

# SPIN@USPAS Summer 2021

## Graduate “Spin Dynamics” Homework

### HOME WORK 3 / ex. 2.3: SPIN CLOSED ORBIT SOLUTION

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The three  $G\gamma$  values concerned will be treated simultaneously, using a single simulation input data file.

1/ Finding the spin closed orbits for 200 keV, 108.412 MeV and 370.082556 MeV.

Particles have to be on the closed orbit - a mere circular trajectory, in the present uniform field. Radii are as follows:

From  $R = B\rho/B_0$ , with R the  $Y_0$  coordinate under OBJET,  $B_0 = 5$  kG the field value, one gets,

$$Y_0(200 \text{ keV}) = 12.924889 \text{ cm}, \quad Y_0(108.412 \text{ MeV}) = 309.47295 \text{ cm}, \quad Y_0(370.082556 \text{ MeV}) = 608.30878 \text{ cm}$$

The data file takes a reference  $B\rho = 64.624444037$  kG cm (BORO quantity under OBJET), hence the relative  $D = \frac{B\rho(E)}{\text{BORO}}$  values:

$$D(200 \text{ keV}) = 1, \quad D(108.412 \text{ MeV}) = 23.943951797, \quad D(370.082556 \text{ MeV}) = 47.06491129$$

A FIT is used to find the periodic spin solution:

- variables are the spin components at the start of the sequence,
- constraints are the spin components: equal initial and final values, and spin modulus, one particle orbit downstream (*i.e.*, at the end of the sequence)

Input data file; two problems are stacked, they are completed in sequence

- the first problem (down to FIT) finds the spin closed orbits,
  - the second problem (beyond FIT) continues tracking over one more turn
- (the “6\* ./60degSector.inc[#S\_60degSectorUnifB:#E\_60degSectorUnifB]” INCLUDE; the factor “6” can be increased for more turns. REBELOTE could be used instead, as well).

```
Cyclotron, classical. Synchronized spin kick in a uniform field
'OBJET'
64.62444403717985          ! Reference Brho ("BORO" in the users' guide) -> 200keV proton.
2
3 1
12.9248888074 0. 0. 0. 0. 1.      'm'          ! Ggamma=1.793229,      0.200MeV
3.0947295453790e2 0. 0. 0. 0. 23.9439548880185 'm'      ! Ggamma=2,          108.411628MeV
6.08308775712857e2 0. 0. 0. 0. 47.0649136542578 'm'      ! Ggamma=2.5         370.082556MeV
1 1 1          ! For any particle: set to 1 to enable ray-tracing, or -9 to ignore.

'PARTICUL'
PROTON          ! This is required for spin motion to be computed,
                ! otherwise, by default zgoubi only requires rigidity.
'SPNTRK'
4              ! Request spin tracking.
                ! Individual initial spins taken (arbitrarily) parallel to Z axis.
0. 0. 1.
0. 0. 1.
0. 0. 1.

'INCLUDE'
1
6* ./60degSector.inc[#S_60degSectorUnifB:#E_60degSectorUnifB]          ! 360 degree sector.
```

```

'FAISCEAU'

'SPINR'
1                                     ! Spin rotation,
0. 30.                               1 about the X-axis, by 30 degree here.

'FIT'
9                                     ! Vary initial spin coordinates of the 3 particles.
3 10 0 [-1.,1.]
3 11 0 [-1.,1.]
3 12 0 [-1.,1.]
3 20 0 [-1.,1.]
3 21 0 [-1.,1.]
3 22 0 [-1.,1.]
3 30 0 [-1.,1.]
3 31 0 [-1.,1.]
3 32 0 [-1.,1.]
12 1e-15                               ! 12 constraints:
10.1 1 1 #End 0. 1. 0                 ! Particle 1: SX_initial==SX_final,
10.1 1 2 #End 0. 1. 0                 ! SY_initial==SX_final,
10.1 1 3 #End 0. 1. 0                 ! SZ_initial==SX_final,
10 1 4 #End 1. 1. 0                   ! |S| == 1.
10.1 2 1 #End 0. 1. 0                 ! Particle 2: same.
10.1 2 2 #End 0. 1. 0
10.1 2 3 #End 0. 1. 0
10 2 4 #End 1. 1. 0
10.1 3 1 #End 0. 1. 0                 ! Particle 3: same.
10.1 3 2 #End 0. 1. 0
10.1 3 3 #End 0. 1. 0
10 3 4 #End 1. 1. 0

'SPNPRT'

'OBJET'
64.62444403717985                     ! Reference Brho ("BORO" in the users' guide) -> 200keV proton.
2
3 1
12.9248888074 0. 0. 0. 1. 'm'         ! Ggamma=1.793229, 0.200MeV
3.0947295453790e2 0. 0. 0. 0. 23.9439548880185 'm' ! Ggamma=2, 108.411628MeV
6.08308775712857e2 0. 0. 0. 0. 47.0649136542578 'm' ! Ggamma=2.5 370.082556MeV
1 1 1                                   ! For any particle: set to 1 to enable ray-tracing, or -9 to ignore.

'PARTICUL'
PROTON                                  ! This is required for spin motion to be computed,
                                       ! otherwise, by default zgoubi only requires rigidity.

'INCLUDE'
1
6* ./60degSector.inc[#S_60degSectorUnifB:#E_60degSectorUnifB]           ! 360 degree sector.
'FAISCEAU'

'SPINR'
1                                     ! Spin rotation,
0. 30.                               1 about the X-axis, by 30 degree here.

'SPNPRT'
'SYSTEM'
1
gnuplot <./gnuplot_Zplt_SDance_vectors.gnu

'END'

