

## Chapter 14

### Spin Dynamics Tutorial: Numerical Simulations

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The next two Sections provide the material of a miniworkshop which extended over the two weeks of the Summer 2021 USPAS Spin Class, and was integral part of the teachings.

The work proposed to the attendees essentially consisted in the numerical simulation of polarized beam manipulations in the AGS injector - “AGS Booster” -, starting from basic principles (computation of resonance strengths, resonance crossing, effect of synchrotron radiation, etc.), and extending to the application of polarization preservation techniques (harmonic orbit correction or excitation, ac dipole, snakes, spin matching, etc.). As a matter of fact, for simplicity the same lattice, the AGS Booster, with magnetic rigidity adapted in consequence, was used for simulations concerning indifferently hadron polarization, or electron polarization and the effect of synchrotron radiation.

The miniworkshop covers many of the theoretical aspects addressed during the lectures, and most of the game in performing these numerical simulations consists in comparisons between their outcomes and theoretical expectations.

Section 14.1 gives the assignments. A first part addresses hadron (precisely, helion) beams (Secs. 14.1.1.1-14.1.1.17), a second part deals with electrons and synchrotron radiation (Secs. 14.1.2.1-14.1.2.5).

Section 14.2 gives detailed solutions of these numerical simulation exercises.

Finally, everything starts from a single input data file, “superA.inc”, short enough to be given in its entirety in Tab. 14.4 and subsidiary Tabs. 14.5 and 14.6, the latter two being a series of 6 cells constitutive of AGS Booster ring. Nothing else is needed but the code executable, “zgoubi”, downloadable from sourceforge [1]. The execution for instance of the said input data file is a mere

zgoubi -in superA.inc

All gnuplot [2] and zpop<sup>1</sup> graphs in the present assignments and in their solutions (Sec. 14.2) derive from ancillary files produced, upon request and during zgoubi

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<sup>1</sup> Zgoubi’s plotting and data treatment companion [3].

execution, by this simple execution instruction. The input data file taken from these Tabs. 14.4, 14.5 and 14.6 is developed further when needed, namely very little, as required in one or the other of these various problems.

However, simulation input and output data files of many of the exercises have been saved in Zgoubi development repository, where they can be downloaded from:

[https://sourceforge.net/p/zgoubi/code/HEAD/tree/trunk/exemples/uspasSpinClass\\_2021/mini-workshop/](https://sourceforge.net/p/zgoubi/code/HEAD/tree/trunk/exemples/uspasSpinClass_2021/mini-workshop/)

Besides, users may want to consider the use of python interfaces to zgoubi, subject to continuing development and available on web, pyZgoubi [4] and zgoubidoo [5], or the *ad hoc* HPC environment interface in Sirepo[6] where as well AGS Booster input data files may be found.

## 14.1 Numerical Simulations: Problems

The numerical simulations proposed in this Section address many of the theoretical aspects of polarized hadron beam acceleration and of electron polarization, introduced in the lectures. They use stepwise ray-tracing techniques (*i.e.*, a step-by-step integration of the equations of particle and spin motion), the reason for this is that the method allows detailed inspection of motion across optical elements, whether using analytical magnetic/electric field models or field maps, and it allows accurate Monte Carlo simulations such as stochastic emission of photons (synchrotron radiation) and its effects on particle and spin dynamics.

Three different classes of problems regarding the manipulation of polarized beams in circular accelerators are addressed:

- excitation of depolarizing resonances, and their effect on bunch polarization,
- preservation of polarization of hadron beams during acceleration,
- maximization of polarization and polarization life-time, in an electron storage ring.

Both series of simulation problems will use the same lattice, namely the AGS Booster ring. This means in particular, the same optical sequence input data files, *mutatis mutandis*.

### Hadron polarization simulations

Beam-beam collisions involving polarized helion ( $^3\text{He}^{2+}$ ) are part of the physics programs at the EIC. Polarized helion beams are produced using an EBIS source. Prior to injection in the EIC HSR (Hadron Storage Ring, an evolution of RHIC collider rings), helion beams are accelerated in the AGS Booster and in the AGS (Fig. 14.1).

At low rigidity, the cold snakes in the AGS cause harmful optical distortions, including linear coupling. A path to overcome this issue is by injecting  $^3\text{He}$  beams

at a high enough energy that these distortions become negligible. On the other hand, under the effect of two partial snakes, the stable spin direction  $\mathbf{n}_0$  in the AGS is at an angle to the guiding field, with a least magnitude every  $|G\gamma| = 3n + 1.5$  (Fig. 14.2). As a result, it is foreseen to extract  $^3\text{He}$  beams from the Booster at  $|G\gamma| = -10.5$ . These will be the conditions considered in these exercises, regarding hadron polarization.

**Table 14.1** Parameters of helion,  $^3\text{He}^{2+}$ , the particle considered in the exercises. Proton data are given for comparison. Note: atomic mass unit:  $931.4940954 \text{ MeV}/c^2$ ;  $m\mu_N = e\hbar/2m$

	units	proton	$^3\text{He}^{2+}$
mass, $m$	$\text{MeV}/c^2$	938.27208	2808.39158
	amu	1.00727646	3.0149322
charge	$ e $	+1	+2
number of nucleons		1	3
number of protons, $Z$		1	2
spin angular momentum, $I$	$\hbar$	1/2	1/2
magnetic moment, $\mu$	$\mu_N$	2.7928474	-2.1276253
gyromagnetic ratio $g = \frac{\mu}{\mu_N} \times 2 \frac{m\mu_N}{Ze\hbar I}$	$\mu/\mu_N$	2	2.99315281
anomalous magnetic moment $G = \frac{g-2}{2}$		1.7928474	-4.18415382
Imperfection resonance interval $ mc^2/G $	$\text{MeV}$	523.3	671.2

