Spin rotators and Snakes Homework

June 9, 2021. Due date: June 12, 2021

1 Homework 1:Helical Snake Design Options

Points:4

We need to design a Siberian Snake for a proton accelerator ring based on helical dipole modules. Three schemes have been proposed , which are summarized in the Table.

	Scheme 1	Scheme 2	Scheme 3
1st Helix	$B = 3.5$ T, $R = +1$, $N = 1$	$B = 1.3$ T, $R = +1$, $N = 1$	$B = 2.5$ T, $R = +1$, $N = 1$
2nd Helix	$B = -1.1$ T, $R = +1$, $N = 2$	$B = -4$ T, $R = +1$, $N = 1$	$B = -2.5$ T, $R = +1$, $N = 1$
3rd Helix	$B = 1.1$ T, $R = +1$, $N = 2$	$B = 4$ T, $R = +1$, $N = 1$	$B = 2.4$ T, $R = -1$, $N = 1$
4th Helix	$B = -3.5$ T, $R = +1$, $N = 1$	$B = -1.3$ T, $R = +1$, $N = 1$	$B = -2.4$ T, $R = -1$, $N = 1$

All snakes use helical magnets with twist period 2.4m. N characterizes a number of helical twist periods in each magnet Using a formula for the orbit shift on one twist period (slide 15), find the maximum orbit excursion inside each Snake design at $E = 25GeV$.. Also calculate the absolute field integral for each design option. On the basis of these calculations, and, may be, some other considerations: which one of three design options you would recommend for the accelerator ring?

Note: when using the orbit shift formula, use $\beta = 1$ and substitute (E/e) with 25 (GeV)

2 Homework 2: Solenoid spin rotator

Points: 3

Consider the rotator system which includes solenoidal magnet followed by bending magnet (with vertical field).

We would like to convert the vertical spin to longitudinal, using such rotator. Calculate required solenoidal and bending field integrals, as well as bending field angles to provide this spin conversion at the energies 1 GeV, 5 GeV and 10 GeV for electrons and for protons. Summarize results in a table.

3 Homework 3: Spinor Math Relations for Siberian Snakes

Points: 4

Using general spinor transformation matrix in the form:

$$M = \exp[-i(\vec{\sigma} \cdot \vec{b})\frac{\varphi}{2}] = I\cos(\varphi/2) - i(\vec{\sigma} \cdot \vec{b})\sin(\varphi/2). \quad (1)$$

where φ is the spin rotation angle and $\vec{b}$ is the rotation axis unit vector.

The matrix of the Siberian Snake with the Snake axis angle α_s is:

$$M_{snake} == -i(\sigma_1 \cos\alpha_s + \sigma_2 \sin\alpha_s). \tag{2}$$

And the matrix of spin rotation in the ring arcs, where the spin rotates around the vertical guiding magnetic field of dipole magnets is:

$$M = \exp[-i(\sigma_3 \frac{G\gamma\theta}{2}] = I\cos(G\gamma\theta/2) - i(\sigma_3 \sin(G\gamma\theta/2). \tag{3}$$

where θ is the arc bending angle.

Then, first, prove that the Snake matrix can be presented as the product of two consecutive rotations (around horizontal and vertical axes):

$$M_{snake} = \exp(-i\alpha_s\sigma_3)\cdot(-i\sigma_1) \tag{4}$$

Second, prove the relation between arc and Snake matrices

$$M_{arc}M_{snake} = M_{snake}M_{arc}^{-1} \tag{5}$$

4 Homework 4: Multiple Siberian Snakes

Points: 6

Consider system of 2N Siberian Snakes placed at the azimuths $\theta_1, \theta_2, \ldots, \theta_{2N}$ in the accelerator ring Each Snake is characterized by its own Snake axis angle $\alpha_{s,i}$.

Show that the one turn matrix is the matrix of the spin rotation around the vertical axis and confirm the spin tune expression shown on slide 32.

$$\begin{aligned} M_{turn} &= M_{sn,2N}V_{\theta_{2N},\theta_{2N-1}}M_{sn,2N-1}V_{\theta_{2N-1},\theta_{2N-2}}\ldots M_{sn,1}V_{\theta_1,\theta_0} \\ &= \ldots \\ &= V(\phi) = \exp(-i\sigma_3\frac{\phi}{2}) \end{aligned}$$

and

$$\phi = G\gamma\sum_{i=1}^{2N}(-1)^{i-1}(\theta_i - \theta_{i-1}) + 2\sum_{i=1}^{N}(\alpha_{s,2i} - \alpha_{s,2i-1}). \tag{6}$$

5 Homework 5: Snake axis imperfection

Points: 2

Consider a system of two full Snakes separated precisely by 180 degree bending angle . The Snake axes are chosen such that $\alpha_{s,2} - \alpha_{s,1} = \pi/2$, so the spin tune is equal to one half.

1. Now let's assume that there is some error in the Snake axis orientation. Find the tolerance on the Snake axis angle to have the spin tune deviation less than 0.1.

2. Now let's assume that one snake was place imperfectly, shifted by 1 degree of bending angle from the perfect location. Evaluate the maximum spin tune shift when accelerating protons from 240 to 250 GeV.

6 Homework 6: Snake resonances

Points: 2

Consider system of two full Snakes separated by 180 degree bending angle. The Snake axes are chosen such that $\alpha_{s,2} - \alpha_{s,1} = \pi/2$. Spin tune for such system is 0.5.

Calculate betatron tune values corresponding to the resonance conditions of 2nd, 3rd,4th and 5th order spin resonances.

Now consider that snake axes were retuned to get the spin tune 0.25. Find the required orientation of the snake axes and calculate betatron tune values corresponding to the locations of 2nd, 3rd,4th and 5th order spin resonances.

How the number of the resonances compare in two cases?