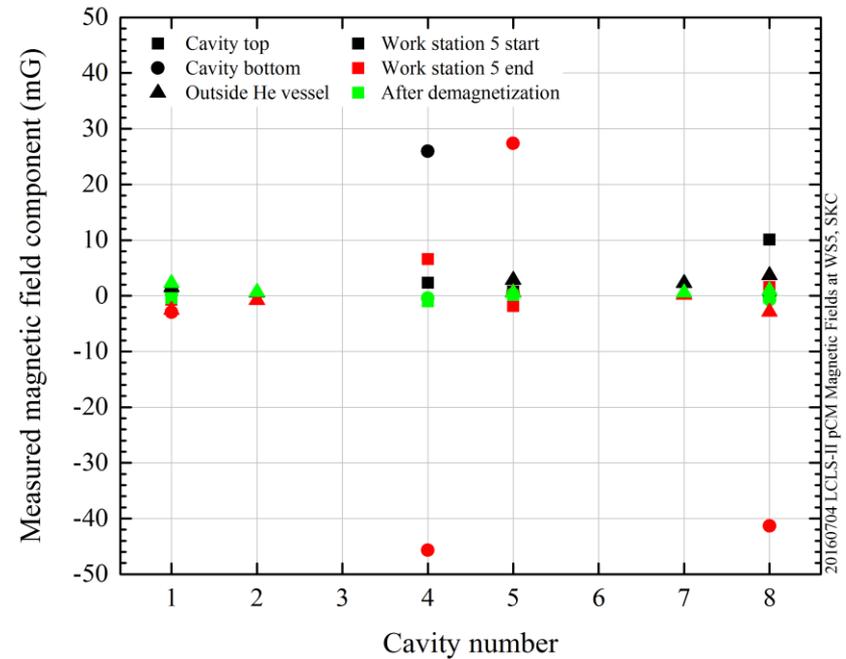


Helmholtz coils wound onto vessel directly



2-layer magnetic shields
manufactured from Cryoperm 10

S. Chandrasekaran, Linac 2016, TUPLR027



Prototype Cryomodule Latest Preliminary Results

- Cryomodule remnant field ≈ 1 mG
- Fast cool down in a cryomodule demonstrated
- $Q_0 \approx 2.7e10$ in a CW cryomodule

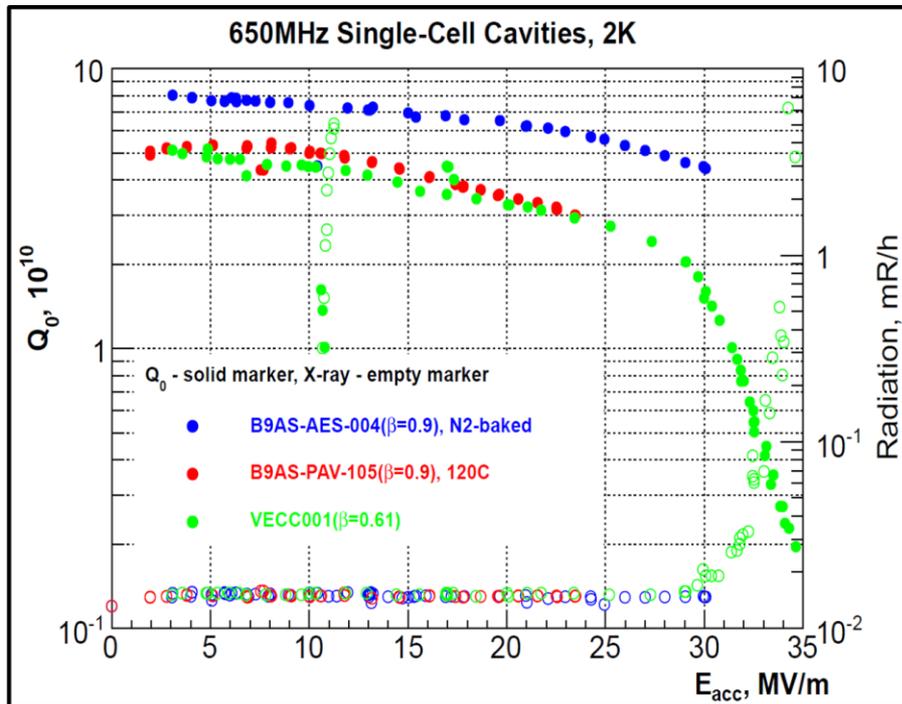
| Cavity | VTS | | pCM after RF_Conditioning | | | |
|----------------------|---------------------|----------------|---------------------------|-------------------------|-----------------|------------------------------|
| | Max Gradient [MV/m] | Q0 @16MV/m | Max Gradient*** [MV/m] | Usable Gradient* [MV/m] | FE onset [MV/m] | Q0 @16MV/m 2K** extrapolated |
| TB9AES021 | 23 | 3.1E+10 | 19.6 | 18.2 | 14.6 | 2.6E+10 |
| TB9AES019 | 19.5 | 2.8E+10 | 19 | 18.8 | 15.6 | 2.6E+10 |
| TB9AES026 | 21.4 | 2.6E+10 | 17.3 | 17.2 | 17.4 | 2.7E+10 |
| TB9AES024 | 22.4 | 3.0E+10 | 21 | 20.5 | 21 | 2.5E+10 |
| TB9AES028 | 28.4 | 2.8E+10 | 14.9 | 14.2 | 13.9 | 2.4E+10 |
| TB9AES016 | 18 | 2.8E+10 | 17.1 | 16.9 | 14.5 | 2.9E+10 |
| TB9AES022 | 21.2 | 2.8E+10 | 20 | 19.4 | 12.7 | 3.2E+10 |
| TB9AES027 | 22.5 | 2.8E+10 | 20 | 17.5 | 20 | 2.5E+10 |
| Average | 22.1 | 2.8E+10 | 18.6 | 17.8 | 16.2 | 2.7E+10 |
| Total Voltage | 183.1 MV | | 154.6 | 148.1 | | |

*Usable Gradient: demonstrated to stably run CW, FE < 50 mR/h, no dark current

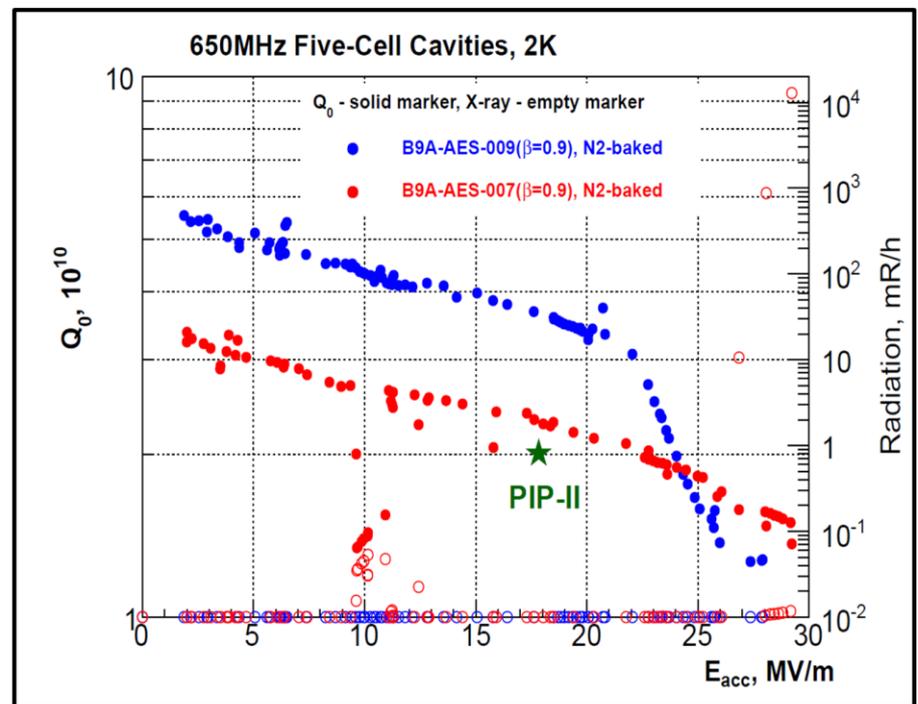
**Fast cooldown from 45K, >40 g/sec, extrapolated from 2.11K

G. Wu, FNAL SRF Department meeting, 24 October 2016, <https://indico.fnal.gov/conferenceDisplay.py?confId=13185>

650 MHz elliptical cavity performance testing at FNAL



Single-cell $B=0.90$ and $B=0.61$ test results.



Five-cell $B=0.90$ test results.

*A. M. Rowe, et al, CAVITY PROCESSING AND PREPARATION OF 650 MHz ELLIPTICAL CELL CAVITIES FOR PIP-II, LINAC 2016

Microphonics and LFD:

Narrow bandwidth of the cavities caused by low beam loading:

- $Q_{\text{load}} = U / (R/Q) / I_{\text{beam}}$ - very high for small beam current of few mA, $Q_{\text{load}} \sim 1e7-1e8$;
- Cavity bandwidth: $f / Q_{\text{load}} \sim$ tens of Hz.



• Pressure variation in the surrounding He bath:

$$\Delta f_{\text{He}} = df/dP \times \Delta P, \Delta P \sim 0.05-0.1 \text{ mbar at 2 K.}$$

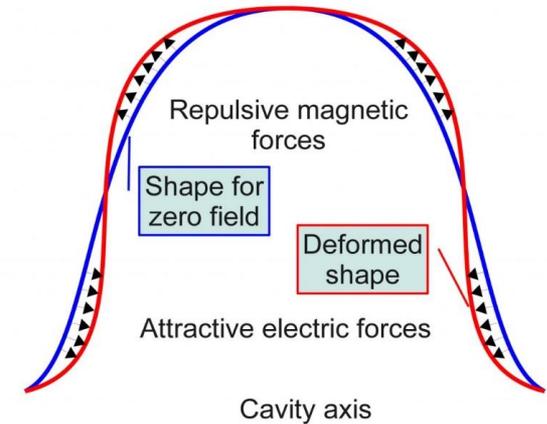
$$df/dP = 30-130 \text{ Hz/mbar (ILC)}$$

• Internal and external vibration sources (microphonics);

• Radiation pressure from the RF field, Lorentz Force Detuning (in pulsed mode).

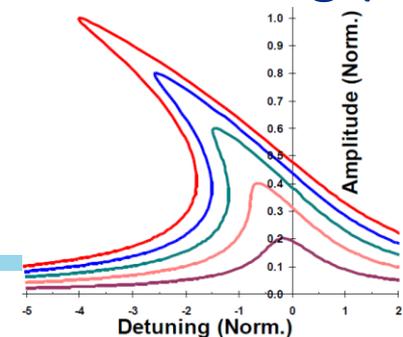
$$\Delta f_{\text{LFD}} = k_L E^2, k_L - \text{Lorentz coefficient,}$$

For typical elliptical cavities $k_L \sim -1 \text{ Hz}/(\text{MeV/m})^2$.



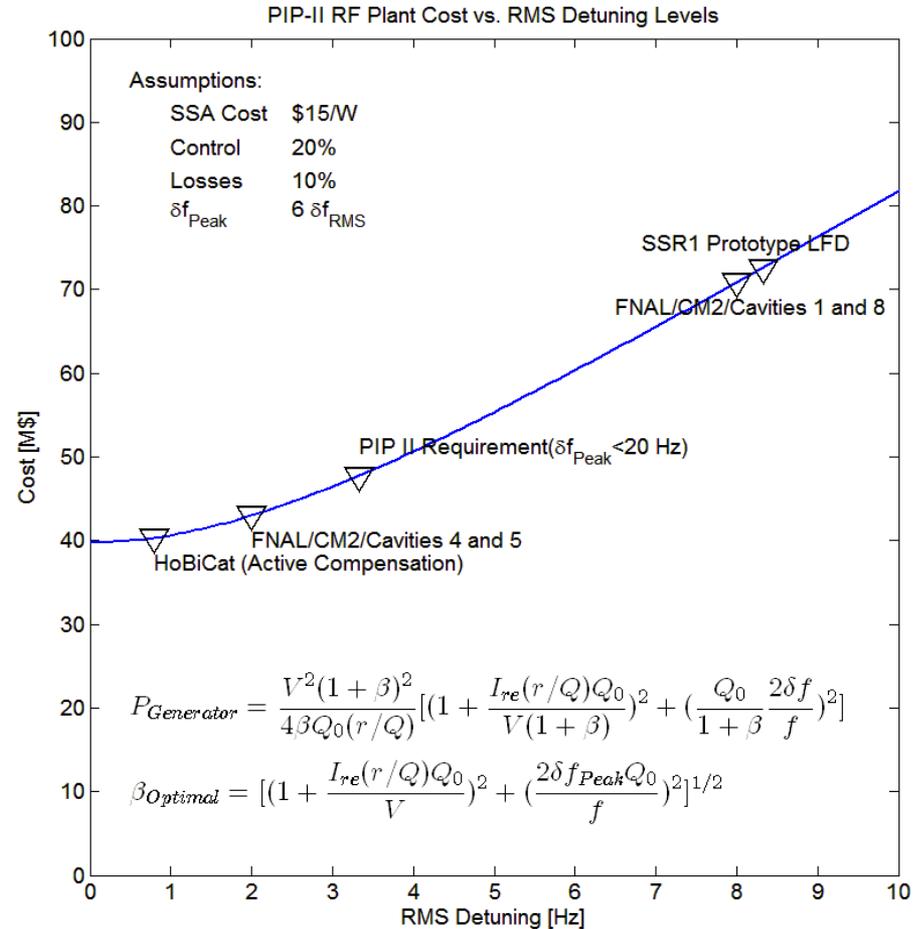
$$P_s = \frac{1}{4} (\mu |\vec{H}|^2 - \epsilon_0 |\vec{E}|^2)$$

$$\Delta f_0 = (f_0)_2 - (f_0)_1 = -K E_{\text{acc}}^2$$



Microphonics:

- Detuned cavities require more RF power to maintain constant gradient
- Providing sufficient reserve increases both the capital cost of the RF plant and the operating cost of the machine
- **PEAK** detuning drives the RF costs
- Beam will be lost if RF reserve is insufficient to overcome PEAK detuning



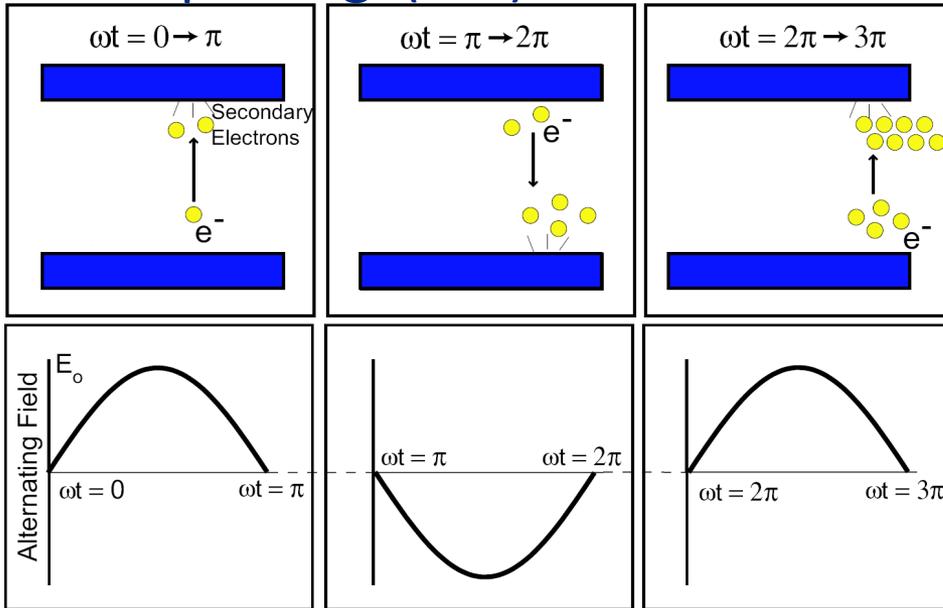
Microphonics Control Strategies

Microphonics can be mitigated by taking some combination of any or all of the following measures:

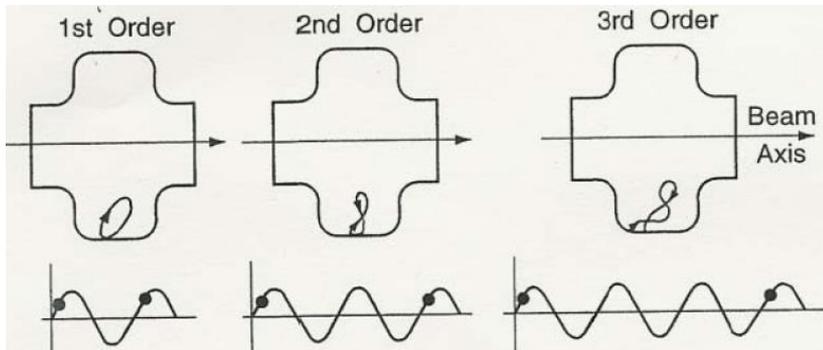
- Providing sufficient reserve RF power to compensate for the expected peak detuning levels.
- Improving the regulation of the bath pressure to minimize the magnitude of cyclic variations and transients.
- Reducing the sensitivity of the cavity resonant frequency to variations in the helium bath pressure (df/dP).
- Minimizing the acoustic energy transmitted to the cavity by external vibration sources.
- Actively damping cavity vibrations using a fast mechanical or electromagnetic tuner driven by feedback from measurements of the cavity resonant frequency.

