Polarization Preservation Homework

June 14, 2021. Due date: June 16, 2021

1(2 grade points). A beam bunch is composed of particles with different betatron amplitudes and phases. Let ε be the emittance of a particle and the distribution function be $\rho(\varepsilon)$. The polarization of the beam after passing through an isolated resonance is given by:

$$< P_f/P_i >= \int_0^\infty (2\exp[\frac{-\pi|\epsilon(\varepsilon)|^2}{2\alpha}] - 1)\rho(\varepsilon)d\varepsilon, \rho(\varepsilon) = \frac{1}{2\varepsilon_0}e^{-\varepsilon/2\varepsilon_0}. \quad (1)$$

Using the fact that the intrinsic resonance strength is proportional to the square of the particle emittance,

$$|\epsilon(\varepsilon)|^2 = |\epsilon(\varepsilon_0)|^2\frac{\varepsilon}{\varepsilon_0}, \quad (2)$$

prove Froissart-Stora formula for a Gaussian distribution is given as

$$P_f/P_i = \frac{1 - \frac{\pi\epsilon^2}{\alpha}}{1 + \frac{\pi\epsilon^2}{\alpha}}. \quad (3)$$

2(6 grade points). Fit Booster harmonic scan data (cos3v and sin3v) with Froissart-Stora formula in unit of harmonic current. Since they are orthogonal, they can be scanned separately. For a given corrector current, the effective resonance strength is the combination of both the original imperfection resonance and the corrector resonance strength. Namely, the Froissart-Stora formula is in the form of

$$P_f = P_i(2\exp[\frac{-\pi|\epsilon_1 - \epsilon_2|^2}{2\alpha}] - 1), \quad (4)$$

Where ϵ_1 and ϵ_2 are the resonance strengths of the original imperfection resonance and the one introduced by the correctors. The resonance strength is a complex number. It has real and imaginary parts or two orthogonal components: cosine and sine. At proper current of the two orthogonal components, the effective resonance strength is zero and polarization is fully preserved in this case. Since we are going to scan the corrector current, we rewrite above formula in a slightly different form:

$$P_f = P_i(2\exp[\frac{-\pi(I_s - I_{s0})^2}{2\sigma_s^2}]\exp[\frac{-\pi(I_c - I_{c0})^2}{2\sigma_c^2}] - 1), \quad (5)$$

where I_s (I_c) is the corrector current for sine (cosine) component, I_{s0} (I_{c0}) is the sine (cosine) corrector current corresponding to optimized polarization, and σ_s (σ_c) will provide the width of the sine (cosine) current scan, or sensitivity of the current variation. During the current scan, only one component of either I_c and I_s is varied. The other component needs to be set as a constant. In other words, the fitting is done with following format for cosine and sine components separately:

$$P_f = p_0(2\exp[\frac{-\pi(I-p_1)^2}{2p_2^2}] - p_3), \tag{6}$$

where p_0, p_1, p_2 and p_3 are the fitting parameters. For the cosine component scan, $p_0 = P_i \exp[\frac{-\pi(I_s - I_{s0)}^2}{2\sigma_s^2}]$, $p_1 = I_{c0}$, $p_2 = \sigma_c$ and $p_3 = 1/\exp[\frac{-\pi(I_s - I_{s0)}^2}{2\sigma_s^2}]$. The terms related to sine are absorbed into the fitting parameters.

Tables 1 and 2 provided the experimental data of the vertical 3rd harmonic scan for the Booster. The polarization is given in arbitrary unit. Actually, it is called asymmetry and needs to be divided by the so-called analyzing power to give a polarization value between -1 and +1. The exercise is to find the I_0 to be used for full correction of the 3rd harmonics. For this purpose, we don't care about the unit of the polarization. There are three parameters for the data fitting: P_i, I_0 and σ. Among the three parameters, I_0 is the most important one and σ provides sensitivity of the polarization on the variation of the particular harmonic component. If possible, plot the fitted curve and experiment data together on one plot.

Table 1: 3rd harmonic sine current scan

cos3v I (A)	Asymmetry	error bar
10	270.471	4.096
9	187.052	4.719
8	123.548	4.211
7	40.727	4.15
6	-39.088	4.184
5	-124.047	4.225
4	-205.082	4.084
3	-303.959	5.397
2	-374.321	4.215
1	-438.749	4.064
0	-510.413	4.076
-1	-579.386	4.07
-2	-628.054	4.051
-3	-676.508	4.087
-4	-701.441	4.121
-5	-727.928	4.312
-6	-730.051	4.902
-7	-733.544	4.089
-8	-719.381	4.495
-9	-683.714	4.318
-10	-626.98	4.124
-11	-578.39	4.194
-12	-523.706	4.167
-13	-454.92	4.242

Table 2: 3rd harmonic cosine current scan

sin3v I (A)	Asymmetry	error bar
5.2	-164.591	3.963
4.2	-297.504	4.098
3.2	-438.518	3.999
2.2	-569.737	4.015
1.2	-672.803	4.032
0.2	-719.652	3.988
-0.8	-756.134	4.035
-1.8	-737.453	4.095
-2.8	-694.489	4.086
-3.8	-595.165	4.006
-4.8	-481.373	4.162