```

The result of the FIT procedure can be found in zgoubi.res listing:

```

24 Keyword, label(s) : FIT
IPASS= 1

FIT variables and constraints in good order, FIT will proceed.
STATUS OF VARIABLES (Iteration # 31 / 999 max.)
LMNT VAR PARAM MINIMUM INITIAL FINAL MAXIMUM STEP NAME LBL1 LBL2
3 1 10 -1.00 0.323 0.32253242 1.00 4.182E-09 SPNTRK - -
3 2 11 -1.00 -0.245 -0.24498407 1.00 4.182E-09 SPNTRK - -
3 3 12 -1.00 0.914 0.91430610 1.00 4.182E-09 SPNTRK - -
3 4 20 -1.00 1.00 1.00000000 1.00 4.182E-09 SPNTRK - -
3 5 21 -1.00 -6.939E-18 -6.93889390E-18 1.00 4.182E-09 SPNTRK - -
3 6 22 -1.00 1.254E-08 1.25445089E-08 1.00 4.182E-09 SPNTRK - -
3 7 30 -1.00 0.00 0.00000000 1.00 4.182E-09 SPNTRK - -
3 8 31 -1.00 -0.259 -0.25881905 1.00 4.182E-09 SPNTRK - -
3 9 32 -1.00 0.966 0.96592584 1.00 4.182E-09 SPNTRK - -
STATUS OF CONSTRAINTS (Target penalty = 1.0000E-15)
TYPE I J LMNT# DESIRED WEIGHT REACHED KI2 NAME LBL1 LBL2 Nb param. [value]
10 1 1 23 0.000000E+00 1.000E+00 2.433528E-09 3.36E-02 SPINR - - 0
10 1 2 23 0.000000E+00 1.000E+00 2.844524E-09 4.59E-02 SPINR - - 0
10 1 3 23 0.000000E+00 1.000E+00 7.892301E-10 3.54E-03 SPINR - - 0
10 1 4 23 1.000000E+00 1.000E+00 1.000000E+00 0.00E+00 SPINR - - 0
10 2 1 23 0.000000E+00 1.000E+00 0.000000E+00 0.00E+00 SPINR - - 0
10 2 2 23 0.000000E+00 1.000E+00 1.639380E-09 1.53E-02 SPINR - - 0
10 2 3 23 0.000000E+00 1.000E+00 9.941455E-10 5.61E-03 SPINR - - 0
10 2 4 23 1.000000E+00 1.000E+00 1.000000E+00 0.00E+00 SPINR - - 0
10 3 1 23 0.000000E+00 1.000E+00 3.368637E-09 6.44E-02 SPINR - - 0
10 3 2 23 0.000000E+00 1.000E+00 1.573686E-09 1.41E-02 SPINR - - 0
10 3 3 23 0.000000E+00 1.000E+00 1.200110E-08 8.18E-01 SPINR - - 0
10 3 4 23 1.000000E+00 1.000E+00 1.000000E+00 0.00E+00 SPINR - - 0
Fit reached penalty value 1.7616E-16

