Spin Matching Homework

June 16, 2021. Due date: June 18, 2021

1 Vector k_0

Points:3

Evaluation of spin matching conditions in the integral form relies on the vector $\mathbf{k_0}$. This vector is a complex vector which is based on real spin vectors $\mathbf{l_0}$ and $\mathbf{m_0}$:

$$\mathbf{k_0} = \mathbf{l_0} - i\mathbf{m_0}$$

The vectors $\mathbf{l_0}$ and $\mathbf{m_0}$ are the solutions of spin motion on the design orbit, and thy are orthogonal to the vector $\mathbf{n_0}$ and to each other. The set $(\mathbf{l_0}, \mathbf{m_0}, \mathbf{n_0})$ presents right-handed orthonormal triad.

Consider one solenoidal magnet. Let's assume at given particle energy it rotates spin by the 90 degrees.

Let's accept that at the solenoid entrance the vector components in the laboratory frame $(\mathbf{e_x}, \mathbf{e_s}, \mathbf{e_y})$ (which is also the right-handed orthonormal triad)

$$\begin{aligned} \mathbf{n_0} &= (0,0,1) \\ \mathbf{l_0} &= (1,0,0) \end{aligned} \tag{1}$$

Using this input data, please, find the components of vector $\mathbf{k_0}$ in the laboratory frame at the entrance and at the exit of the solenoid.

2 Spin Matching for Dipole Magnet Spin Rotators

Points:5

Spin Rotators based on Dipole magnets are shown on slide 18 in the Spin Matching lecture.

Can you show how one gets the spin matching condition, involving dispersion functions D_x and $_y$ on slide 19?

For this rotator system you should use the precession vector components, listed on slide 18.

Then, using that precession vector and the expressions for uncoupled orbital motion (slide 4), evaluate the spin-orbital integral:

$$\int_{-s_r}^{s_r} [w_x k_{0x} + w_s k_{0s} + w_y k_{0y}] ds$$

and leave only terms proportional to momentum offset δ. Finally, show how to arrive to the resulting formula:

$$\int_{-s_r}^{s_r} [\nu_0 (g_x(s) D_x k_{0y} - g_y(s) D_y k_{0x}] ds$$