? Exercise 2: Difference in Precession Between Two Experimental Halls

Show that the difference in precession between two experimental halls can be written as $\Phi_{n1}^{h1} - \Phi_{n2}^{h2} = \frac{a}{m_e} f(h_1, n_1; h_2, n_2)\pi$ where h_1, h_2 are the halls A, B or C and n_1, n_2 are the passes at which the beam is extracted.

Write a program to find the combinations of energies in Hall A and Hall B for which the difference in precession between the two halls is exactly an integer number of π. This should allow to reproduce figure 8.5.

Solution:

Starting from equation 8.3, we introduce the ratio $\alpha = \frac{E_0}{E_1}$ of the injector energy to the linac energy and recast it in this form:

$$\Phi_n^h = E_1 \left(\frac{g-2}{2m_e}\right) \left[2n^2 - n\left(1 - 2\alpha - \frac{2\theta_h}{\pi}\right) - \alpha\left(1 - \frac{\theta_h}{\pi}\right)\right] \tag{8.6}$$

We also assumed that both linacs produce the same acceleration ($E_1 = E_2$) to simplify the formula.

From there, we can write the difference between halls h1 at pass n1 and h2 at pass n2 and obtain the solution.

When the quantity $E_1 \left(\frac{g-2}{2m_e}\right) (h1, n1, h2, n2)$ is an integer multiple of π, both halls have the maximum polarization, this occurs for specific values of E_1, the so-called magic energies.

One can write a simple python script [9] which generates all these combinations and plot it to reproduce the figure.