```

## 2/ Propagating the spin closed orbit over more turns.

The goal is to check the periodicity, for instance by superposition of motions turn by turn.

The 2 different types of figures below at the bottom have been obtained with the following gnuplot files:

```

# CHOOSE PARTICLE TO PLOT, FROM HERE:
trj = 3; R=6.08308775712857e2 ; Gg=2.5 ; mag =800
trj = 1; R=12.92488888074 ; Gg= 1.793229; ; mag =20
trj = 2; R=3.0947295453790e2 ; Gg=2. ; mag =200
# Positioning of 1st and last dipoles in zgoubi.dat sequence, and increment
dip1=5; dip2=20; dd=3 ; pi3 = 4.*atan(1.)/3

set xlabel "orbital angle"; set ylabel "S_X, S_Y, S_Z"; set yrange [-1:+1]
set label "Vert. bars: every 2{/Symbol p} spin precession" at 1, 0.4
do for [nPrec=1:4] {
set arrow from nPrec* 2*pi/Gg, -1 to nPrec* 2*pi/Gg, 1 nohead
}

# spin components versus orbital angle:
set title "Six dipoles over a turn, each dipole its color \n Spin vector tip (angle to Z is magnified)"
plot \
"zgoubi.plt" u ($19==trj && $50==1 ? $14/R :1/0):($33) w lp pt 4 ps .4 , \
"zgoubi.plt" u ($19==trj && $50==1 ? $14/R :1/0):($34) w lp pt 6 ps .4 , \
"zgoubi.plt" u ($19==trj && $50==1 ? $14/R :1/0):($35) w lp pt 8 ps .4

set terminal postscript eps blacktext color enh
set output "gnuplot_Zplt_theta-SZ.eps"