The optimal combination of measures may differ for different cavity types.

Multipacting (MP) in SRF cavities

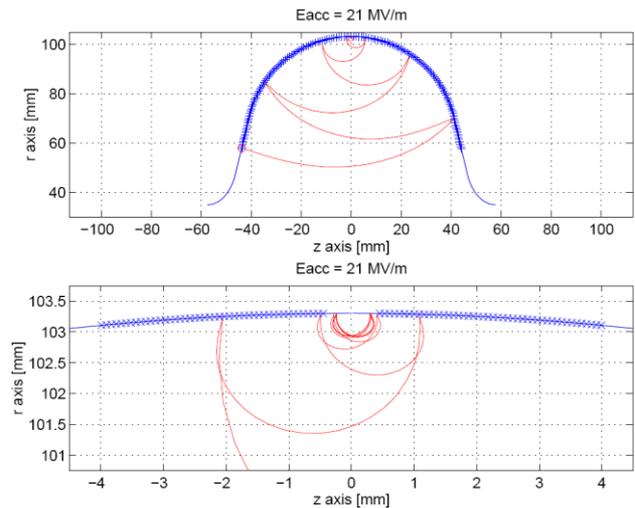
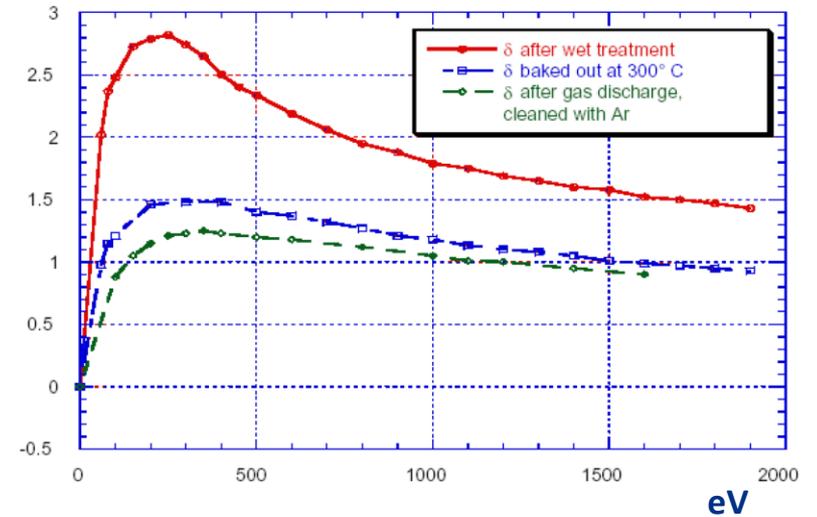


Multipactor discharge with an electric field oscillating between two metal electrodes.



Typical one-point multipactor trajectories for orders 1, 2 and 3.

Secondary emission coefficient for Nb

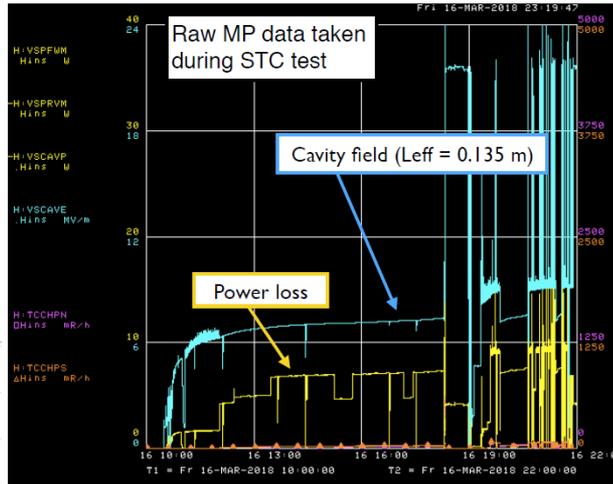


Two point MP in 1.3GHz TESLA cavity. 2D simulations

Multipacting in SRF cavities



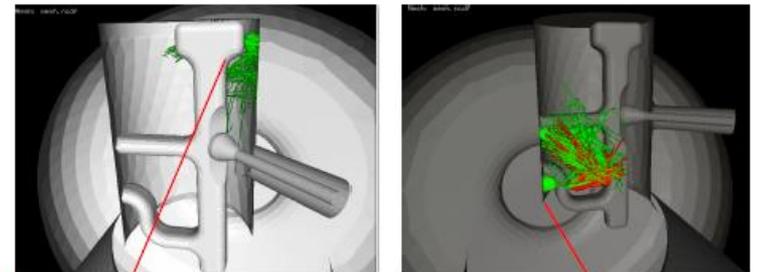
Strong MP in SSR1 at 5, 6.5 and 7 MV/m. 120 C bake for 48 h helps to reduce MP conditioning time



3.9 GHz HOM coupler failure due to overheating caused by MP: redesigned to shift MP barriers above operating gradients



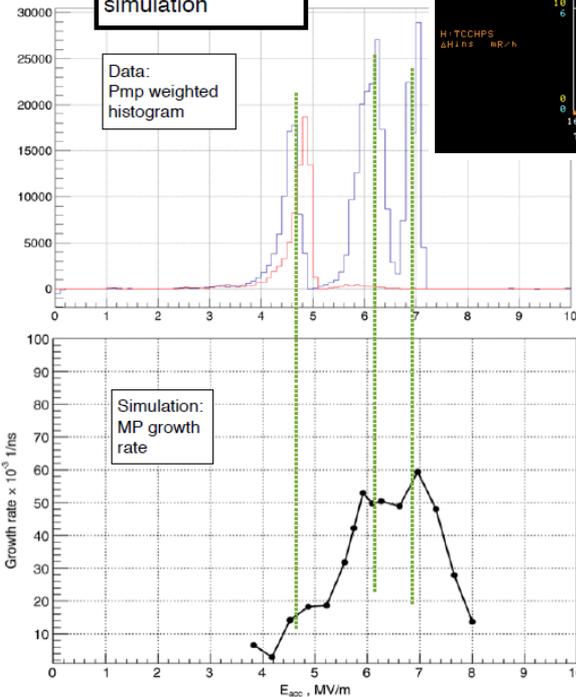
Multipacting in HOM2 at SNS



SSR1 cavity

Comparison to MP simulation

Data: Pmp weighted histogram



- QWR, HWR and SSR are prone to MP, need up to 10 -15 hours to process;
- Elliptical cavities have much better performance.

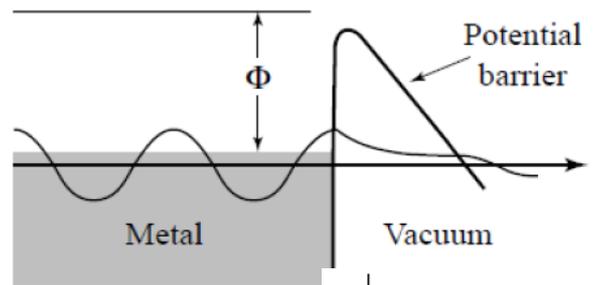
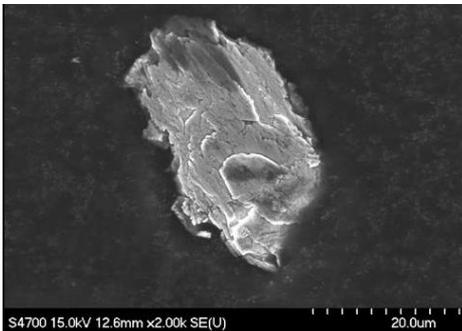
Good agreement between MP conditioning data (Pmp weighted histogram) and MP simulation (growth rate)

Field emission (FE) and dark currents in SRF cavities

- ❖ FE in SRF cavities is originated from *localized sites* on the inner cavity surface.
- ❖ The predominant source emitters are microscopic particulates adhering to the inner cavity surface, chemical residuals, and geometrical flaws.
 - Field emitters introduced by the necessary chemical surface processing → post chemistry ultrasonic cleaning and high pressure water rising.
 - Field emitters introduced through the cavity opening ports onto the cavity surface, at a time beyond the completion of final cleaning, from external sources → SRF cavities are assembled in large-sized high-quality Class 10 cleanliness clean rooms into cavity strings; critical assembly steps are done with the opening port facing down; cavity strings are evacuated slowly etc.

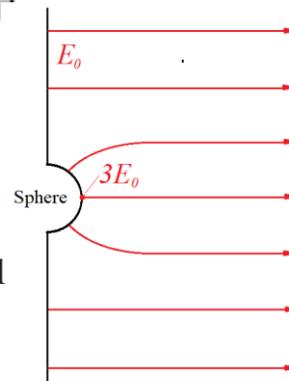
❖ Diagnostics:

- X-ray monitoring/mapping
- Temperature monitoring/mapping
- Electron detecting
- Optical imaging:



Field enhancement factor β :

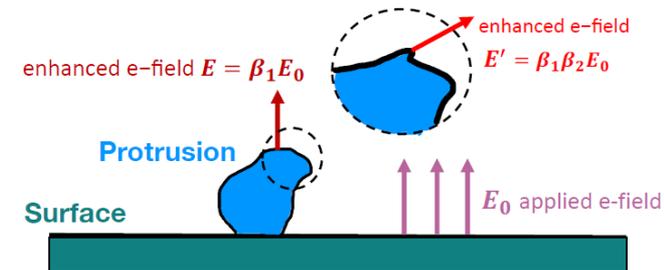
For a spherical protrusion in a metal wall $\beta = 3$



The tunneling current density, $j(E)$

$$J(E) = k \frac{1.54 \times 10^{-6} (\beta E)^{5/2}}{\Phi} \exp\left(-\frac{6.83 \times 10^9 \Phi^{3/2}}{\beta E}\right)$$

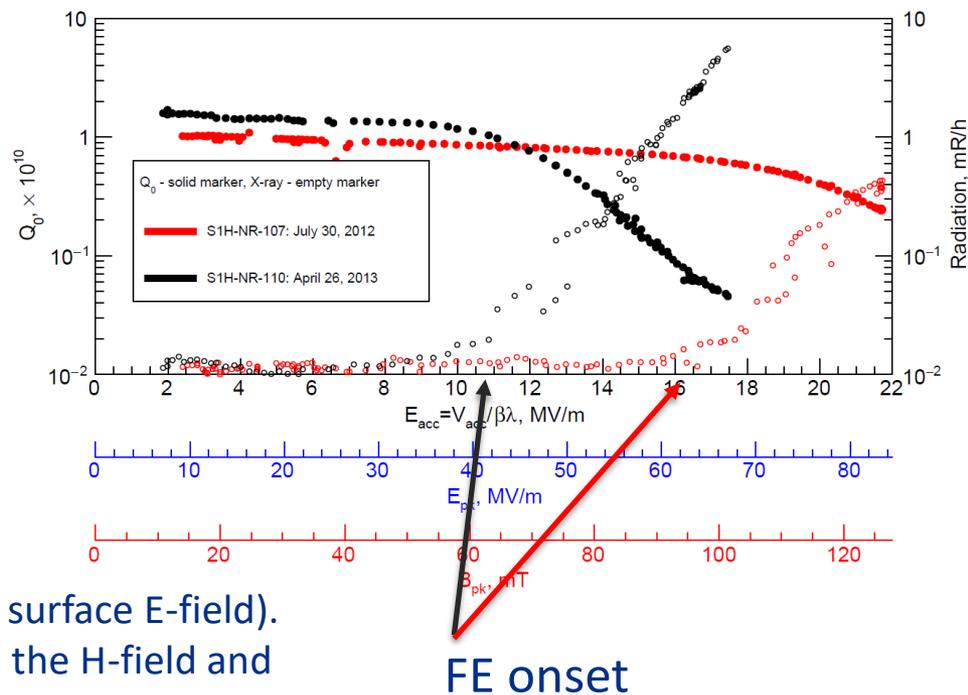
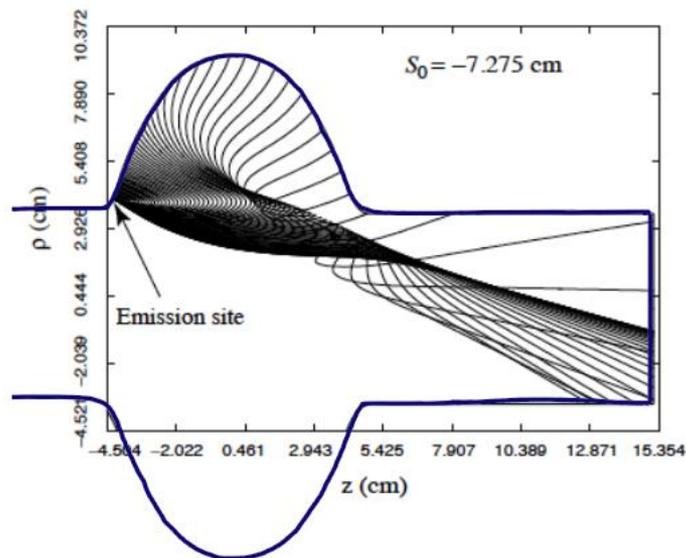
j – current density in A/m²,
 E – surface electric field in MV/m,
 Φ – work function in eV,
 β – field enhancement factor (10-100)
 k – effective emitting surface area.