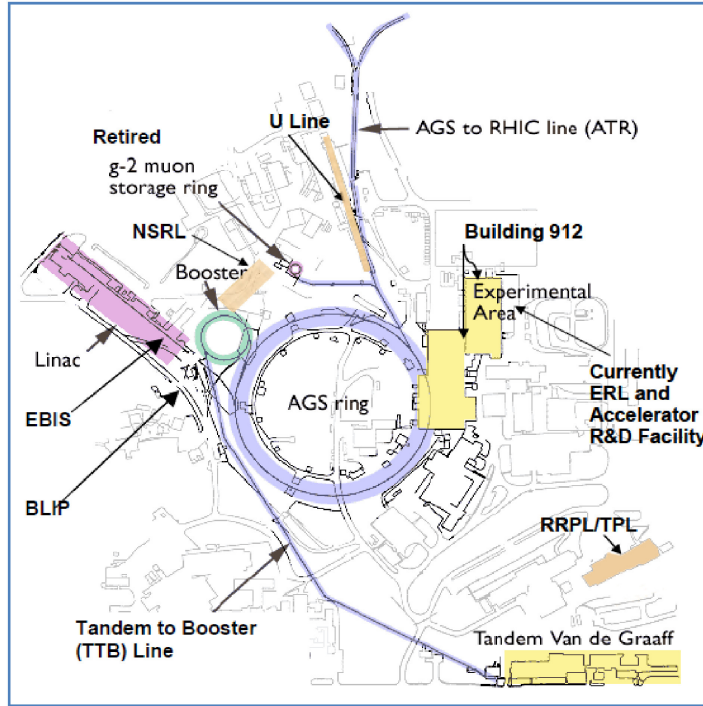
### Electron polarization simulations

High polarization of the electron beam at the collision points is required by the EIC physics program. Relativistic electrons emit photons, and a small fraction of these radiated photons contribute to spin flip, which builds up beam polarization through the Sokolov-Ternov effect. In the vast majority of cases, photon emission is associated with noisy orbital motion causing extra spin diffusion, *i.e.* depolarization.

In the present simulations regarding high energy electrons in a storage ring, rather than using the EIC electron storage ring (ESR) lattice, the very ring considered for hadrons, the AGS Booster, is used. There are various reasons for that: the Booster is a short ring (200 m), whereas the ESR is 3.8 km, this results in quicker tracking; it allows, quite efficiently, for dealing with a single lattice for both studies, hadron and electron spin dynamics; the AGS Booster lattice is much simpler than the ESR one, input data files are easier to handle; moving from hadron to electron simulations (or vice versa) reduces to essentially a matter of changing the reference rigidity and the nature of the particle.

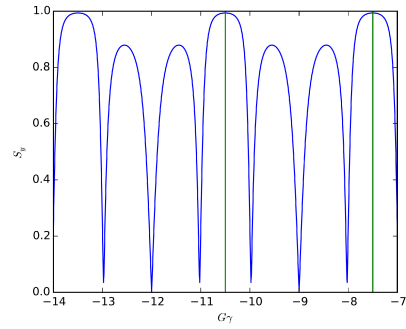
The goals in the electron polarization simulations are to

- establish stable particle motion in the AGS Booster lattice for an electron beam energy of 10 GeV, checking the damping parameters;
- calculate the invariant spin field  $\mathbf{n}_0$ ;
- understand the depolarization through the deviation of  $\mathbf{n}_0$  from the vertical in arc dipoles, and
- practice the spin matching mechanism, which maximizes electron polarization.



**Fig. 14.1** RHIC injector cascade, the future EIC hadron injector system, in 2021 (RHIC itself is not shown):  $H^-$  200 MeV linac, EBIS ion sources, AGS booster (which also accelerates ions for the NSRL, NASA Space Research Lab), the AGS, and the AGS to RHIC (AtR) injection line

**Fig. 14.2** Vertical component of AGS  $\mathbf{n}_0$  spin eigenvector, in the presence of two partial snakes. The angle to the vertical guide field is minimal every three units,  $G\gamma = -7.5, -10.5, -13.5$ , etc.



### Practical aspects regarding these numerical simulations

When developing simulation input data files, or when using existing ones, in order for what's computed, and the physics behind it, to be clear to the user, it can not be avoided to refer to the Users' Guide [1]. Having it at hand, ready to use, and consulting it whenever something happens which looks weird, or looks like going awry, is recommended.

A good thing to do when questions arise—and many will, is to navigate to the INDEX section of the Users' Guide. Note the two main parts in the Users' Guide: PART A which comments on the physics content and capabilities of the various optical elements and commands, and PART B which details the formatting of the data in an input data file. Two bold numbers generally appear in the guide INDEX, for any item; the first points to Part A, and the second to Part B. Additional considerations that may usefully be given some attention, are documented in the Appendix.

The computation of spin and orbital motion in this code uses stepwise ray-tracing techniques. This means that it solves the Lorentz and T-BMT differential equations proper, with no approximations on particle or spin dynamics, step-by-step. This allows accurate field modeling (and that does not preclude approximate field models if desired anyway) and detailed insight regarding their effect on spin motion.

In any event, one should not lose sight of the goal of the present simulation exercises, which is not especially to learn about a computer code. It is rather to play with, and learn about, spin dynamics in electric and magnetic fields, snakes, rotators, synchrotron radiation and spin diffusion, as a complement to the various theoretical chapters.

## 14.1.1 Polarized Helion in AGS Booster

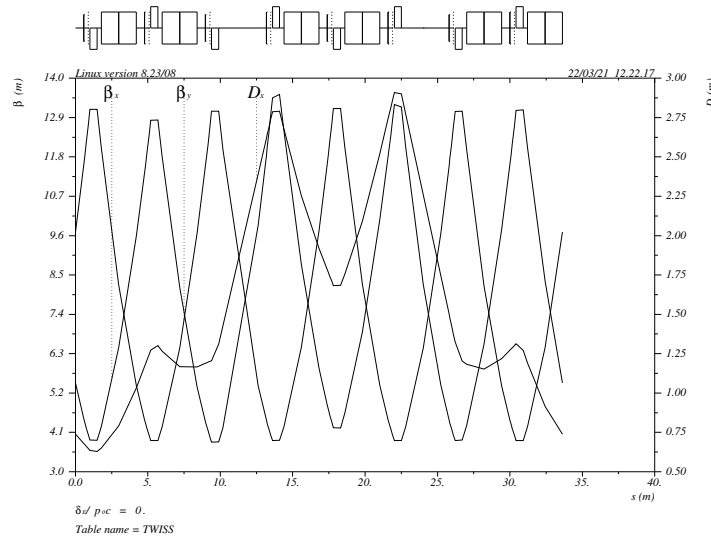
### 14.1.1.1 AGS Booster Parameters

The ring lattice used for these exercises is a simplified version of the AGS Booster, composed of 6 superA super cells. Lattice parameters are given in Tab. 14.2, the table needs to be completed as part of the exercises.

The superA sequence (Tab. 14.4) is taken from the MAD8 [7] model used for Booster operation. The resulting optical functions of superA are displayed in Fig. 14.3.

Question 14.1.1.1-1: complete Tab. 14.2 with the missing numerical values. These parameters will be used throughout the simulations, in particular in setting proper data values in the simulation input data files.

### 14.1.1.2 Cell and Lattice Optics



**Fig. 14.3** Booster superA cell optical functions, as per MAD8 [7] model used for Booster operation. A simplified Booster lattice, comprised of 6 such super cells (parameters given in Tab. 14.2), is used in the present simulations

It is necessary to first check lattice parameters, *viz* simulation input data files, prior to engaging in fancy spin tracking simulations.