replot; set terminal X11; unset output

pause 1

# 2D: spin projected in (X,Y) plane, first:
set title "Six dipoles over a turn, each dipole its color \n Thick line: trajectory; thin line: spin vector tip (angle to Z is magnified)"
unset yrange; unset label; unset arrow
set xlabel "X_(Lab)"; set ylabel "Y_(Lab)"; set zlabel "S_Z"; set xtics; set ytics; set ztics; set size ratio -1

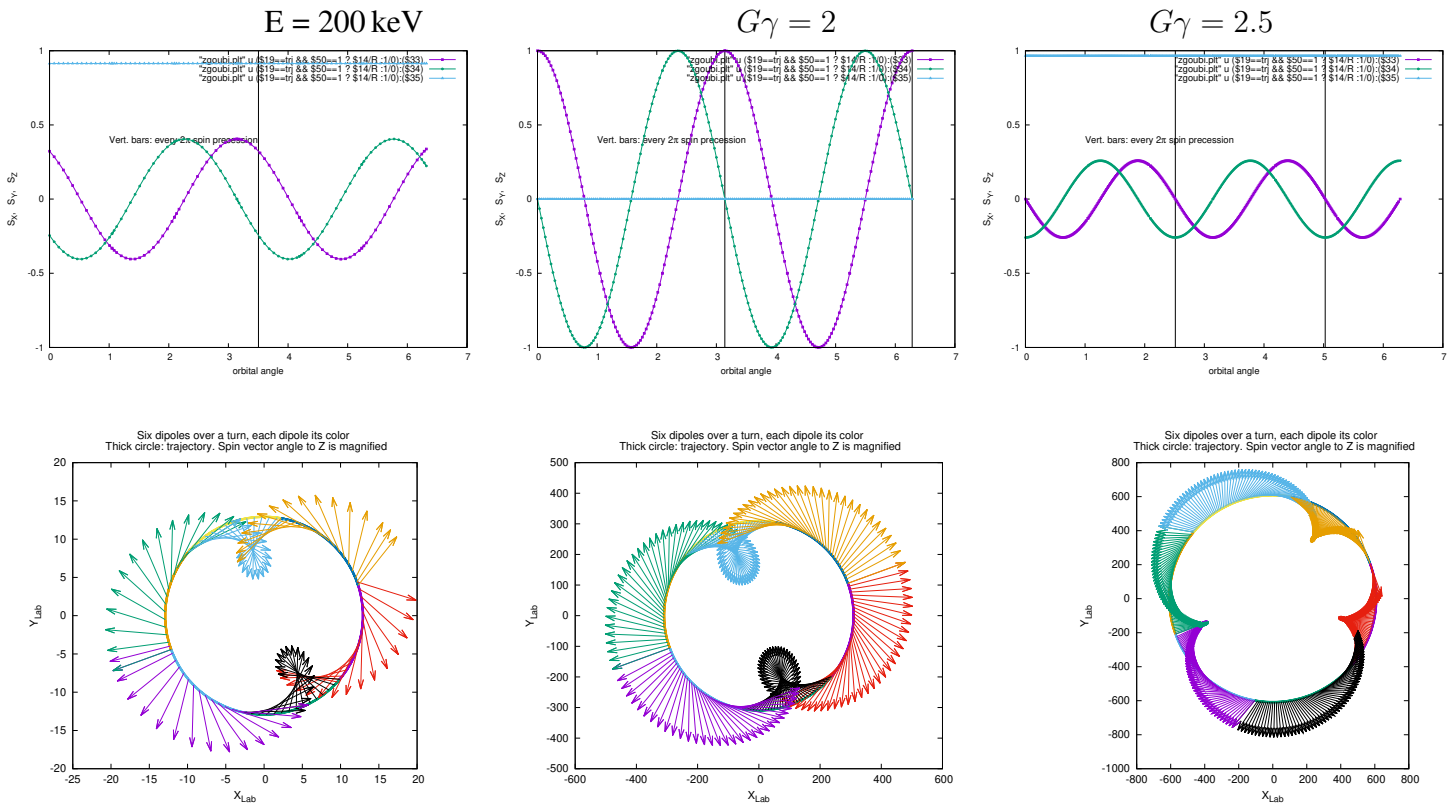
plot \
for [dip=dip1:dip2:dd] \
"zgoubi.plt" u ($19==trj && $42==dip? $10*cos(-$22-pi3*(dip-6.)/3.) :1/0):($10*sin(-$22-pi3*(dip-6.)/3.) ) w l lw 6 notit , \
for [dip=dip1:dip2:dd] \
"zgoubi.plt" u ($19==trj && $42==dip? $10*cos(-$22-pi3*(dip-6.)/3.) :1/0):($10*sin(-$22-pi3*(dip-6.)/3.) ) :($19==trj && $42==dip? mag*(cos(-$22-pi3*(dip-6.)/3.))*$

pause 1

set terminal postscript eps blacktext color enh ; set output "gnuplot_Zplt_spinDance_2D.eps"
plot \
for [dip=dip1:dip2:dd] \
"zgoubi.plt" u ($19==trj && $42==dip? $10*cos(-$22-pi3*(dip-6.)/3.) :1/0):($10*sin(-$22-pi3*(dip-6.)/3.) ) w l lw 6 notit , \
for [dip=dip1:dip2:dd] \
"zgoubi.plt" u ($19==trj && $42==dip? $10*cos(-$22-pi3*(dip-6.)/3.) :1/0):($10*sin(-$22-pi3*(dip-6.)/3.) ) :($19==trj && $42==dip? mag*(cos(-$22-pi3*(dip-6.)/3.))*$
set terminal X11; unset output

exit