Field emission (FE) and dark currents in SRF cavities

Effect of dark current

- heat and RF loading of the cavity
- production of avalanches of secondary electrons
- accelerating to hundreds of MeV before being kicked out by down stream quadrupoles
- originating electromagnetic cascade showers in the surrounding materials

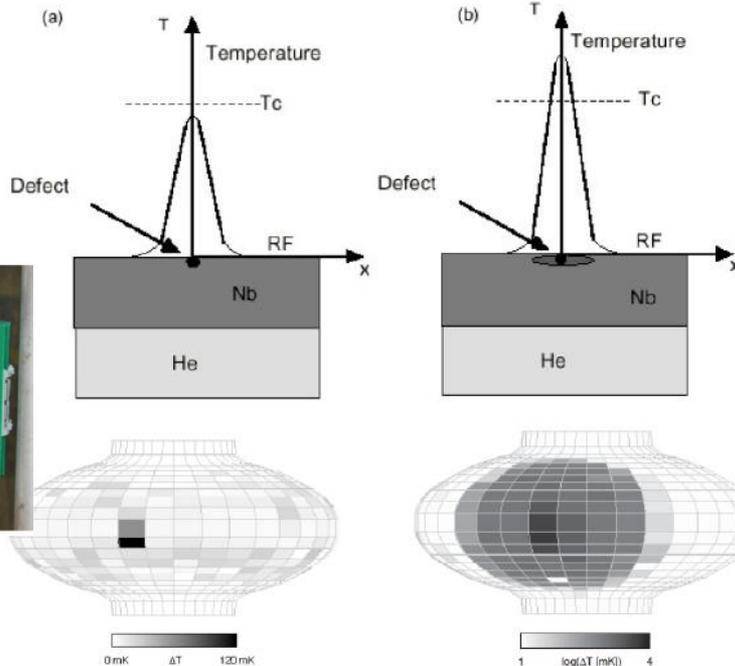


- The emitter is located at the cell entrance (high surface E-field).
- Significant number of FE electrons bend back in the H-field and strike the wall

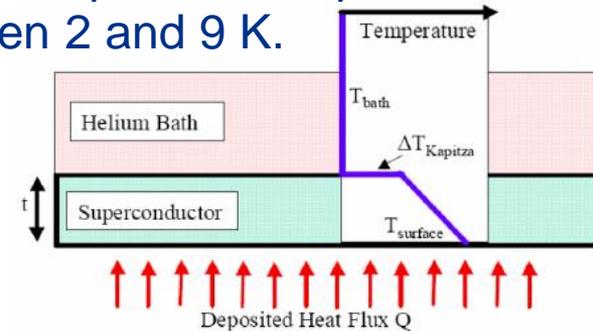
Thermal breakdown

- If there is a localized heating, the hot area will grow with field. At a certain field there is a thermal runaway and the field collapses (loss of superconductivity or quench).
- Thermal breakdown occurs when the heat generated at the hot spot is larger than that can be evacuated via Nb wall to the helium bath.

Temperature mapping



- Both the thermal conductivity and the surface resistivity of Nb are highly temperature dependent between 2 and 9 K.



$$H_b^2 = \frac{T_0^3}{2 \cdot R_s(T_0) \cdot (\Delta \cdot T_c - T_0)} \cdot \left(\frac{k \cdot h}{k + h \cdot d} \right)$$

T_0 - He bath temperature,

T_c - critical temperature,

Δ - energy gap,

$h(T_0)$ - Kapitza resistance,

$k(T_0)$ - thermal conductivity,

$$R_s(T) = R_0 \cdot \left[\frac{f(\text{GHz})}{1.3} \right]^2 \cdot \left(\frac{T_c}{T} \right) \cdot e^{-\Delta \frac{T_c}{T}}$$

$$R_0 = 10^{-5} [\Omega]; \quad \Delta = 1.8; \quad T_c = 9.2^\circ\text{K}$$

Summary:

- SRF technology allows 10^6 less surface losses than RT technology and consequently, much high acceleration gradient at high duty cycle or in CW regime;
- Losses at SRF are determined mainly by BCS resistance (inertia), flux trapping and intrinsic residual resistance;
- The acceleration gradient is limited mainly by thermal breakdown, field emission, etc., but not by breakdown.
- Modern cavity processing techniques (N-doping, N-infusion) allow very high Q_0 .
- To achieve high Q_0 small residual magnetic field may be required, and therefore, good shielding and degaussing. The cryo-system should allow fast cooling for flux expulsion.
- Resonance discharge (multipacting) may be an issue; cavity processing is required; the cavity shape should be optimized.
- Field emission may limit the gradient; large-scale clean rooms are necessary among other means.

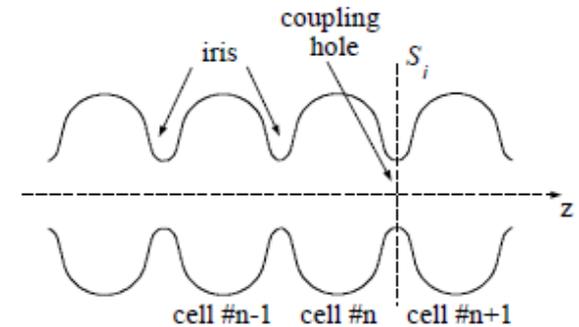
Chapter 6.

Multi-cell SRF cavities.

- a. Multi-cell SRF cavities;
- b. Why π -mode?
- c. Equivalent circuit and normal modes;
- d. Parameters of the SRF SW cavity;
- c. Cavity efficiency at different particle velocity versus the number of cells;
- d. Why elliptical multi-cell cavity does not work at low particle velocity.

Multi-cell SRF cavity:

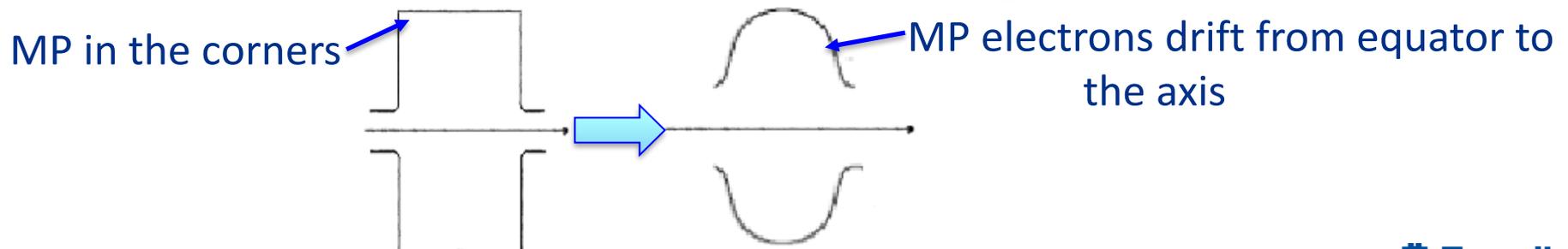
- Single – cell cavities are not convenient in order to achieve high acceleration: a lot of couplers, tuners, etc.
- Multi-cell cavities are used in both RT and SRF accelerators.
- Multi-cell SRF cavity is a standing–wave periodic acceleration structure, operating at the phase advance per period equal to π (i.e, the fields in neighboring cells have the same distribution, but opposite sign).
- In order to provide synchronism with the accelerated particle, period is $\beta\lambda/2$ (in general case it is $\varphi\beta\lambda/2\pi$; φ is phase advance per period).
- The end cells have special design (full length, not half) in order to provide field flatness along the structure for operation mode with the phase advance π .



Why SW π - mode?

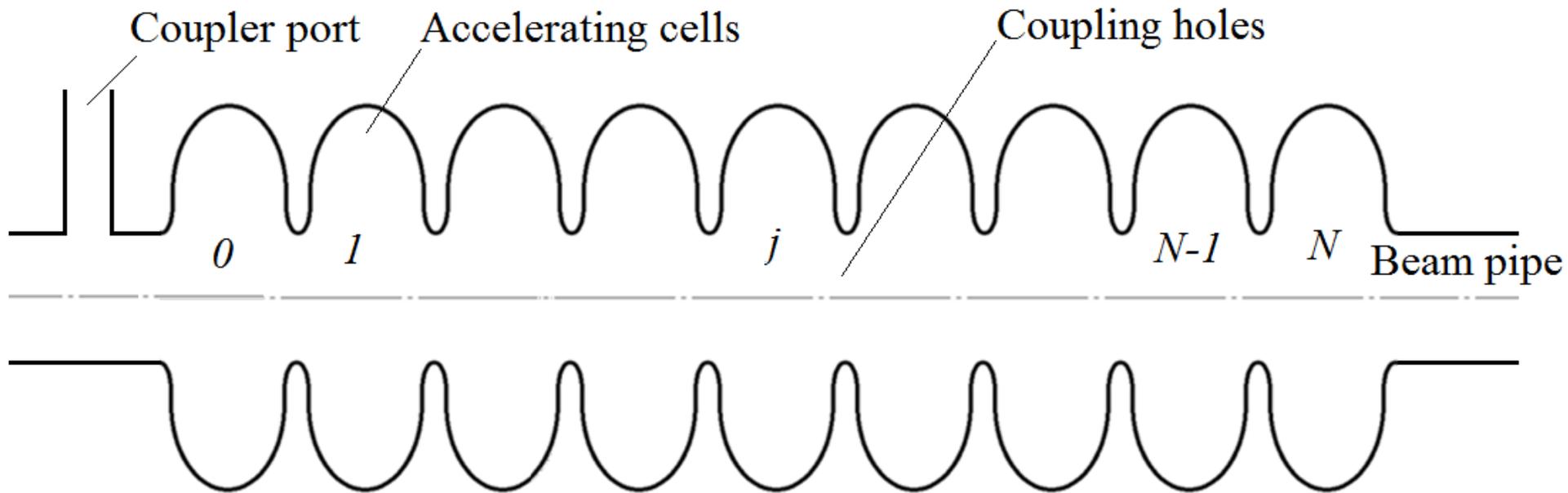
- The SW modes except π have small acceleration efficiency because most of the cavities have small field (in ideal case $X_{ni} \sim \cos(\pi qj/N)$, q - mode number, j - cell number).
- Bi-periodic structure $\pi/2$ -mode does not work because it is prone to multipacting in the empty coupling cells and difficult for manufacturing (different cells) and processing (narrow coupling cells).
- π -mode structure is simple, easy for manufacturing and processing.
- Drawback (see Appendix 12):
 - Big aperture to provide big coupling;
 - Considerably small number of cells N (5-9).

□ Elliptical cavity is not prone to multipacting in contrast to a pillbox.



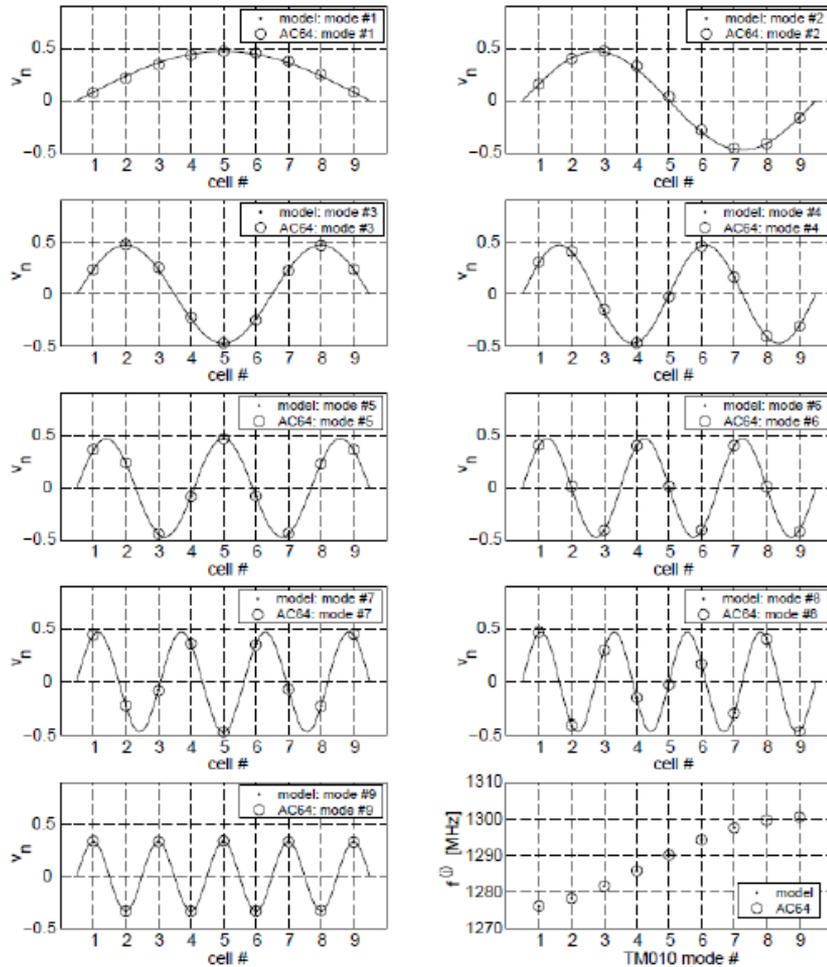
Why SW π – mode?

Schematic of the SRF multi-cell cavity



- The cells have elliptical shape to get rid of multipacting;
- The end cells have full length, but the shape is different.;
- The coupler is placed in the beam pipe.

Multi-cell RF cavity:

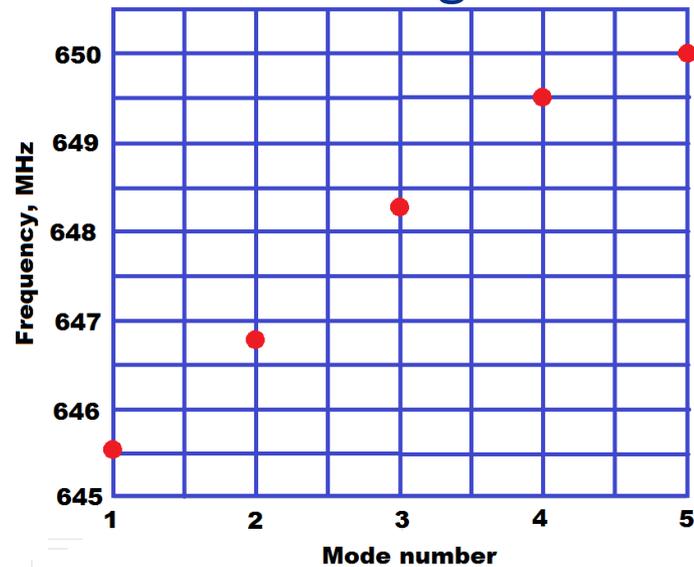


An example of calculated eigen modes amplitudes in a 9-cell TESLA cavity compared to the measured amplitude profiles. Also shown are the calculated and measured eigen frequencies. The cavity has full size end cells especially tuned in order to get field flatness for the operating mode.