The Zgoubi input data file `superA.inc` (Tab. 14.4) has been translated from the MAD model (Sec. 14.1.1.1); this and other Zgoubi input data files used in subsequent exercises can be found in [8].

Question 14.1.1.2-1: run that file, namely (with `[pathTo]/` being the address of the folder that contains the Zgoubi executable on your computer):

```
[pathTo]/zgoubi -in superA.inc
```

This produces the transport matrix of super cell A, and its periodic beam matrix and tunes (logged in `zgoubi.res`).

Explain the role of the FIT procedure.

Question 14.1.1.2-2: check the lattice parameters against Tab. 14.2 data (they are logged at the end of the sequence, bottom of `zgoubi.res` result listing).

Question 14.1.1.2-3: check the periodic optical functions (logged in `zgoubi.res`) against MAD8 results (logged in MAD8 “*print*” file).

**Table 14.2** Booster parameters. The table is to be completed in answer to Question 14.1.1.1-1, accounting for  ${}^3\text{He}^{2+}$  parameters given in Tab. 14.1. The injector DTL (drift tube linac) determines the ion velocity at Booster entrance, namely,  $\beta = v/c = 0.0655$  value

injection $\beta$		0.0655
injection energy	MeV/u	2.0146
injection $G\gamma$	MeV	
injection $B\rho$	T m	
extraction energy	MeV	
extraction $G\gamma$		-10.5
extraction $B\rho$	T m	
<i>Lattice</i>		
Orbit length	m	201.78
tunes, $\nu_x, \nu_y$		4.73, 4.82
chromaticities, $\xi_x, \xi_y$		-4.8, -5.2
momentum compaction $\alpha$		0.043998
transition $\gamma$		*
<i>RF system</i>		
revolution frequency, $f_{\text{rev}}$	MHz	to
RF harmonic, h		4
peak voltage	MV	0.4
frequency, $f_{\text{rf}}$	MHz	to
<i>Spin</i>		
$dG\gamma/d\theta$		

**Table 14.3** Beam parameters

normalized emittances, $\varepsilon_x, \varepsilon_y$ ( $\pi\mu\text{m}$ )	2.5
longitudinal emittance	eV s 0.5
momentum spread	$\pm 3 \cdot 10^{-4}$

Question 14.1.1.2-4: run a TWISS command (replace MATRIX command in superA.inc) to produce the optical functions along the super cell (TWISS logs these in the file zgoubi.TWISS.out). Produce a graph of the latter, compare to Fig. 14.3 from MAD8.

### 14.1.1.3 Spin Optics

Injection energy (see Tab. 14.2) is considered in this question. Tracking is needed in some of the questions, it is performed using the input data file given in Tab. 14.7, in which, compared to Tab. 14.4, OBJET[BORO] and SCALING coefficients have been set to injection rigidity, leaving the optics unchanged (cell optical functions as in Fig. 14.3).

Question 14.1.1.3-1: the spin closed orbit in the ideal ring (six superA cells, planar, no defects) is vertical everywhere.

Tracking shows that this is also the case for off-momentum particles. Is it what's expected? Please explain.

**Table 14.4** The superA cell sequence in this superA.inc data file features the markers `MARKER[LABEL1=superA_S]` and `MARKER[LABEL1=superA_E]` at its extremities. These labeled MARKERS allow that very section of superA.inc, and that section only, to be INCLUDED in further job files - INCLUDE has a similar function to CALL in MAD job files. This input file superA.inc itself INCLUDEs the LA1, LA2, LA3, LA6 subsections (Tabs. 14.5-14.6). Comments are provided for guidance, consulting the Users' Guide is unavoidable

```

superA.inc data file
'OBJET'
1000.000000                                ! Reference rigidity (kG.cm).
5                                           ! An option to generate 11 particles (convenient for MATRIX computation),
.01 .001 .01 .001 0. .001                ! with the sampling specified in this line,
0. 0. 0. 0. 0. 1.                        ! and centered on these values, Y, T, Z, P, s, D.
'FAISCEAU'                                ! Local particle coordinates.

'SCALING'                                ! A sort of "power supplies rack", allows tweaking fields
1 5                                       ! (a field scaling factor), in 7 different families of magnets, here.
BEND
-1
1.
1
MULTIPOL
-1
1.
1
MULTIPOL QHA*                             ! These two families, QHA* and QVA* (* is a wild card)
-1                                       ! control the tunes.
1.0864492                               ! FIT variable #12.
1
MULTIPOL QVA*
-1
1.0657342                               ! FIT variable #16.
1
MULTIPOL DVCA*                             ! Make sure all vertical kickers are zero-ed.
-1
0.
1

'MARKER' superA_S
'INCLUDE'
1
LA1.inc[LA1S:LA1E]
'INCLUDE'
1
LA2.inc[LA2S:LA2E]
'INCLUDE'
1
LA3.inc[LA3S:LA3E]
'INCLUDE'
1
LA2.inc[LA2S:LA2E]
'INCLUDE'
1
LA1.inc[LA1S:LA1E]
'INCLUDE'
1
LA6.inc[LA6S:LA6E]
'INCLUDE'
1
LA1.inc[LA1S:LA1E]
'INCLUDE'
1
LA2.inc[LA2S:LA2E]

'MARKER' superA_E

'FIT'                                ! Find the 4D closed orbit (nil, in the present case of perfect cell)
6 nofinal                            ! and set the (fractional) cell tune values.
1 30 0 [-1,1]
1 31 0 [-10,10]
1 32 0 [-1,1]
1 33 0 [-10,10]
3 12 0 1.
3 16 0 1.
6 1e-15                               ! 6 constraints (and penalty = 1e-15):
3.1 1 2 #End 0. 1. 0                 ! Y_0= Y(end of sequence),
3.1 1 3 #End 0. 1. 0                 ! T_0= T(end of sequence),
3.1 1 4 #End 0. 1. 0                 ! Z_0= Z(end of sequence),
3.1 1 5 #End 0. 1. 0                 ! P_0= P(end of sequence).
0.1 7 7 #End 0.78833333333333 1. 0   ! 6-cell Qy=4.73/6.
0.1 8 8 #End 0.80333333333333 1. 0   ! 6-cell Qy=4.82/6.
'FAISCEAU'                                ! Local particle coordinates.
'MATRIX'
1 11
'END'

```



**Table 14.5** LA1.inc and LA2.inc optical sequences. These are two of the 4 different files (LA1.inc, LA2.inc, LA3.inc, LA6.inc) that make up the LA1.inc-LA2.inc-LA3.inc-LA2.inc-LA1.inc-LA6.inc-LA1.inc-LA2.inc superA.inc super cell sequence of Booster. These LA\*.inc are subject to INCLUDE in superA.inc data file (Tab. 14.4)