```



Top row: spin coordinates versus turn; middle row: projection in the median plane.

3/ A simple way is to check the angle between the precessing spin and the closed orbit vector.

The goal is to check the periodicity, for instance by superposition of motions turn by turn:

- a first job, spinClosedOrbit.dat, computes the latter, for the three energies, and saves zgoubi.plt to zgoubi\_closedSOorbit.plt. Note that the spin kick by SPINR is 30 deg, here;

- a second job, spinPrecess.dat, computes the precessing spin motion, for the three energies, and saves zgoubi.plt to zgoubi\_precess.plt. spinPrecess.dat essentially differs from spinClosedOrbit.dat by the initial spin values (under SPNTRK), which are taken off the pin closed orbit in the former.

#### ◇ spinClosedOrbit.dat:

```
Cyclotron, classical. Synchronized spin kick in a uniform field
'OBJET'
64.62444403717985          ! Reference Brho ("BORO" in the users' guide) -> 200keV proton.
2
3 1
12.9248888074 0. 0. 0. 0. 1.      'm'          ! Ggamma=1.793229,    0.200MeV
3.0947295453790e2 0. 0. 0. 0. 23.9439548880185 'm'      ! Ggamma=2,        108.411628MeV
6.08308775712857e2 0. 0. 0. 0. 47.0649136542578 'm'    ! Ggamma=2.5      370.082556MeV
1 1 1          ! For any particle: set to 1 to enable ray-tracing, or -9 to ignore.

'PARTICUL'
PROTON          ! This is required for spin motion to be computed,
                ! otherwise, by default zgoubi only requires rigidity.
'SPNTRK'          ! Request spin tracking.
4                ! Individual initial spins taken parallel to Z axis.
0.32253242 -0.24498407 0.91430610
1. 0. 0.
0. -0.25881905 0.96592584

'INCLUDE'
1
6* ./60degSector.inc[#S_60degSectorUnifB:#E_60degSectorUnifB]      ! 360 degree sector.
'FAISCEAU'

'SPINR'
1
0. 30.          ! Spin rotation,
                1 about the X-axis, by 30 degree here.

'SPNPRT'

'SYSTEM'
1
cp zgoubi.plt zgoubi_closedSOorbit.plt

'END'
```

#### ◇ spinPrecess.dat:

```
Cyclotron, classical. Synchronized spin kick in a uniform field
'OBJET'
64.62444403717985          ! Reference Brho ("BORO" in the users' guide) -> 200keV proton.
2
3 1
12.9248888074 0. 0. 0. 0. 1.      'm'          ! Ggamma=1.793229,    0.200MeV;
3.0947295453790e2 0. 0. 0. 0. 23.9439548880185 'm'      ! Ggamma=2,        108.411628MeV;
6.08308775712857e2 0. 0. 0. 0. 47.0649136542578 'm'    ! Ggamma=2.5      370.082556MeV.
1 1 1          ! For any particle: set to 1 to enable ray-tracing, or -9 to ignore.

'PARTICUL'
PROTON          ! This is required for spin motion to be computed,
                ! otherwise, by default zgoubi only requires rigidity.
'SPNTRK'          ! Request spin tracking.
4                ! Individual initial spins taken parallel to Z axis.
0. 0. 1.
.9 0. 0.435889894354
0. 0. 1.

'INCLUDE'
1
6* ./60degSector.inc[#S_60degSectorUnifB:#E_60degSectorUnifB]      ! 360 degree sector.
'FAISCEAU'

'SPINR'
1
0. 30.          ! Spin rotation,
                1 about the X-axis, by 30 degree here.

'SPNPRT'

'SYSTEM'
1
cp zgoubi.plt zgoubi_precess.plt

'END'
```