Normal modes in a standing-wave elliptical cavity:

PIP II 650 MHz cavity

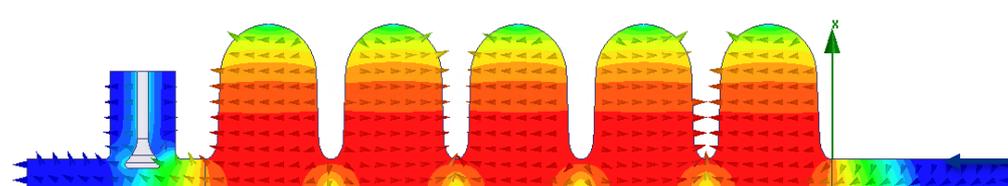
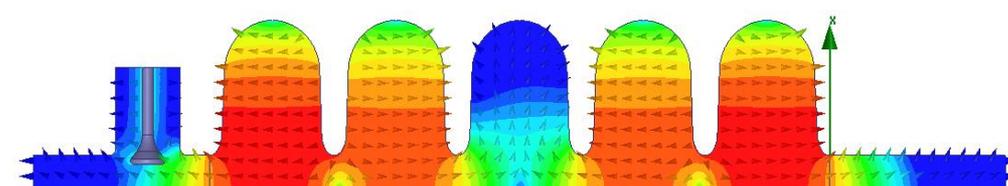
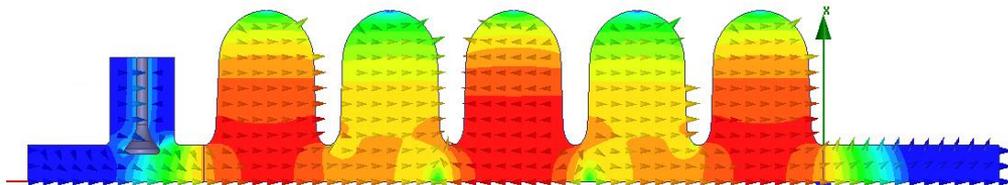
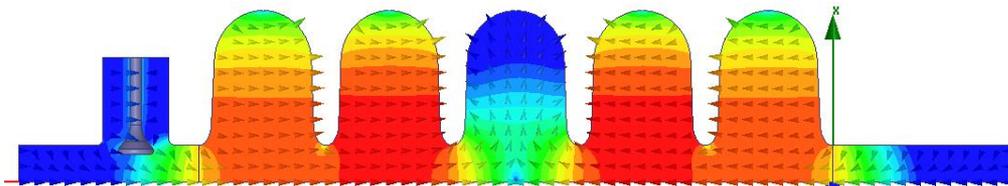
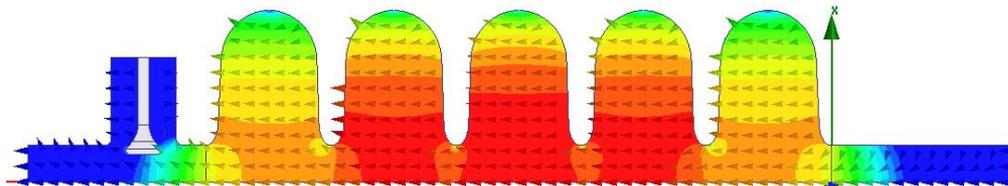
“Brillouin diagram”:



$$f(q) \approx f_0 \left(1 - \frac{k}{2} \cos \frac{\pi(q-1)}{(N-1)} \right)$$

q —the mode number;
 N — the number of cells;
 k — coupling: $k=0.7\%$

Operation mode “ π ”



Mode 1
“0”

Mode 2
“ $\pi/4$ ”

Mode 3
“ $\pi/2$ ”

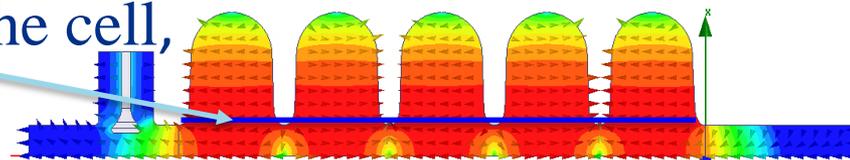
Mode 4
“ $3\pi/4$ ”

Mode 5
“ π ”

Axial acceleration field distribution

At the aperture, $E_z(a, z) \sim \text{const}$ over the cell,

$$E_z(a, z) \sim \sum A_{2n} \cos(2nk_0 z/\beta);$$



$$E_z(0, z) \sim \sum A_{2n} \cos(2nk_0 z/\beta) / I_0 [ak_0 a (1 - 4n^2/\beta^2)^{1/2}] = \sum B_{2n} \cos(2nk_0 z/\beta)$$

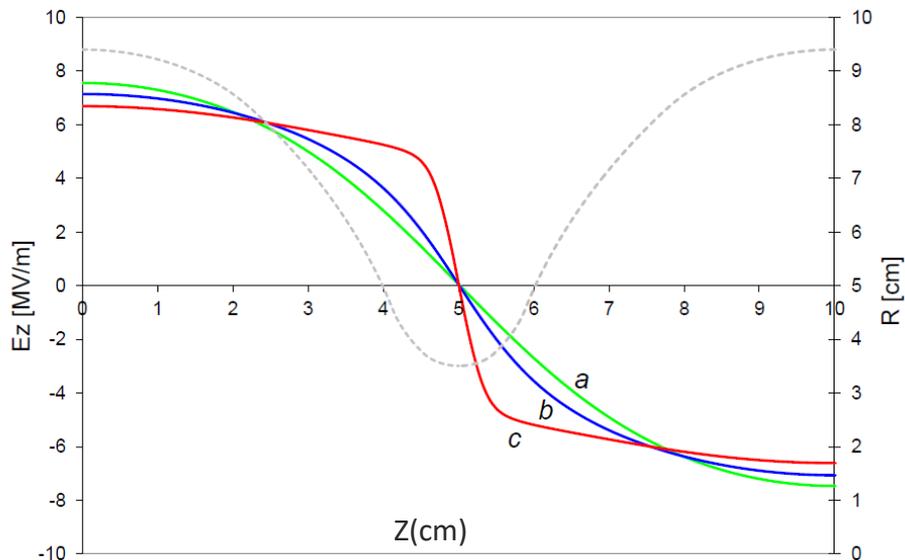


$$B_{2n} = A_{2n} I_0 [ak_0 a (1 - 4n^2/\beta^2)^{1/2}];$$

for example for $\beta = 1$ $B_0 = A_0$ and $B_{2n} \approx A_{2n} \exp(-2nk_0 a) \ll B_0$



$E_z(0, z) \sim A_0 \cos(k_0 z)$ – sinusoidal distribution on the axis! Valid for $\beta < 1$.



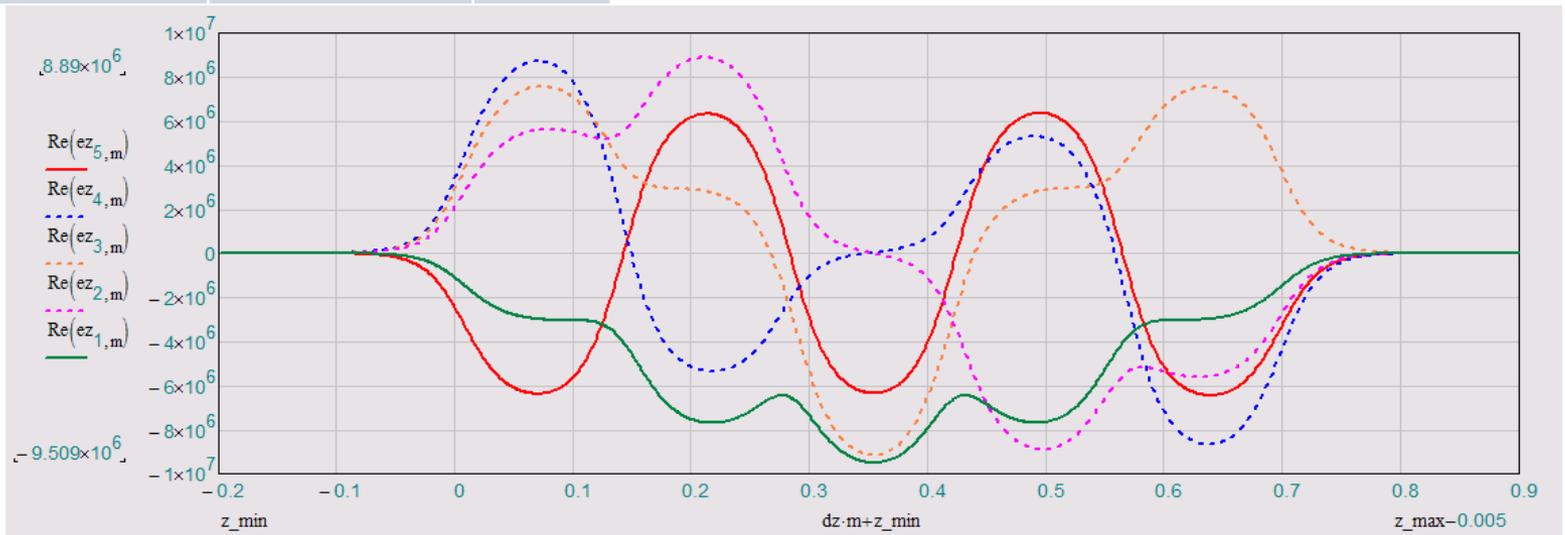
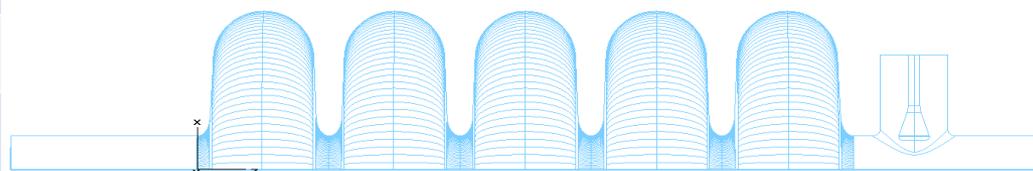
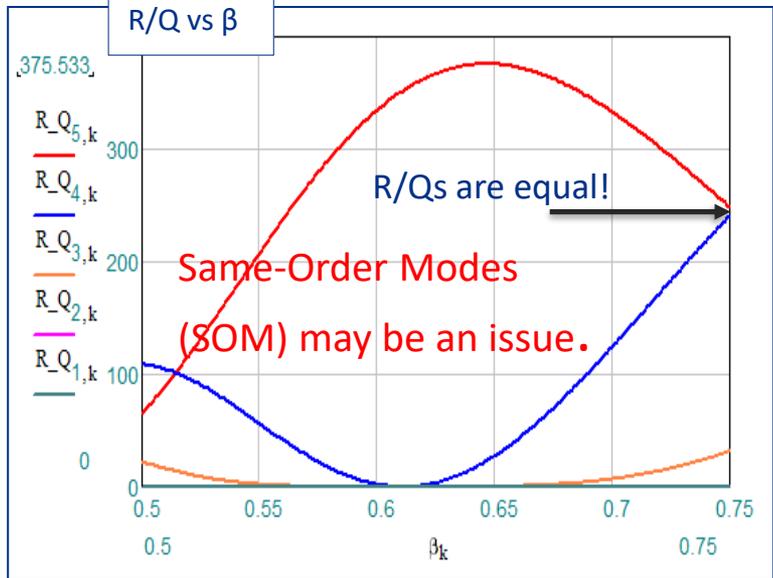
Geometry of an iris of a CEBAF multi-cell cavity (gray line). Longitudinal electric field at a different radial position: $r = 0$ cm (green line), $r = 2.5$ cm (blue line), $r = 3.45$ cm (red line). Fields are normalized to 4 MeV/m accelerating gradient.

- Field at the aperture close to rectangular
- Field on the axis is close to sinusoidal

PIP II $\beta_G=0.61$, 650 MHz elliptical cavity:

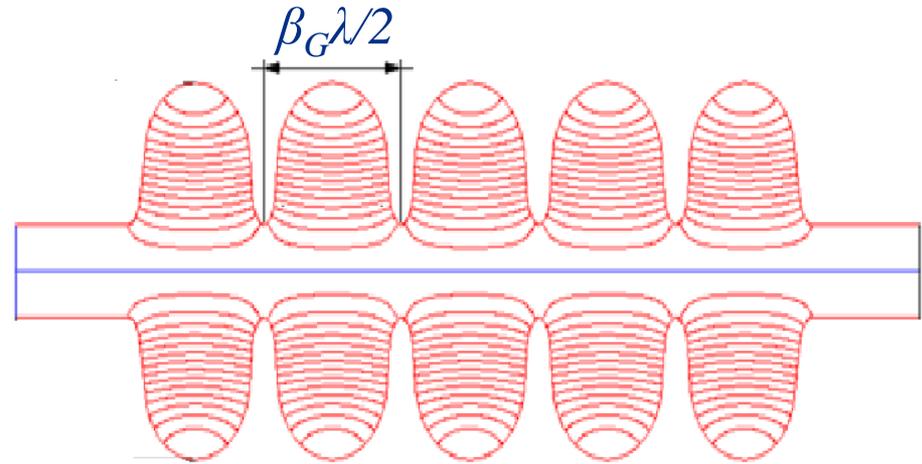
elliptical cavity:

| Mode | Freq [GHz] | (R/Q) _{opt} [Ω] | β_{opt} |
|-------------------|------------|--------------------------|---------------|
| 0 | 0.6456 | 0.5 | >0.75 |
| $\frac{1}{4} \pi$ | 0.6468 | 0.4 | 0.69 |
| $\frac{1}{2} \pi$ | 0.6483 | 32.1 | >0.75 |
| $\frac{3}{4} \pi$ | 0.6495 | 241.0 | >0.75 |
| π | 0.6500 | 375.5 | 0.65 |



Parameters of a multi-cell cavity:

- “Geometrical beta”: $\beta_G = 2l/\lambda$,
 l is the length of a regular cell,
 λ is wavelength.



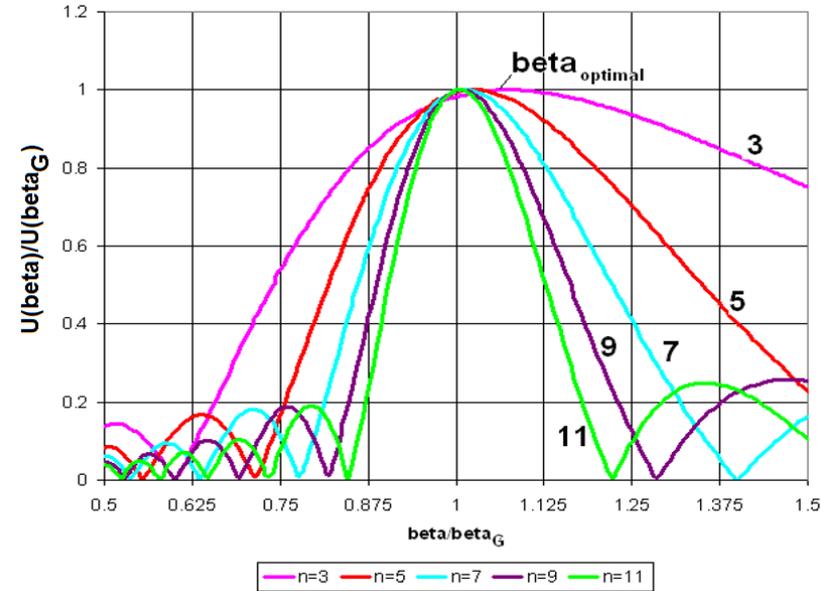
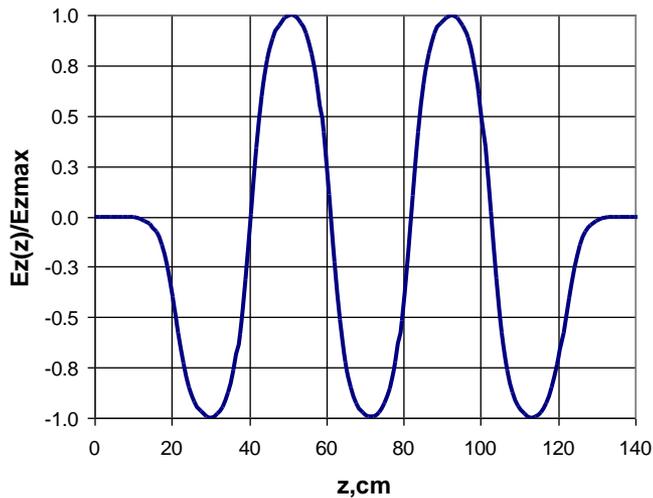
- $R/Q = V^2/\omega U$, V is the energy gain per cavity (in optimal acceleration phase), $V = V(\beta)$; ω – cyclic operation frequency; U is EM energy stored in a cavity; R/Q is a function of β , as well as V . R/Q is the same for geometrically similar cavities. Decreases when the cavity aperture a increases.
- “Optimal β ”: value of β , where V (and R/Q) is maximal.
- Acceleration gradient: $E = V/L_{eff}$, $L_{eff} = n\beta_G \lambda / 2$ – effective length, n is the number of cells.