'MARKER'	LA1S		'MARKER'	LA2S	
'DRIFT'	DRIF	L057	'DRIFT'	DRIF	L057
57.0400			57.0400		
'MULTIPOL'	DVCA1	VKIC	'MULTIPOL'	DHCA2	HKIC
0 .kicker			0 .kicker		
0.1E-03 10.0000 -0.0E+00 0. 0. 0. 0. 0. 0. 0. 0.			0.1E-03 10.0000 -0.0E+00 0. 0. 0. 0. 0. 0. 0. 0.		
.0 0 1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.			.0 0 1. 0. 0. 0. 0. 0. 0. 0. 0. 0.		
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.			4 .1455 2.2670 -.6395 1.1558 0. 0. 0.		
.0 0 1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.			.0 0 1. 0. 0. 0. 0. 0. 0. 0. 0. 0.		
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.			4 .1455 2.2670 -.6395 1.1558 0. 0. 0.		
1.570796327 0. 0. 0. 0. 0. 0. 0. 0.			0.00 0. 0. 0. 0. 0. 0. 0. 0.		
#2013120 Kick			#2013120 Kick		
1 0. 0. 0.			1 0. 0. 0.		
'DRIFT'	DRIF	L007	'DRIFT'	DRIF	L007
6.9600			6.9600		
'MULTIPOL'	SVA1	SEXT	'MULTIPOL'	SHA2	SEXT
0 .Sext			0 .Sext		
0.1E+02 10. 0. 0. 0.000 0. 0. 0. 0. 0. 0. 0.			0.1E+02 10. 0. 0. 0.000 0. 0. 0. 0. 0. 0. 0.		
0.00 0.00 1.00 0. .0 0. .0 0. .0 0. .0			0.00 0.00 1.00 0. .0 0. .0 0. .0 0. .0		
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723			6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723		
0.00 0.00 1.00 0. .0 0. .0 0. .0 0. .0			0.00 0.00 1.00 0. .0 0. .0 0. .0 0. .0		
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723			6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723		
0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.			0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.		
2. ! cm MultSVA1			2. ! cm MultSHA2		
1 0. 0. 0.			1 0. 0. 0.		
'DRIFT'	DRIF	L014	'DRIFT'	DRIF	L014
13.8050			13.8050		
'DRIFT'	MONI	PUEVA1	'DRIFT'	MONI	PUEHA2
0.0000			0.0000		
'DRIFT'	DRIF	L011	'DRIFT'	DRIF	L012
11.6867			12.1317		
'MULTIPOL'	QVA1	QUAD	'MULTIPOL'	QHA2	QUAD
0 .Quad			0 .Quad		
0.493916E+02 10. 0. -0.5472896982 0. 0. 0. 0. 0. 0. 0.			0.485016E+02 10. 0. 0.5256342158 0. 0. 0. 0. 0. 0. 0.		
0. 0. 6.00 3.00 1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.			0. 0. 6.00 3.00 1.00 0.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.		
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723			6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723		
0. 0. 6.00 3.00 1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.			0. 0. 6.00 3.00 1.00 0.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.		
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723			6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723		
0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.			0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.		
2. ! cm MultQVA1			2. ! cm MultQHA2		
1 0. 0. 0.			1 0. 0. 0.		
'DRIFT'	DRIF	L029	'DRIFT'	DRIF	L031
29.4917			29.9367		
'BEND'	DHA1T	SBEN	'BEND'	DHA2T	SBEN
0 .Bend			0 .Bend		
1.2096161E+02 0.00E+00 7.2121043E-01			1.2096161E+02 0.00E+00 7.2121043E-01		
0.00 0.00 0.00000000			0.00 0.00 0.00000000		
4 .2401 1.8639 -.5572 .3904 0. 0. 0.			4 .2401 1.8639 -.5572 .3904 0. 0. 0.		
0.00 0.00 0.00000000			0.00 0.00 0.00000000		
4 .2401 1.8639 -.5572 .3904 0. 0. 0.			4 .2401 1.8639 -.5572 .3904 0. 0. 0.		
1.00000E+00 cm Bend			1.00000E+00 cm Bend		
3 0. 0. 0.			3 0. 0. 0.		
'BEND'	DHA1Z	SBEN	'BEND'	DHA2Z	SBEN
0 .Bend			0 .Bend		
1.2096161E+02 0.00E+00 7.2121043E-01			1.2096161E+02 0.00E+00 7.2121043E-01		
0.00 0.00 0.00000000			0.00 0.00 0.00000000		
4 .2401 1.8639 -.5572 .3904 0. 0. 0.			4 .2401 1.8639 -.5572 .3904 0. 0. 0.		
0.00 0.00 0.00000000			0.00 0.00 0.00000000		
4 .2401 1.8639 -.5572 .3904 0. 0. 0.			4 .2401 1.8639 -.5572 .3904 0. 0. 0.		
1.00000E+00 cm Bend			1.00000E+00 cm Bend		
3 0. 0. 0.			3 0. 0. 0.		
'MARKER'	LA1E		'MARKER'	LA2E	
'END'			'END'		

Question 14.1.1.3-2: provide the following simulation: track the spin closed orbit over a turn, for an on-momentum particle, and for off-momentum particles at  $dp/p = -10^{-4}$  and  $dp/p = +10^{-4}$ . Provide a graph of the spin components for these 3 particles.

Add the computation of the spin matrix to get the 1-turn spin map and the spin tune.

Explain the value of the 1-turn spin precession angle (as found for instance under SPNPRT in zgoubi.res).