A gnuplot script, below, plots the scalar product between the precessing spin  $\vec{S}(\theta)$  and the periodic solution  $\vec{n}(\theta)$ , as a function of  $\theta$  around the ring (displayed in the bottom graphs), expected to be constant.

```

set xlabel "orbital angle [rad]"; set ylabel "cos({/Symbol f})"

set key c r

# Change particle number here
# trj = 1: 12.9248888074 0. 0. 0. 1. 'm' ! Ggamma=1.793229, 0.200MeV
# trj = 2: 3.0947295453790e2 0. 0. 0. 0. 23.9439548880185 'm' ! Ggamma=2, 108.411628MeV
# trj = 3: 6.08308775712857e2 0. 0. 0. 0. 47.0649136542578 'm' ! Ggamma=2.5 370.082556MeV

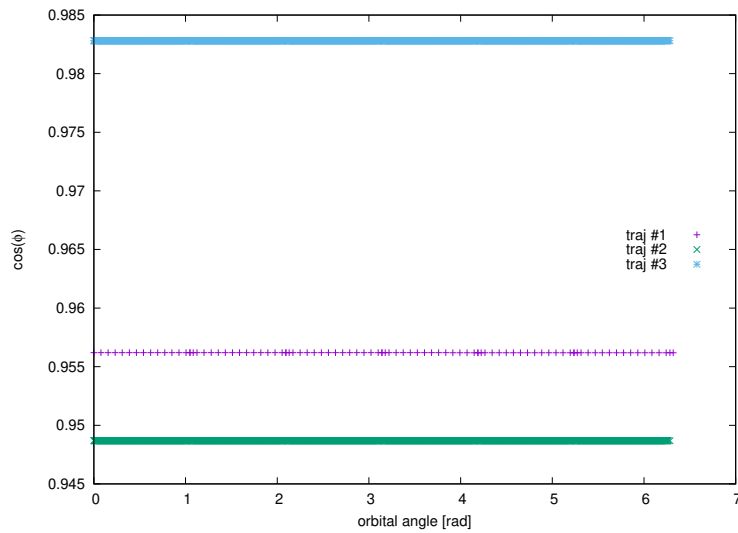
array R[3]
trj = 1; R[1]=12.9248888074 ; Gg= 1.793229; ; mag =200
trj = 2; R[2]=3.0947295453790e2 ; Gg=2. ; mag =200
trj = 3; R[3]=6.08308775712857e2 ; Gg=2.5 ; mag =800

plot for [trj=1:3] \
"< paste zgoubi_closedSOrbit.plt zgoubi_precess.plt" u ($19==trj ? $14/R[trj] :1/0):(sqrt($33*$85 + $34*$86 + $35*$87)) w p tit 'traj #'.trj

pause 1

set terminal postscript eps blacktext color enh
set output "gnuplot_Zplt_spinAngle.eps"
#
plot for [trj=1:3] \
"< paste zgoubi_closedSOrbit.plt zgoubi_precess.plt" u ($19==trj ? $14/R[trj] :1/0):(sqrt($33*$85 + $34*$86 + $35*$87)) w p tit 'traj #'.trj
#
set terminal X11; unset output

```



Scalar product of the periodic spin vector and the precessing spin vector, as a function of orbital angle, for the three cases  $G\gamma = 1.7932$ ,  $G\gamma = 2$ ,  $G\gamma = 2.5$