Parameters of a multi-cell cavity (cont)

- Surface electric field enhancement: $K_e = E_{peak}/E$, E_{peak} is maximal surface electric field.
- Surface magnetic field enhancement: $K_m = B_{peak}/E$, B_{peak} is maximal surface magnetic field.
- Unloaded quality factor: $Q_0 = \omega W/P_{loss}$, P_{loss} – surface power dissipation.
- G -factor: $G = Q_0 * R_s$, R_s is the surface resistance. G is the same for geometrically similar cavities. At fixed gain the losses are proportional to $G*(R/Q)$.
- Loaded quality factor: $Q_{load} = \omega W/P$, $P = P_{loss} + P_{load}$; P_{load} – power radiated through the coupling port.
- Coupling: $K = 2(f_\pi - f_0)/(f_\pi + f_0)$,

Multi-cell cavity

A multi-cell SRF elliptical cavity is designed for particular $\beta = \beta_G$, but accelerates in a wide range of particle velocities; the range depends on the number of cells in the cavity N . Field distribution for the tuned cavity has equal amplitudes for each cell; longitudinal field distribution for considerably large aperture is close to sinusoidal (see slide 43):



$$\frac{V(\beta)}{V(\beta_{optimal})} = \frac{2\beta}{\pi N} \left(\frac{\sin\left(\frac{\pi N(\beta - \beta_G)}{2\beta}\right)}{\beta - \beta_G} - (-1)^n \frac{\sin\left(\frac{\pi N(\beta + \beta_G)}{2\beta}\right)}{\beta + \beta_G} \right)$$

$$\beta_{optimal} \approx \beta_G \left(1 + \frac{6}{\pi^2 N^2} \right)$$

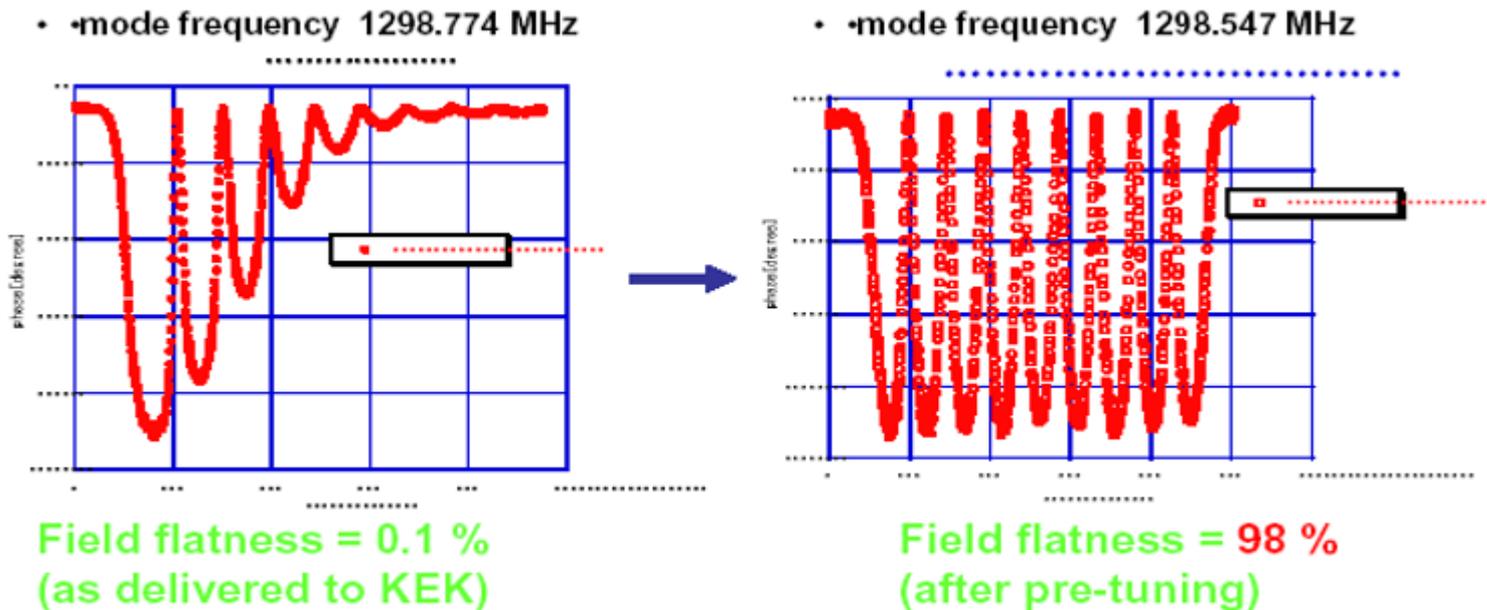
V is the energy gain per cavity.

The cavity containing more cells provides effective acceleration in more narrow particle velocity range!

Why SW π – mode?

Cavity tuning:

- Compensation of the errors caused by manufacturing
- Compensation of the errors caused by cool-down.
- Field flatness
- Tuning the operating mode frequency to resonance.



Field flatness in ILC – type cavity before and after pre-tuning.

Elliptical cavities:

INFN Milano, 700 MHz, $\beta_G = 0.5$



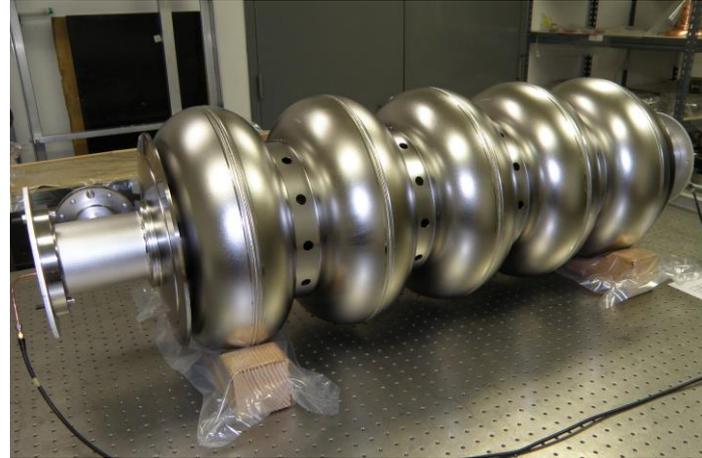
SNS, 805 MHz, $\beta_G = 0.61$



SNS, 805 MHz, $\beta_G = 0.81$



PIP II, 650 MHz, $\beta_G = 0.9$



XFEL, 1300 MHz, $\beta_G = 1$



XFEL, 3900 MHz, $\beta_G = 1$



Multi-cell cavity is not effective for low β :

During acceleration a particle interacts with cylindrical EM waves,

$$E_z(r,z,t) \sim J_0(k_r r) \exp(ik_z z - i\omega t),$$

where $J_0(x)$ is Bessel function.

For acceleration, the cylindrical wave should be synchronous, i.e., it should have phase velocity equal to the particle velocity:

$$\omega/k_z = v = c\beta, \text{ or } k_z = \omega/\beta c = k/\beta \text{ (} k \text{ is full wavenumber, } k = \omega/c = 2\pi/\lambda)$$

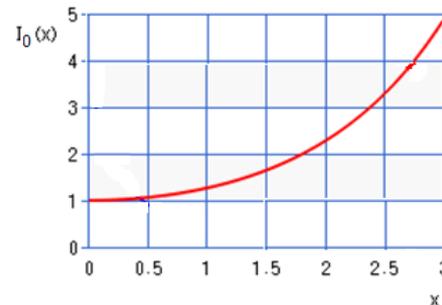
On the other hand, for EM wave one has:

$$(k)^2 = (k_r)^2 + (k_z)^2 \text{ or } (k_r)^2 = (k)^2 - (k_z)^2 = (k)^2(1 - 1/\beta^2). \text{ Thus, } k_r = ik/\beta\gamma.$$

- In ultra-relativistic case $k_r \rightarrow 0$
- In non-relativistic case $k_r = ik/\beta$ and the synchronous cylindrical wave is

$$E_z(r,z,t) \sim I_0(2\pi r/\lambda\beta) \exp(ikz/\beta - i\omega t),$$

$I_0(x)$ is modified Bessel function.



Multi-cell cavity is not effective for low β :

For small β

$$I_0(r) \sim \exp(2\pi r/\lambda\beta).$$

Synchronous EM is concentrated on the cavity periphery, not on the axis! Consequences:

- Small (R/Q):

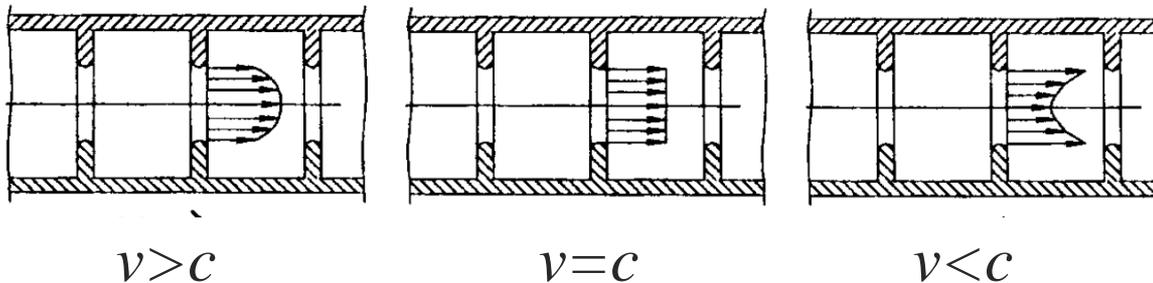
$$(R/Q) \sim \exp(-4\pi a/\lambda\beta), \quad a \text{ is the cavity aperture radius;}$$

- High K_e

$$K_e \sim \exp(2\pi a/\lambda\beta);$$

- High K_m .

$$K_m \sim \exp(2\pi a/\lambda\beta).$$

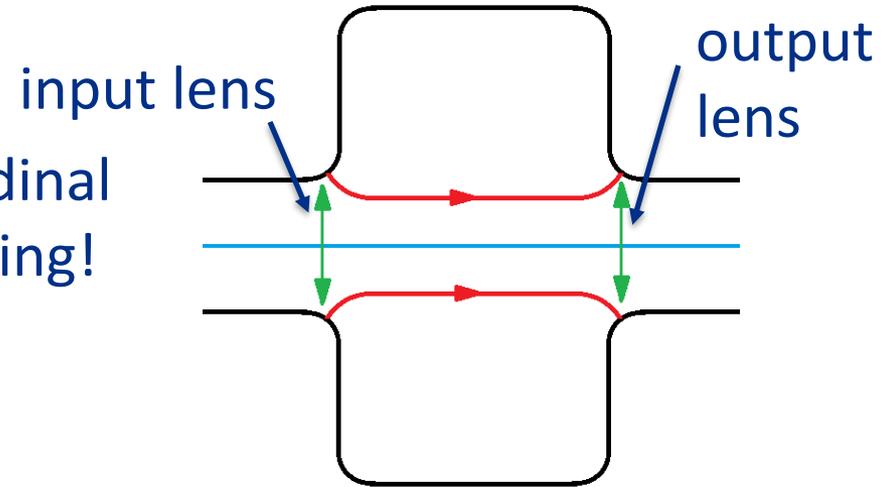


Multi-cell cavity is not effective for low β :

RF cavity provides the beam focusing,

$$\frac{1}{F} \sim \frac{\pi}{\beta^3 \gamma^3} \frac{V}{U_0} \frac{1}{\lambda} \sin(\varphi_s)$$

- For $\varphi_s < 0$ (necessary for longitudinal stability) the cavity provides defocusing!
- Defocusing:
 - $\sim 1/\beta^3$;
 - $\sim 1/\lambda$.



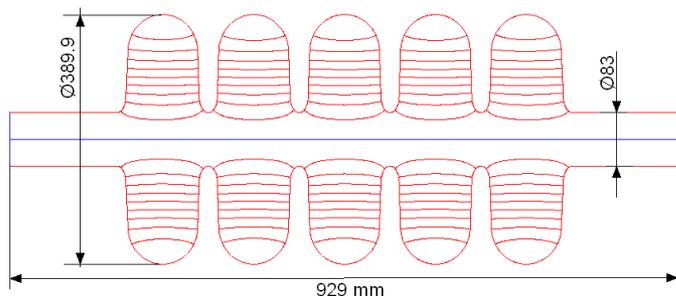
Defocusing should be compensated by external focusing elements,
-solenoids (low energy);
-quads (high energy).

For small β longer RF wavelength (lower frequency) should be used.
But axisymmetric cavity has very big size, $D \sim 3/4 \lambda$

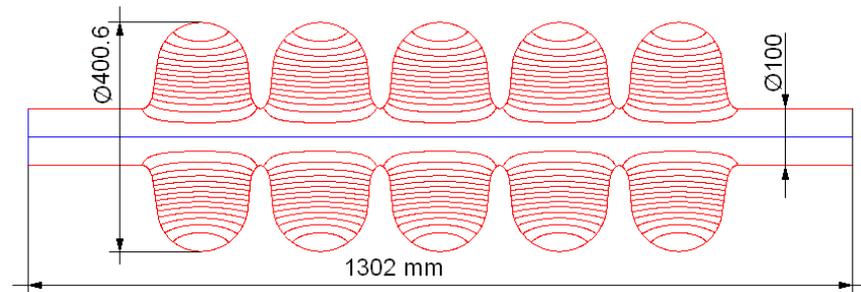
For small β other types of cavities should be used!