**Table 14.6** LA3.inc and LA6.inc optical sequences. These are two of the 4 different files (LA1.inc, LA2.inc, LA3.inc, LA6.inc) that make up the LA1.inc-LA2.inc-LA3.inc-LA2.inc-LA1.inc-LA6.inc-LA1.inc-LA2.inc superA.inc super cell sequence of Booster. These LA\*.inc are subject to INCLUDE in superA.inc data file (Tab. 14.4)

```

'MARKER'      LA3S
'DRIFT'       DRIF      L057
57.0400
'MULTIPOL'    DVCA3     VKIC
0 .kicker
0.1E-03 10.0000 -0.0E+00 0. 0. 0. 0. 0. 0. 0. 0.
.0 0 1. 0. 0. 0. 0. 0. 0. 0.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
.0 0 1. 0. 0. 0. 0. 0. 0. 0.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
1.570796327 0. 0. 0. 0. 0. 0. 0. 0.
#20|3|20 Kick
1 0. 0. 0.
'DRIFT'       DRIF      L007
6.9600
'MULTIPOL'    SVA3      SEXT
0 .Sext
0.1E+02 10. 0. 0. 0.000 0. 0. 0. 0. 0. 0.
0.00 0.00 1.00 0. .0 0. .0 0. .0
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
0.00 0.00 1.00 0. .0 0. .0 0. .0
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
0. 0. 0. 0. 0. 0. 0. 0. 0.
2. ! cm MultSVA3
1 0. 0. 0.
'DRIFT'       DRIF      L014
13.8050
'DRIFT'       MONI      PUEVA3
0.0000
'DRIFT'       DRIF      L011
11.6867
'MULTIPOL'    QVA3      QUAD
0 .Quad
0.493916E+02 10. 0. -0.5472896982 0. 0. 0. 0. 0. 0. 0.
0. 0. 6.00 3.00 1.00 0.00 0.00 0.00 0.00 0.00 0. 0. 0.
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
0. 0. 6.00 3.00 1.00 0.00 0.00 0.00 0.00 0.00 0. 0. 0.
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
0. 0. 0. 0. 0. 0. 0. 0. 0.
2. ! cm MultQVA3
1 0. 0. 0.
'DRIFT'       DRIF      L029
29.4917
'DRIFT'       DRIF      LDHH
121.0000
'MARKER'      CAVITY
'DRIFT'       DRIF      LDHH
121.0000
'MARKER'      LA3E
'END'

'MARKER'      LA6S
'DRIFT'       DRIF      L057
57.0400
'MULTIPOL'    DHCA6     HKIC
0 .kicker
0.1E-03 10.0000 -0.0E+00 0. 0. 0. 0. 0. 0. 0. 0.
.0 0 1. 0. 0. 0. 0. 0. 0. 0.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
.0 0 1. 0. 0. 0. 0. 0. 0. 0.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
0.00 0. 0. 0. 0. 0. 0. 0. 0.
#20|3|20 Kick
1 0. 0. 0.
'DRIFT'       DRIF      L007
6.9600
'MULTIPOL'    SHA6      SEXT
0 .Sext
0.1E+02 10. 0. 0. 0.000 0. 0. 0. 0. 0. 0.
0.00 0.00 1.00 0. .0 0. .0 0. .0
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
0.00 0.00 1.00 0. .0 0. .0 0. .0
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
0. 0. 0. 0. 0. 0. 0. 0. 0.
2. ! cm MultSHA6
1 0. 0. 0.
'DRIFT'       DRIF      L014
13.8050
'DRIFT'       MONI      PUEHA6
0.0000
'DRIFT'       DRIF      L012
12.1317
'MULTIPOL'    QHA6      QUAD
0 .Quad
0.485016E+02 10. 0. 0.5256342158 0. 0. 0. 0. 0. 0. 0.
0. 0. 6.00 3.00 1.00 0.00 0.00 0.00 0.00 0.00 0. 0. 0.
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
0. 0. 6.00 3.00 1.00 0.00 0.00 0.00 0.00 0.00 0. 0. 0.
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
0. 0. 0. 0. 0. 0. 0. 0. 0.
2. ! cm MultQHA6
1 0. 0. 0.
'DRIFT'       DRIF      L031
29.9367
'DRIFT'       DRIF      LDH
242.0000
'MARKER'      LA6E
'END'

```

Question 14.1.1.3-3: what are the spin tune values, on-momentum and at  $dp/p = \pm 10^{-4}$ ?

#### 14.1.1.4 Depolarizing Resonances

Question 14.1.1.4-1: begin filling in Tabs. 14.8 and 14.9, for the moment with the respective locations ( $G\gamma$  values) of

- imperfection resonances,
- systematic intrinsic resonances,

over the energy range of concern (Tab. 14.2). These data will be used in subsequent questions.

Question 14.1.1.4-2: illustrate the crossing of intrinsic resonances (the strengths of which is  $\varepsilon_y$ -dependent) with two graphs of  $S_y(G\gamma)$ , as follows:

**Table 14.7** Input data file to track 3 helion ions, on-momentum and at  $\delta p/p = \pm 10^{-4}$ , and their spins, along Booster. This data file also defines the [SCALING\_S:SCALING\_E] segment, for INCLUDE purpose - for shortness - in subsequent exercises

```

Track 3 particles and their spin vectors, along superA cell.
! Additional non-void lines here require a comment sign '!'

! Periodic dispersion (from prior lattice parameter computation), for chromatic orbit
! calculation: eta_Y=0.7428, eta'_Y=-0.10475

'OBJET'
0.3074552E3      ! Reference rigidity/kG.cm, for 3He++, at injection beta value 0.0655.
2              ! An option to define initial particle coordinates, one by one.
3 1              ! 3 particles are defined, launched on their closed orbit:
0.742818e-2 -0.10475e-1 0. 0. 0. 1.0001 'p' ! +1e-4 off-momentum (chromatic orbit coord.);
0. 0. 0. 0. 0. 1. 'o' ! on-momentum particle (coordinates are zero, D=1);
-0.742818e-2 0.10475e-1 0. 0. 0. 0.9999 'p' ! -1e-4 off-momentum (chromatic orbit coord.).
1 1 1

'PARTICUL' ! Defining the particle species is necessary, in order for the program to solve
HELION      ! the T-BMT equation.
'SPNTRK'    ! Define initial spin vector. Switch on spin tracking.
4
0.866025403784 0. 0.5      ! Initial S_X=S_Z=1/sqrt(2), S_Y=0.
0.707106781187 0. 0.707106781187 ! Initial S_X=S_Z=1/sqrt(2), S_Y=0.
0.5      0. 0.866025403784 ! Initial S_X=S_Z=1/sqrt(2), S_Y=0.

'MARKER' SCALING_S ! A segment, down to "'MARKER' SCALING_E", defined for further INCLUDE.
'SCALING' ! This sets the magnet power supplies, now that the rigidity has been changed
1 5      ! from 1 Tm to 0.3074552 (injection value). This ensures unchanged optics.
BEND
-1
0.3074552
1
MULTIPOL      ! Default value for any ULTIPOL, unless otherwise specified further down.
-1
0.3074552
1
MULTIPOL QH*      ! These two families, QH* and QV* (* stands for whatever),
-1              ! control the tunes.
0.3074552 * 1.0864492      ! FIT variable #12
1
MULTIPOL QV*
-1
0.3074552 * 1.0657342      ! FIT variable #16
1
MULTIPOL DVCA*      ! Make sure all vertical kickers are zero-ed.
-1
0.
1
'MARKER' SCALING_E      ! End of [SCALING_S:SCALING_E] segment.

'OPTIONS'
1 1
.plt 2

'INCLUDE'
1
6* superA.inc[superA_S:superA_E] ! INCLUDE superA segment from superA.inc file, 6 times.

'FAISCEAU'      ! Particle coordinates, here!
'SPNPRT'      ! Print out spin data, at his location along the optical sequence.

'SYSTEM'      ! System call.
2              ! 2 successive commands, as follows:
gnuplot <./gnuplot_Zplt_sSpin.gnu ! Produce a graph of spin components versus distance.
okular gnuplot_Zplt_sSpin.eps &    ! Display that graph.

'END'

```

- take a few particles evenly distributed on the same vertical invariant  $\varepsilon_y$  (OBJET[KOBJ=8] can be used; or initial coordinates may be generated off-line and then OBJET[KOBJ=2] used). The horizontal invariant  $\varepsilon_x$  can be taken null (explain why);
- accelerate (use CAVITE, placed for simplicity at either end of the optical sequence) from injection  $G\gamma$  to some  $G\gamma \lesssim -18 + \nu_y$  in two different cases:  $\varepsilon_y = 2.5 \mu\text{m}$  and 10 times less.