Parameters of an elliptical cavity (cont)

Example for the 650 MHz cavities for PIP II



LB650 ($\beta_G=0.61$)



HB650 ($\beta_G=0.9$)

| Parameter | | LH650 | HB650 |
|---|------------------|-------|-------|
| β_G | | 0.61 | 0.9 |
| β_{optimal} | | 0.65 | 0.94 |
| Cavity Length = $n_{\text{cell}} \cdot \beta_{\text{geom}} \lambda / 2$ | mm | 703 | 1038 |
| R/Q | Ohm | 378 | 638 |
| G-factor | Ohm | 191 | 255 |
| K_e | | 2.26 | 2.0 |
| K_m | mT/(MeV/m) | 4.22 | 3.6 |
| Max. Gain/cavity (on crest) | MeV | 11.7 | 17.7 |
| Acc. Gradient | MV/m | 16.6 | 17 |
| Max surf. electric field | MV/m | 37.5 | 34 |
| Max surf. magnetic field, | mT | 70 | 61.5 |
| Q_0 @ 2K | $\times 10^{10}$ | 2 | 3 |
| P_{2K} max | [W] | 24 | 24 |

Summary:

- ❑ Single – cell cavities are not convenient in order to achieve high acceleration: a lot of couplers, tuners, etc; multi-cell π -mode elliptical cavities are used in SRF accelerators;
- ❑ Why π -mode?
 - The SW modes except π have small acceleration efficiency because most of cavities have small field;
 - Bi-periodic structure $\pi/2$ -mode does not work because it is prone to multipacting in the empty coupling cells and difficult for manufacturing (different cells) and processing (narrow coupling cells).
 - π -mode structure is simple, easy for manufacturing and processing.
- ❑ Elliptical cavities are used to mitigate multipacting;
- ❑ End cells have the same length as regular ones, but a bit different shape to keep field flatness for operation π -mode.
- ❑ Range of acceleration efficiency strongly depends on the number of cells: cavities with smaller number of cells operate in wider β range.
- ❑ Elliptical cavities are not effective for small particle velocity.

Chapter 7.

SRF Cavities for Low β Accelerators .

- a. Why TEM-type cavities work at low particle velocities;
- b. Types of TEM cavities;
- c. Velocity range of TEM-type cavities.

RF cavity types

Quarter-wave resonator (QWR)

concept:

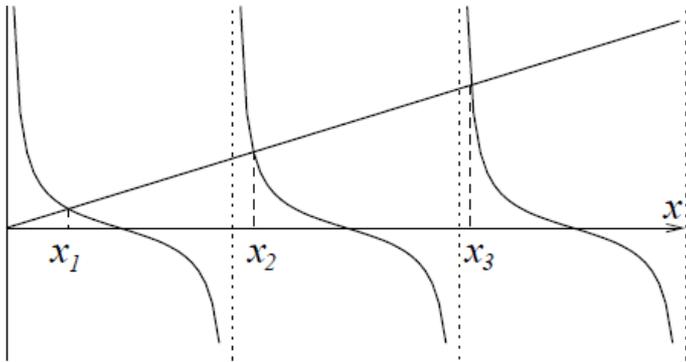
Resonance:

$$\frac{1}{j\omega C} + j\omega Z_c \tan\left(\frac{\omega l}{c}\right) = 0 \quad \text{or} \quad \cot\left(\frac{\omega l}{c}\right) = \omega C Z_c.$$

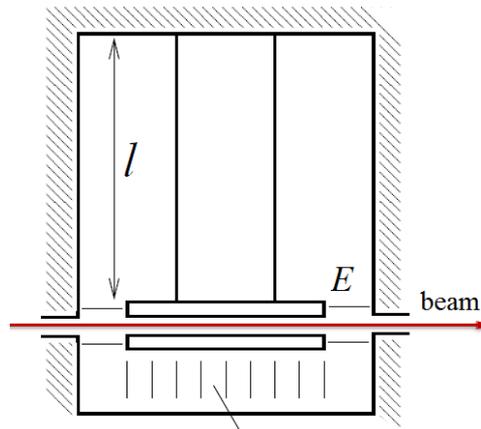
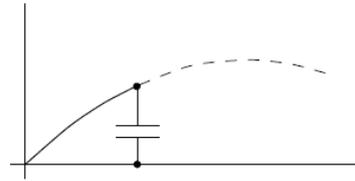
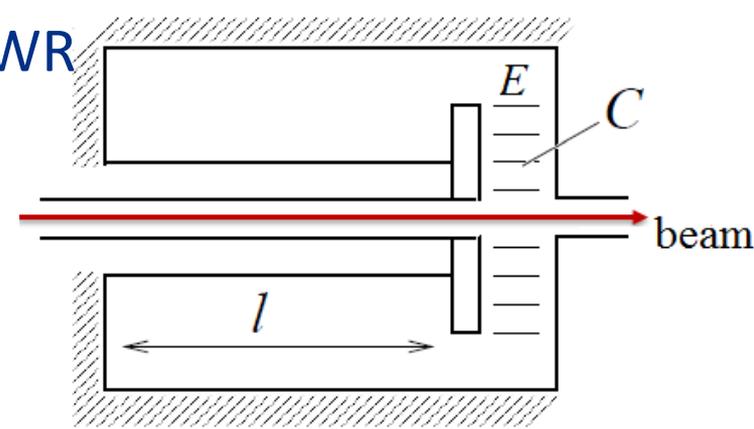
Z_c here is the coaxial impedance

Compact ($L \approx \lambda/4$) compared to pillbox ($D \approx 3/4\lambda$).

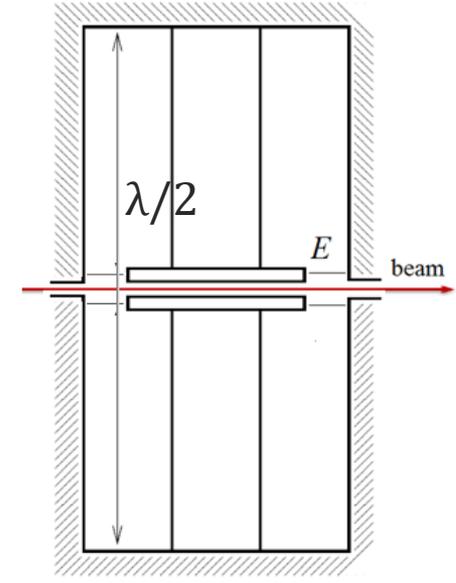
$$\frac{\omega l}{c} = x, \quad \frac{C Z_c c}{l} = A \rightarrow \cot(x) = Ax.$$



One-gap QWR



Two-gap QWR



Half-wave resonator (HWR)

RF cavities for low β :

TEM-like cavities:

- Split-ring resonator;
- Quarter-wave resonator;
- Half-wave resonator;
- Spoke resonator.



Split-ring



QWR



HWR



SSR

(single-spoke)

- Narrow acceleration gap ($\sim \beta\lambda$) allows concentrate electric field near the axis;
- Aperture $\sim 0.02-0.03\lambda$ allows acceptable field enhancement;
- Number of gaps in modern cavities is 2 for small beta which allows operation in acceptably wide beta domain. For $\beta > 0.4$ multi-gap cavities are used –double- and triple-spoke resonators;
- Focusing elements (typically, solenoids) are placed between the cavities.

RF cavities for low β :

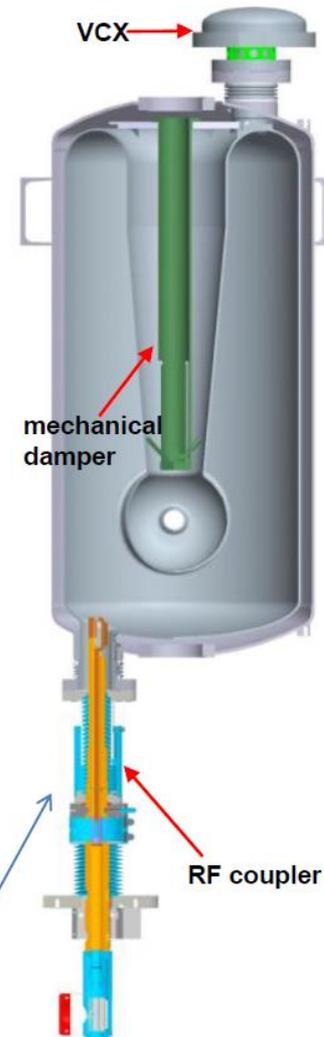
Quarter-wave resonator:

- Allows operate at very low frequency ~ 50 MHz, (and thus, low beta) having acceptable size;
- Has a good (R/Q);
- Low cost and easy access.

But:

- Special means needed to get rid of dipole and quadrupole steering, and
- Provide mechanical stability

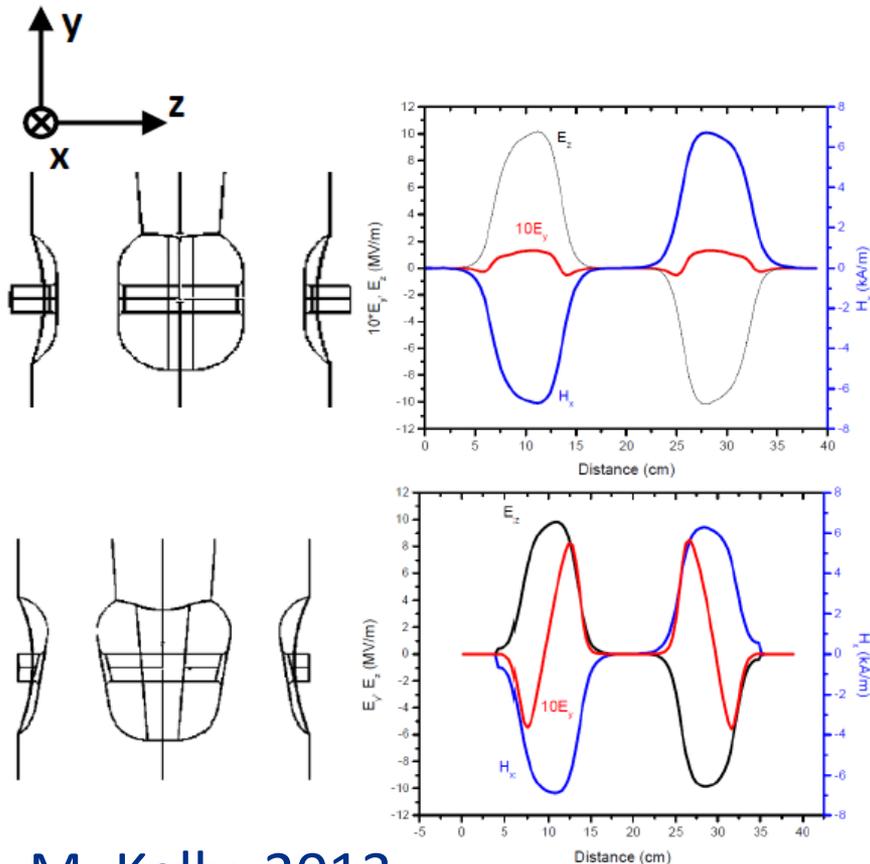
beta=0.14, 109.125 MHz QWR(**Peter N. Ostroumov**)



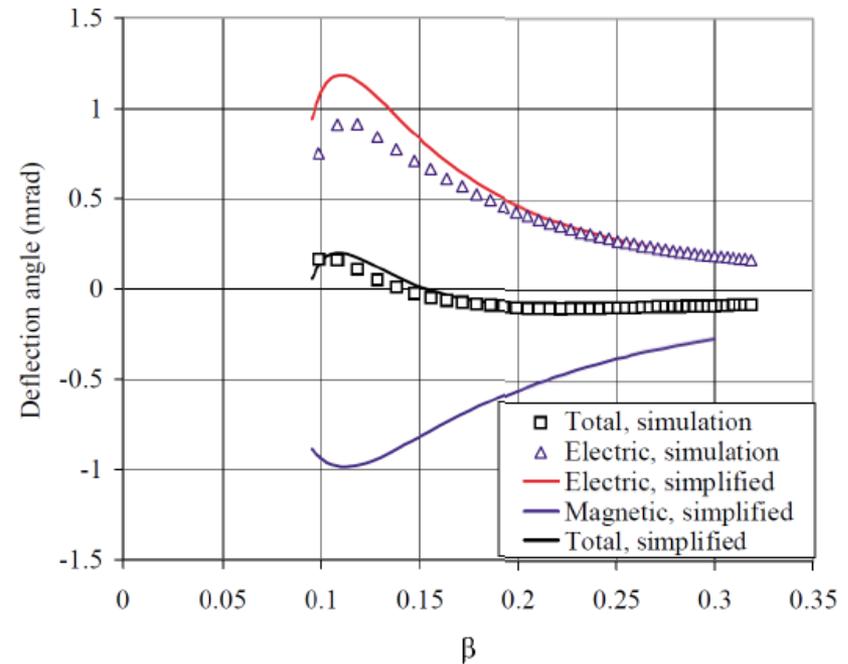
RF cavities for low β

Beam Steering in Quarter-wave Cavities*

- Beam steering due to unavoidable magnetic field on the beam axis.
- One remedy: The vertical field E_y , normally small, may be modified by the cavity geometry to cancel magnetic steering due to H_x .



$$\Delta p_y \sim \frac{1}{\beta^2} \int_{L/2}^{-L/2} E_y \cos(kz + \varphi) + \beta c \cdot B_x \sin(kz + \varphi) dz$$



M. Kelly, 2013

RF cavities for low β :

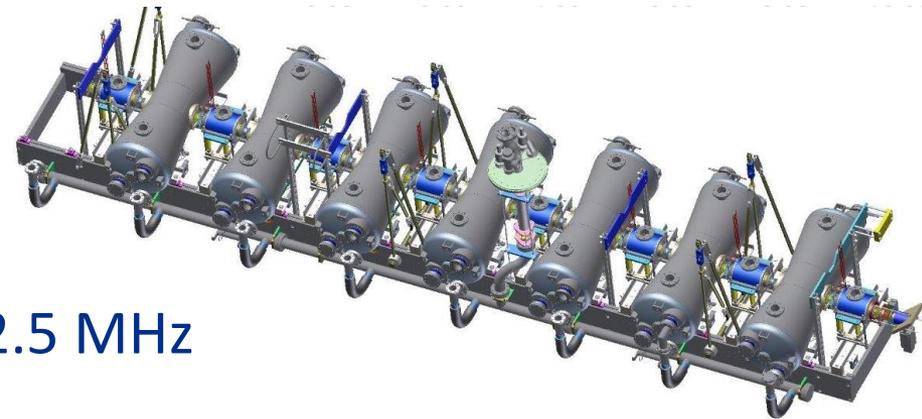
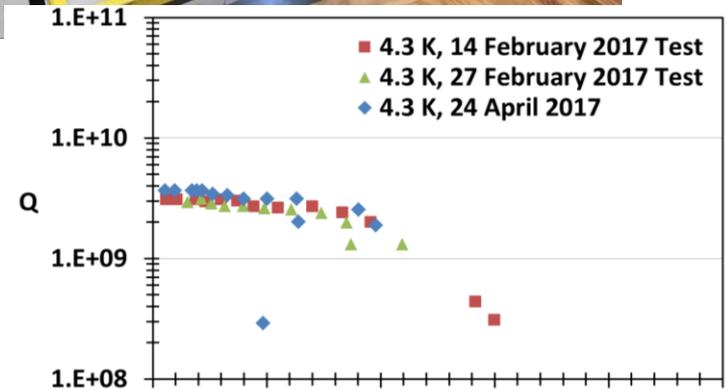
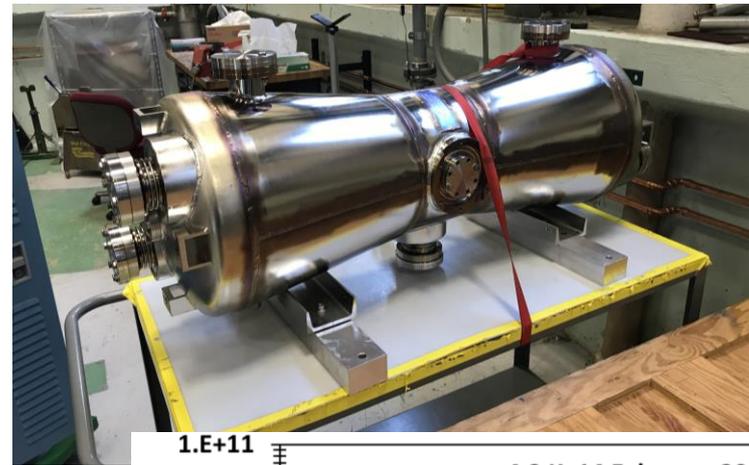
Half-wave resonator (HWR):

- No dipole steering;
- Lower electric field enhancement;
- High performance;
- Low cost;
- Best at ~ 200 MHz.