**Table 14.8** Imperfection resonance locations ( $G\gamma$ ) and strengths ( $\epsilon_n$ ); table to be completed. Note: give resonance strengths normalized to *rms* closed orbit amplitude

$G\gamma$	theory	$ \epsilon_n /y_{co, rms}$	
		tracking stationary	crossing
-5			
-6			
etc.			

**Table 14.9** Systematic intrinsic resonances; table to be completed. Note: give resonance strengths normalized to the square root of the invariant value

$kM \pm \nu_y$	$G\gamma$	$ \epsilon_n /\sqrt{\epsilon_y/\pi}$	
		theory stationary	tracking crossing
$0 - \nu_y$			
$-12 + \nu_y$			
etc.			

Comparing these two graphs, essentially two things are observed: please comment.

Question 14.1.1.4-3: provide a graph showing the span in magnetic field strengths experienced in the vertical quadrupoles by the orbiting particles, depending on their initial betatron phase.

#### 14.1.1.5 Imperfection Resonance Strengths

Introduce a particular series of random vertical misalignments of the 48 quadrupoles around Booster. ERRORS could be used for that, to randomly modify MULTI-POL[KPOS=5] alignment data; however, it is suggested instead to use the misalignment series proposed in Tab. 14.10, for consistency with the detailed solution to this and subsequent questions given in Sec. 14.2 (Secs. 14.2.1.5, 14.2.1.7).

A python script amongst other means can be written to apply this series to the six, now distinct, superA, superB, superC, ..., superF super cells. An example script is shown in Tab. 14.11 that saves a new include file that contains superA through superF as zgoubi\_misaligned.INC. These modified super\*.inc files will be used in place of the six superA.inc in the previous exercises.

Question 14.1.1.5-1: calculate the strengths of the imperfection resonances excited from  $|G\gamma| = 5$  to  $|G\gamma| = 10$ , using the theoretical thin lens model (Eq. 2.29).

**Table 14.10** A possible random vertical misalignments series for the 48 quadrupoles around Booster (this is the data series used in the solutions, Sec. 14.2, Sec. 14.2.1.5). It is now necessary to distinguish the 48 quadrupoles of six, now distinct, super cells: essentially a matter of renaming each quadrupole, for instance with suffixes A1 to A8 (superA cell), B1 to B8 (superB cell), ..., F1 to F8 (superF cell)

# superA:		# superD:	
0.6723850	QVA1	-0.0082470	QVD1
0.7345750	QHA2	0.0172110	QHD2
0.7262320	QVA3	0.0307940	QVD3
0.5887510	QHA4	0.0098080	QHD4
0.4926790	QVA5	-0.0629690	QVD5
0.4026180	QHA6	-0.2462660	QHD6
0.3593190	QVA7	-0.5099270	QVD7
0.3802380	QHA8	-0.7531260	QHD8
# superB:		# superE:	
0.3218880	QVB1	-0.9371040	QVE1
0.2246450	QHB2	-1.1170480	QHE2
0.1578100	QVB3	-1.1903730	QVE3
0.1243620	QHB4	-1.2219470	QHE4
0.0929490	QVB5	-1.2518560	QVE5
0.0385940	QHB6	-1.2414150	QHE6
-0.0469160	QVB7	-1.2676770	QVE7
-0.0152930	QHB8	-1.2564210	QHE8
# superC:		# superF:	
-0.0345980	QVC1	-1.2332150	QVF1
-0.0029100	QHC2	-1.1096020	QHF2
0.0949500	QVC3	-0.9485780	QVF3
0.0761360	QHC4	-0.6493420	QHF4
0.0156440	QVC5	-0.4064320	QVF5
0.0002140	QHC6	-0.1635210	QHF6
-0.0111360	QVC7	0.0906590	QVF7
-0.0080890	QHC8	0.3853270	QHF8

Hint: produce a zgoubi.TWISS.out file using TWISS command, accounting for the now non-zero vertical closed orbit excursion using FIT (preceding TWISS), to evaluate Eq. 2.29.

Complete Tab. 14.8 accordingly (“theory” column).

#### 14.1.1.6 Intrinsic Resonance Strengths

Assume an invariant value equal to the transverse beam emittance (Tab. 14.3). Use your input file without quadrupole misalignments.

Question 14.1.1.6-1: calculate the strengths of the intrinsic resonances, using the theoretical thin lens model (Eq. 2.35).

Hint: use the optical file zgoubi.TWISS.out produced in Sec. 14.1.1.2 to evaluate Eq. 2.35.

Complete Tab. 14.9 accordingly (“theory” column).

#### 14.1.1.7 Spin Motion Through Imperfection Resonances

It is suggested here to use the input file with misaligned quadrupoles of Sec. 14.1.1.5 (this is the case for the solutions provided in Sec. 14.2, Sec. 14.2.1.7).

Question 14.1.1.7-1: stationary case (fixed energy).

[illegible]

Complete Tab. 14.8, “stationary” column, by tracking at various distances near resonance, using Eq. 2.49.

Question 14.1.1.7-2: accelerating through the resonance.

CAVITE[IOPT=3] may be used for acceleration.

Complete Tab. 14.8, “crossing” column, by tracking through every integer resonance from  $G\gamma = -5$  to  $G\gamma = -10$ .

### 14.1.1.8 Spin Motion Through Intrinsic Resonances

Question 14.1.1.8-1: stationary case (fixed energy).

Consider one of the strong resonances found in 14.1.1.5.

For various distances to the resonance (say,  $\Delta_n = N \times |\epsilon_n|$ ,  $N$  is a small integer), produce a graph of the motion of the spin components  $S_{x,s,y}(\text{turn})$ , over a few hundred turns - assume vertical initial spin vector orientation for simplicity.