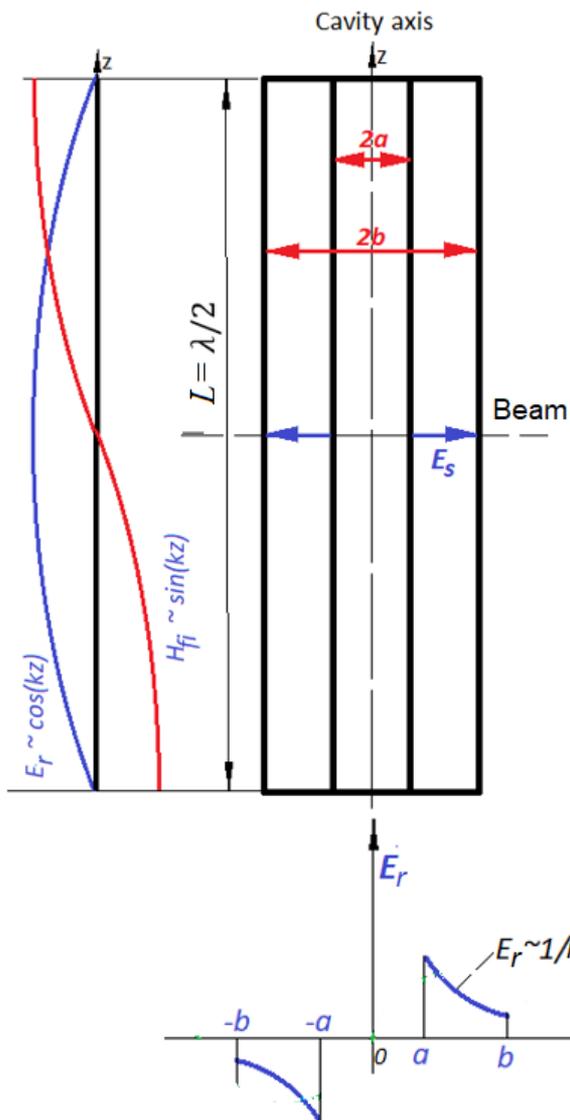
But:

- Special means needed in some cases to get rid of quadrupole effects;
- Two times lower R/Q

PIP II HWR cavity, 162.5 MHz
(M.Kelli, Z. Conway)



Ideal HWR



- The cavity is a TEM coaxial line shortened at $z = \pm L/2$
- Electric field in a coaxial line:

$$E_r(r, z) = \frac{C}{r} \cos(kz), \quad a \leq r \leq b;$$

$$E_r(r, z) = 0, \quad r < a;$$

$$E_r(r, z) = 0, \quad r > b, \quad k = \frac{\omega}{c}$$

$$E_r(r, z) = \frac{C}{r} \cos(kz),$$

$$U = \int_a^b E_r(r, 0) dr = C \cdot \ln\left(\frac{b}{a}\right)$$

$$\rightarrow C = \frac{U}{\ln\left(\frac{b}{a}\right)}$$

$$E_r(r, z) = \frac{U}{\ln\left(\frac{b}{a}\right)} \cdot \frac{1}{r} \cos(kz),$$

- Magnetic field:

From Maxwell equations:

$$H_\phi(r, z) = \frac{i}{\omega\mu_0} \cdot \frac{\partial E_r}{\partial z} = \frac{iC}{Z_0 r} \sin(kz) =$$

$$= \frac{iU}{2\pi Z_c r} \sin(kz); \quad Z_c = \frac{1}{2\pi} Z_0 \ln\left(\frac{b}{a}\right) \text{ - the line impedance; } Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi \text{ Ohm}$$

- Resonance frequency:

$$kL = \pi \rightarrow k = \frac{\pi}{L} \rightarrow \omega = \frac{\pi c}{L} \rightarrow f = \frac{c}{2L}, \quad c \text{ is speed of light.}$$

- Stored energy:

$$W = \frac{\mu_0}{2} \int |H|^2 dV = \frac{\pi}{4} \frac{U^2}{\omega Z_c}$$

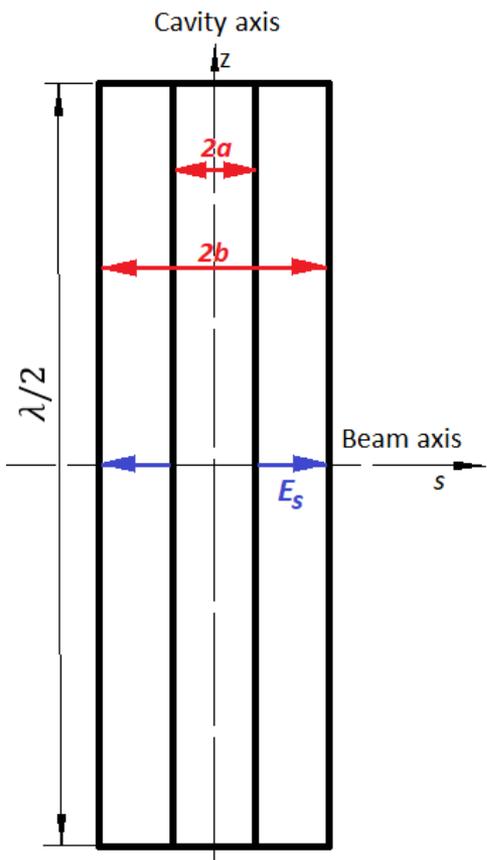
- Ohmic loss:

$$P = \frac{1}{2} \oint R_s H^2 dS = \frac{R_s U^2}{8\pi Z_c^2} \left[L \left(\frac{1}{b} + \frac{1}{a} \right) + 4 \ln\left(\frac{b}{a}\right) \right]$$

- Unloaded quality factor:

$$Q_0 = \frac{\omega W}{P}.$$

Ideal HWR



- Acceleration field on the beam axis: $E_s(s) = \frac{U}{\ln\left(\frac{b}{a}\right)} \cdot \frac{1}{s}$

- Acceleration voltage:

$$V = \frac{2U}{\ln\left(\frac{b}{a}\right)} \int_a^b \frac{\sin\left(\frac{ks}{\beta}\right)}{s} ds = \frac{2U}{\ln\left(\frac{b}{a}\right)} \left[Si\left(\frac{kb}{\beta}\right) - Si\left(\frac{ka}{\beta}\right) \right]$$

where $Si(x) = \int_0^x \frac{\sin(x)}{x} dx$. We have two gaps \rightarrow factor "2" in the nominator.

Optimal acceleration:

$$\frac{dV}{d\beta} = 0 \rightarrow \sin\left(\frac{kb}{\beta}\right) - \sin\left(\frac{ka}{\beta}\right) = 2\sin\left(\frac{k(b-a)}{2\beta}\right) \cos\left(\frac{k(a+b)}{2\beta}\right) = 0$$

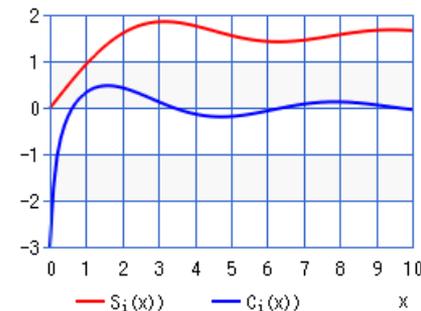
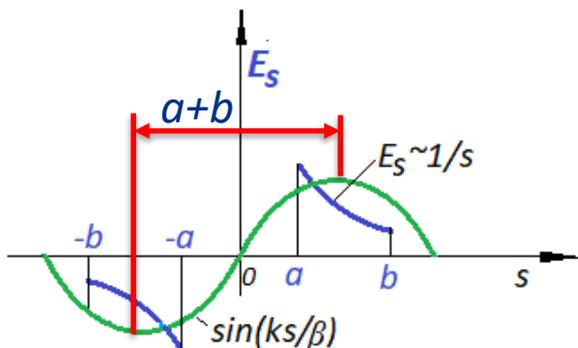
$$\rightarrow \frac{k(a+b)}{2\beta} = \frac{\pi}{2} \rightarrow \boxed{\frac{a+b}{2} = \frac{\beta\lambda}{4}}$$

- "Effective cavity length": $L_{eff} = \beta\lambda$

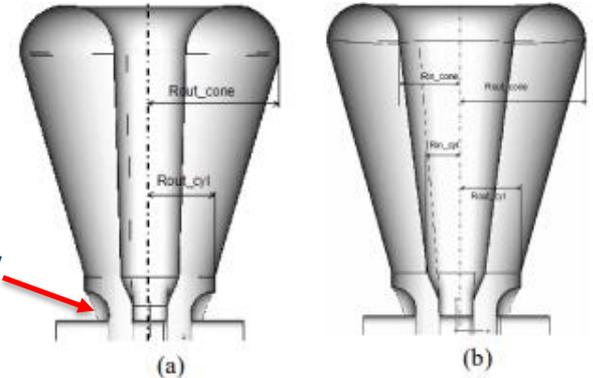
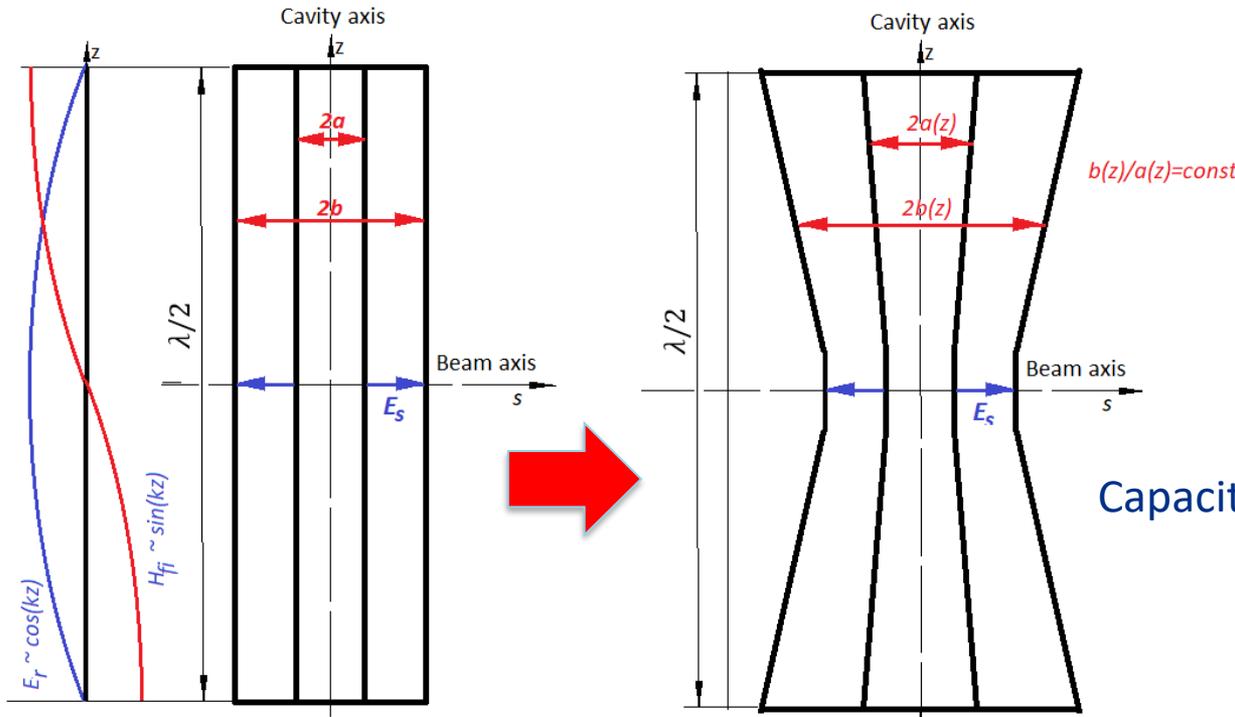
(compare to multi-cell elliptical cavity: $L_{eff} = \frac{\beta\lambda}{2} n$, n is number of gaps)

$Si(x)$ calculator:

<https://keisan.casio.com/exec/system/1180573420>

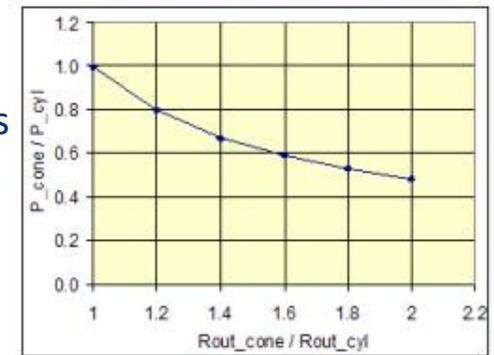


Loss reduction in HWR: conical HWR*



HWR with enlarged outer (a) and central conductor (b) dome diameters.

Capacity



Power dissipation in conical HWR relative to cylindrical shape.