Produce a Fourier spectrum of the horizontal components (Fourier amplitude versus  $\text{frac}(\nu_{sp})$ ); explain the frequency components observed in the spectrum.

Produce tables, or graphs, which compare theory with the following results:

(i) average values of the vertical component  $S_y$ ; of the horizontal components  $S_x$ ,  $S_s$ .

(ii) dependence of spin component precession frequencies upon distance to the resonance.

Question 14.1.1.8-2: complete Tab. 14.9, “stationary” column, by tracking at various distances near the resonance, using Eq. 2.49.

Question 14.1.1.8-3: accelerating through the resonance.

CAVITE[IOPT=3] may be used for acceleration.

Consider the strong resonance of Question 14.1.1.8-1.

Deduce the resonance strength from the Froissart-Stora formula; check against the theoretical value obtained in Question 14.1.1.6-1 and against the numerical value(s) in Question 14.1.1.8-2.

Question 14.1.1.8-4: complete Tab. 14.9, “crossing” column, by tracking through every systematic intrinsic resonance.

### 14.1.1.9 Spin Motion Through a Weak Resonance

Consider a weak intrinsic resonance (take for instance a random one, or otherwise a systematic with a small enough invariant value), such that  $P_f \approx 0.99 P_i$ .

Question 14.1.1.9-1: compute the turn-by-turn spin motion  $S_y(\text{turn})$  across that resonance, produce  $S_y(\text{turn})$  graph.

Match that spin motion  $S_y(\text{turn})$  with the Fresnel integral model. From this match, obtain the resonant  $G\gamma$  value, vertical tune, and resonance strength.

#### 14.1.1.10 Beam Depolarization Using a Solenoid

Depolarization of the beam while it is still in the accelerator may be a method for calibrations. A longitudinal field can be introduced locally in the lattice for that. Depolarization is obtained by crossing an integer resonance. This is the object of the present simulation.

Question 14.1.1.10-1: introduce a  $L = 1$  meter solenoid, field  $B_s$  (SOLENOID may be used for that, or a 1-D axial field map using BREVOL), in a straight section in the defect free Booster lattice.

Determine  $B_s$  from theory for proper value of the strength  $|\epsilon_n|$  of an appropriate integer resonance. Plot  $P_f(B_s L)$ .

Accelerate (using CAVITE[IOPT=3]) a particle with vertical initial spin through that resonance, check that spin motion ends up in the vicinity of the median plane, asymptotically.

Repeat the simulation using SPINR, a pure spin rotation, in lieu of SOLENOID.

Question 14.1.1.10-2: check depolarization of a beam with Gaussian coordinate distributions in transverse coordinates and momentum spread, with the following parameters:

$$\varepsilon_x = \varepsilon_y = 1 \pi \mu m, \quad \sigma_{\frac{\delta p}{p}} = 10^{-3}$$

#### 14.1.1.11 Introduce a Partial Snake

A partial Siberian snake makes imperfection resonances strong, so causing complete adiabatic spin flip at every imperfection resonance crossing (Chap. 1). The forbidden spin tune band it induces near integer  $G\gamma$  allows for placing the fractional part of the vertical betatron tune inside this gap, so forbidding crossing of intrinsic resonances  $\nu_{sp} = n \pm \nu_y$ .

The goal in this exercise is to assess the efficiency of a partial snake in overcoming integer resonances, and the necessary partial snake strength for preservation of polarization during acceleration.

Question 14.1.1.11-1: create a vertical closed orbit around Booster lattice. This can use ERRORS to generate random vertical misalignment of lattice quadrupoles (this is the case for the solutions provided in Sec. 14.2, Sec. 14.2.1.11; another possibility would be to re-use the input file with misaligned quadrupoles of Sec. 14.1.1.5). Only one ring optics, meaning a single set of quadrupole misalignments is considered in the exercise, as it mostly aims at addressing principles (it is not intended to perform statistics on misalignment samples).

Calculate the strengths of the spin resonances so excited.

Accelerate a particle on the vertical closed orbit, over  $G\gamma : -6.5 \rightarrow -10.5$ , provide a graph of  $S_y(\text{turn})$ .

Check the location and spacing of the resonances, confirm theoretical expectations.



Question 14.1.1.11-2: install in a drift a longitudinal-axis partial snake (use SPINR for pure spin rotation, avoiding any orbit and optics perturbation).

Inhibit ERRORS (ERRORS[ONF=0]) and set the snake angle to  $\phi_{\text{snake}} = 2\pi|Jn|$ , with  $|Jn|$  being the strength of the strongest resonance.

Set the lattice rigidity on  $G\gamma_n = 7$  resonance. Find the spin closed orbit for an on-momentum particle. Plot the spin orbit components around the ring,  $S_{x,y,s}(s)$ . Explain what is observed.

Question 14.1.1.11-3: still in the case of a perfect ring, planar closed orbit, compute the  $G\gamma$  dependence of the spin closed orbit vector, observed at the snake. Produce a graph of the spin orbit components  $S_{x,s,y}(G\gamma)$ .

Produce a graph of the spin tune dependence on  $G\gamma$ ,  $\nu_{\text{sp}}(G\gamma)$ .

Question 14.1.1.11-4: add quadrupole misalignments now (ERRORS[ONF=1]). Thus, spin-wise, both effects now apply, a vertical closed orbit distortion and local spin rotation by a snake (SPINR[ $\phi_{\text{snake}} = 1.224^\circ$ ]).

Accelerate a particle on the vertical closed orbit, over  $G\gamma : -6.5 \rightarrow -10.5$ , provide a graph of  $S_y(\text{turn})$ . Explain what is observed.

Question 14.1.1.11-5: increase the spin precession in the snake in steps, observe how it affects spin rotation, confirm theoretical expectations.

Justify a minimal spin precession by the snake for spin flip upon resonance crossing.

#### 14.1.1.12 Introduce Full Snakes

Imperfection resonance strengths increase in proportion to  $\gamma$ , thus full Siberian snakes are used at high energy, in order to overcome integer resonances (Chap 1). A full Snake maintains the stable spin precession direction unperturbed as long as the spin rotation it causes (its strength) is much larger than the spin rotation due to the resonance driving fields (Chap. 1).

Based on the previous exercises, set lattice and beam input data in the following way:

- Set the vertical beam emittance to large enough a value to cause polarization losses at one or more intrinsic resonances.
- Introduce a random closed orbit distortion sufficiently large that some imperfection resonances create polarization loss during acceleration.
- Set the snake to “full” mode,  $\phi_{\text{snake}} = 180^\circ$ , longitudinal-axis rotation.
- $G\gamma : -6.5 \rightarrow -13.5$  acceleration range will be considered, so to include three systematic intrinsic resonances (as comes out of the studies in Sec. 14.1.1.4).