Increase the cavity transverse size at the ends keeping about the same ratio $b(z)/a(z)$ helps to decrease loss without change of the R/Q , which is determined in high degree by Z_c

$$H_\phi(r, z) = \frac{I}{2\pi r} \sin(kz) = \frac{U}{2\pi Z_c r} \sin(kz);$$

$$Z_c = \frac{Z_0}{2\pi} \ln\left(\frac{b}{a}\right);$$

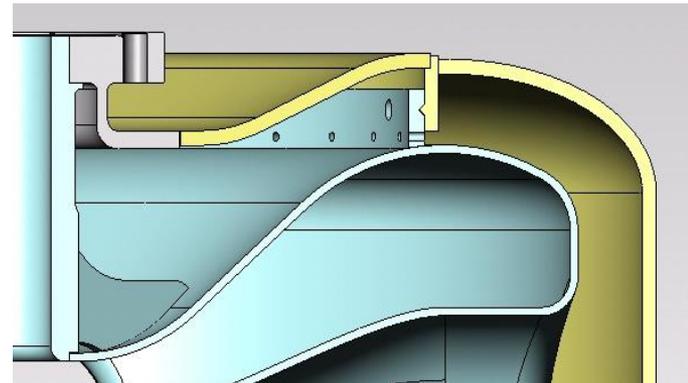
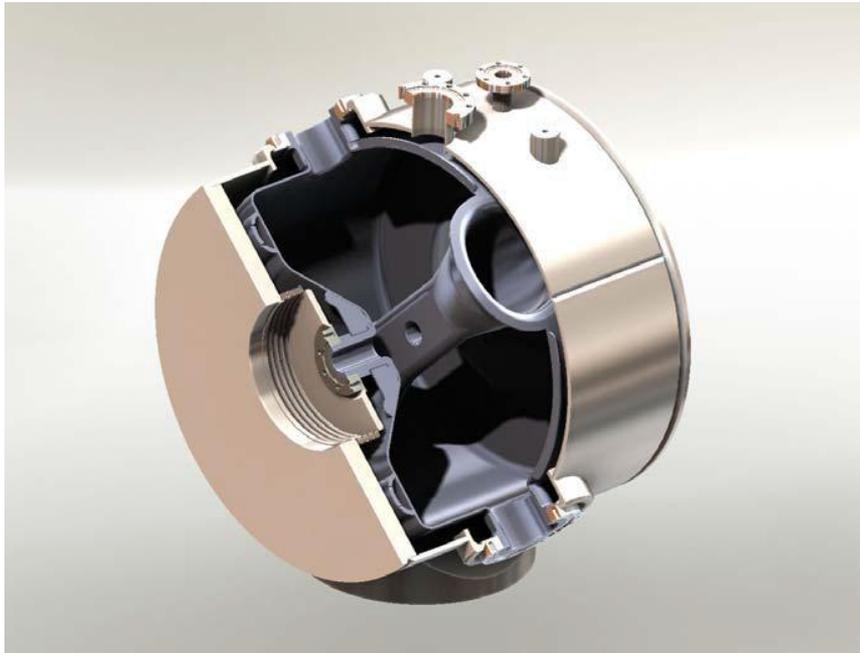
$$P = \frac{1}{2} \oint R_s H^2 dS = \frac{R_s U^2}{8\pi Z_c^2} \left[L \left(\frac{1}{b} + \frac{1}{a} \right) + 4 \ln\left(\frac{b}{a}\right) \right] \propto \frac{\lambda}{b}$$

$L = \lambda/2$ – the cavity size in z - direction

*E. Zaplatin, 2009

RF cavities for low β :

Spoke resonator

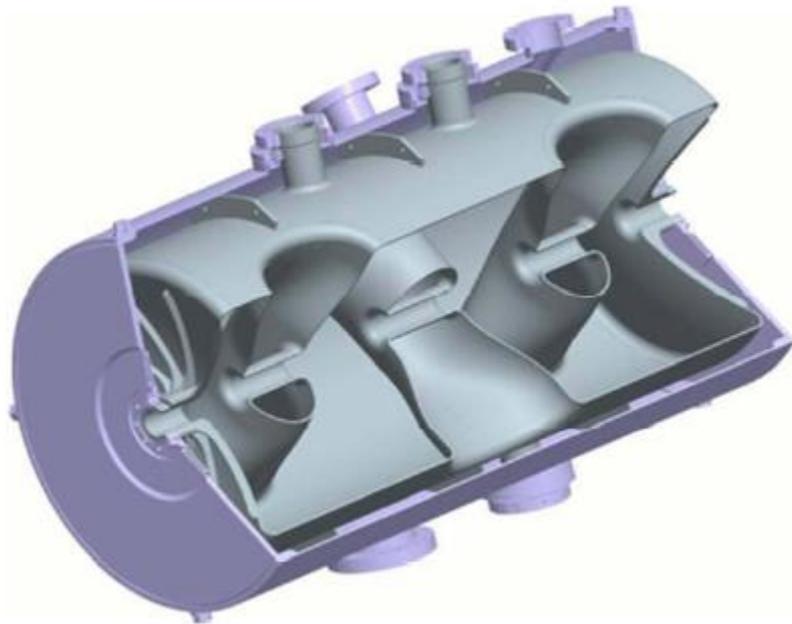


Mechanical coupling of the cavity to the He vessel in order to improve mechanical stability.

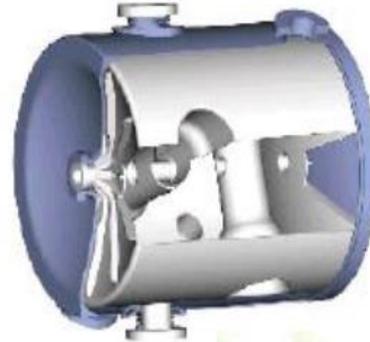
FNAL 325 MHz SSR1 cavity layout and photo. $\beta=0.22$

RF cavities for low β :

Multi-spoke resonators



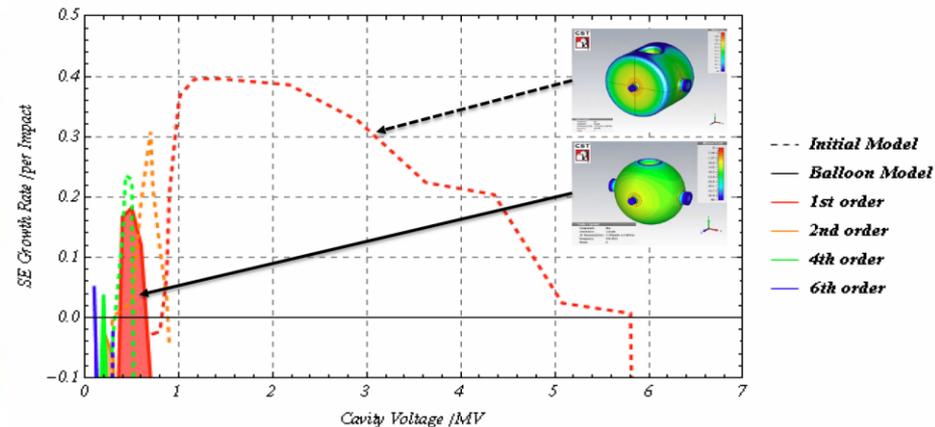
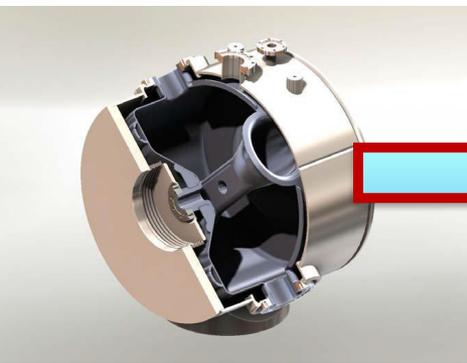
Triple-spoke cavity



345 MHz, $\beta=0.4$,
3-gap spoke cavity
for ion beam acceleration
ANL

RF cavities for low β :

- TEM-type cavities are prone to multipacting;
- Elliptical cavities have much better performance (MP electrons drift towards the axis)
- Idea (R. Laxdal): combine SR and elliptical cavity \rightarrow balloon cavity.

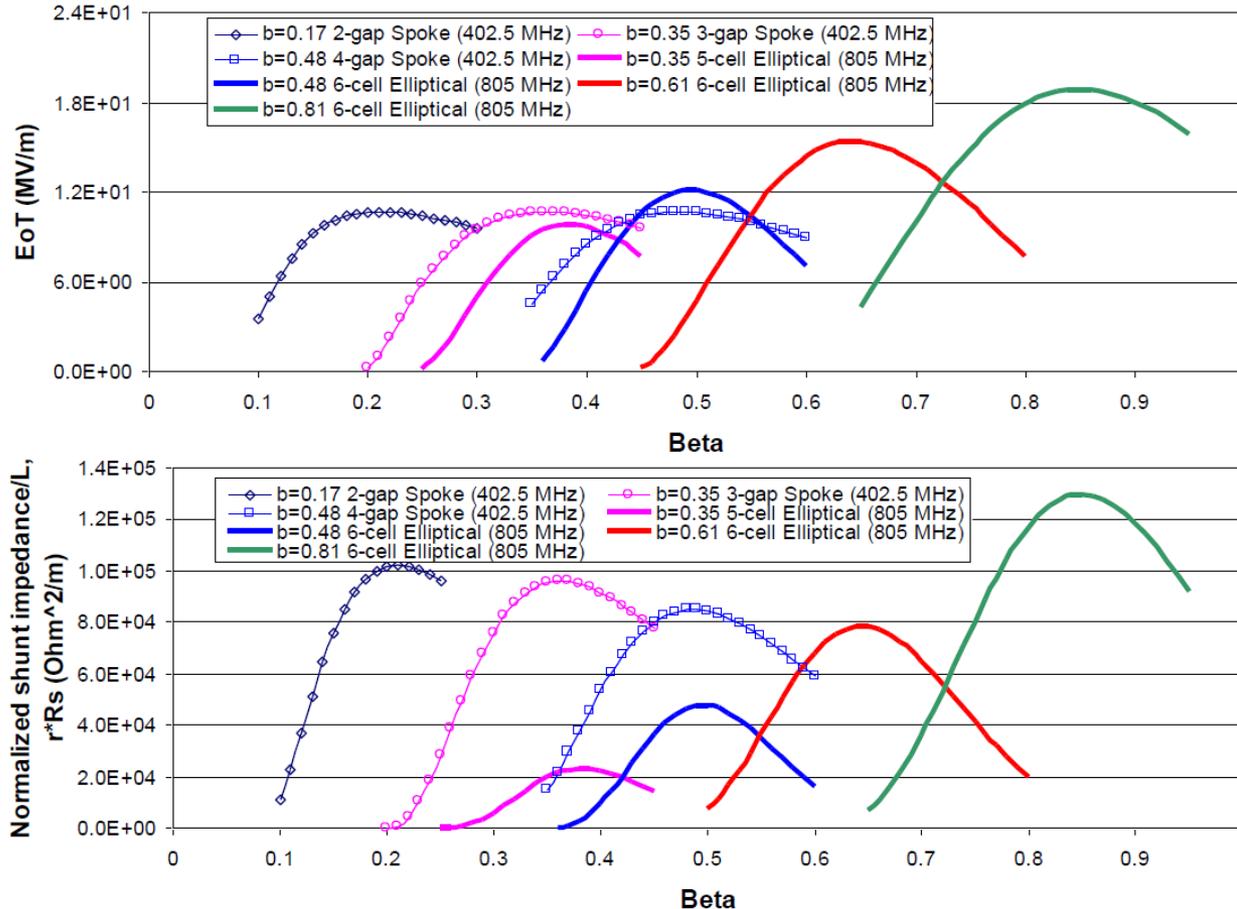


- Balloon cavity is successfully tested:
condition time reduced from ~ 10 hours to ~ 30 mins!

Why not multi-spoke for $\beta > 0.5$?

Comparison of RF properties (elliptical cavity versus spoke cavity)*

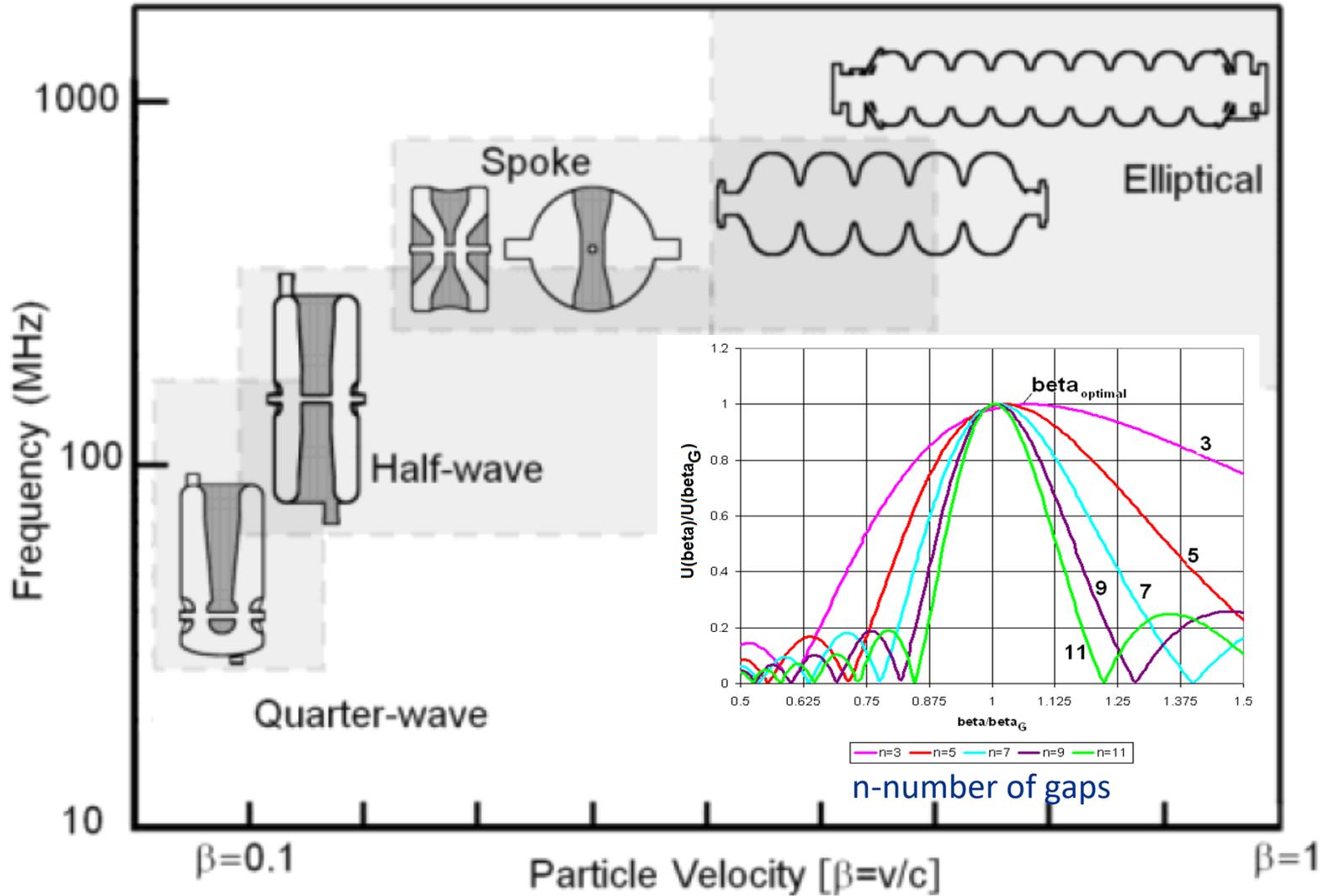
Spoke cavities (402.5 MHz) and elliptical cavities (805 MHz) are optimally designed under the same criteria: $E_{\text{peak}} \approx 40 \text{ MV/m}$ and $B_{\text{peak}} \approx 85 \text{ mT}$. Here EoT is gradient, and r^*Rs is $R/Q \cdot G$ per unit length.



For $\beta > 0.5-0.6$ elliptical cavity is preferable!

*Sang-Ho Kim, Mark Doleans, USPAS, January 2013, Duke University

SRF Cavity types depending on particle velocity



Summary:

- ❑ For acceleration of the particles having low velocity, QWR, HWR and spoke cavities are used in modern RT and SRF accelerators, which have high R/Q at low β .
- ❑ Double and triple-spoke resonators are also used up to $\beta = 0.5$.
- ❑ QWR, HWR and SR are prone to MP; Balloon cavity has no MP.
- ❑ TEM-type cavities are used up to $\beta = 0.5$. For higher β elliptical cavities are used in SRF accelerators;