Question 14.1.1.12-1: compute spin closed orbit and spin tune. Compute spin orientation at opposite azimuth ( $\Delta\theta = 180^\circ$ ) to the snake.

Repeat for  $dp/p = 10^{-4}$  beam momentum offset.

Which parameters depend on the energy and which do not? Check against expectation from theory.

Question 14.1.1.12-2: accelerate over  $G\gamma : -6.5 \rightarrow -13.5$ . Is there any polarization loss?

Question 14.1.1.12-3: now use a horizontal emittance as large as the vertical one. Accelerate over  $G\gamma : -6.5 \rightarrow -13.5$ . Is there any polarization loss?

Question 14.1.1.12-4: add a second snake, at proper location and with proper axis orientation to obtain a spin tune of 0.5 independent of beam energy.

Compute the spin closed orbit around the ring. How is it different from the single snake case?

Compute the spin closed orbit and spin tune for  $dp/p = 10^{-4}$  beam momentum offset. Compare with on-momentum parameters, check against expectation from theory.

Accelerate a beam over  $G\gamma : -6.5 \rightarrow -13.5$ . Is there any polarization loss?

#### 14.1.1.13 High Order Snake Resonances

Set the vertical beam emittance to a value which is large enough to create polarization losses at one or more intrinsic resonances. Introduce a random closed orbit distortion sufficiently large that some imperfection resonances create polarization loss during acceleration. Use the lattice with two snakes. Select the snake axes such that a condition for 2nd order snake resonance with the vertical betatron tune is satisfied.

Question 14.1.1.13-1: accelerate over  $G\gamma : -6.5 \rightarrow -13.5$ , produce a graph of  $\langle S_y(\text{turn}) \rangle$ .

Remove the closed orbit distortion, repeat the acceleration cycle, produce a graph of  $\langle S_y(\text{turn}) \rangle$ .

Compare the results, explain the difference in the polarization loss between the cases with and without the closed orbit distortion.

Question 14.1.1.13-1: select the snake axes orientation such that a condition for 3rd order snake resonance with vertical betatron tune is satisfied.

Accelerate over  $G\gamma : -6.5 \rightarrow -13.5$ , produce a graph of  $\langle S_y(\text{turn}) \rangle$ . Is there polarization loss? Explain the difference in the polarization loss between 2nd and 3rd order resonances.

#### 14.1.1.14 Harmonic Orbit Correction

Using the quadrupole alignment data, perform a harmonic scan for both a strong and a weak imperfection resonance found in Question 14.1.1.5-1. Each corrector magnet is 10 cm long, has an excitation of 9.75 G/A, and a maximum corrector current of 25 A. Power the corrector magnets according to:

$$B_{j,h} = a_h \sin(h\theta_j) + b_h \cos(h\theta_j) \quad (14.1)$$

where  $j$  is the corrector number,  $\theta_j$  is the location in the ring,  $a_h$  and  $b_h$  are the amplitudes for harmonic  $h$ . Fit and provide the resulting  $P_f$  data with a Gaussian to find  $I_{c,0}$  and  $I_{s,0}$ , and the associated  $\sigma_s$  and  $\sigma_c$  values.

At each of the resonances, is it more reasonable to correct the harmonics or exacerbate them?

How accurate must the harmonic corrector currents of the two families be to have a  $< 1\%$  polarization loss at each of the resonances?

Track particles through the two imperfection resonances with your desired corrector current. What is the polarization loss through the two resonances?

#### 14.1.1.15 Preserve Polarization Using Tune-Jump

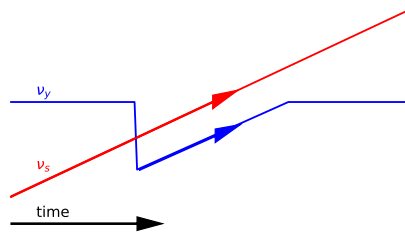
When particles encounter a resonance, if the crossing speed is fast enough, the spin will not be disturbed by the resonance and the polarization will be preserved. The acceleration speed is limited by the RF system and magnet ramping rate, so fast crossing speed needs to come from another method.

The tune jump technique uses dedicated quadrupoles to cause a swift tune change  $\frac{d\nu_y}{d\theta}$ , so increasing the resonance crossing speed according to

$$\alpha = \frac{dG\gamma}{d\theta} \pm \frac{d\nu_y}{d\theta}, \quad (14.2)$$

The  $|G\gamma| = 0 + \nu_y$  resonance is considered in this exercise, to simulate the fast tune jump method as sketched in Fig. 14.4. Booster simulation input data files of Sec. 14.1.1.2 (Tabs. 14.4-14.7) will be used in the following questions, possibly modified as needed.

**Fig. 14.4** Cartoon showing the fast tune jump method where  $\nu_y$  sweeps across  $\nu_s$  quickly, increasing the crossing speed (Eq. 14.2) so minimizing polarization loss as follows from Eq. 2.44



Question 14.1.1.15-1: no tune-jump setting of the quadrupoles in this first question, perfect booster ring optics is considered.

Take an RF cavity voltage of 100 kV ( $30^\circ$  synchronous phase) so an appreciable depolarization can be observed. What is the expected value of  $\frac{dG\gamma}{d\theta}$  with this RF setting?

Take  $B\rho = 2.12998742$  at the start of the tracking, upstream of the resonance (*i.e.*,  $|G\gamma| = 4.59646969$ , 276.7452 MeV kinetic energy). Calculate what turn  $N_{0+\nu_y}$  the resonance is located at.

Consider a particle on  $\varepsilon_y = 1.864 \times 10^{-7} \pi$  m vertical invariant. Assume spin initially vertically aligned. What is the expected asymptotic polarization value,  $P_f$ , upon crossing of this resonance?

Run a numerical simulation of this resonance crossing. Note: SCALING in Tab. 14.12 can be used by simply changing the data under “MULTIPOL QV” so to recover constant tunes  $\nu_x = 4.73$ ,  $\nu_y = 4.82$  all the way (ss in earlier exercises).

Question 14.1.1.15-2: Change the SCALING input parameters so the setpoint of vertical quadrupoles (labeled QV\*) begins to change at turn  $N_{0+\nu_y} - 50$  and continues to  $N_{0+\nu_y} + 50$  with a total change of -5%. Set the change to return the nominal vertical quadrupole field value at turn  $N_{0+\nu_y} + 1050$ . These SCALING settings are detailed in Tab. 14.12.

**Table 14.12** Detailed setting of SCALING for tune jump simulation. Compared to earlier exercises, the change essentially concerns the vertical quadrupole scaling data, namely under “MULTIPOL QV\*”, with SCALING[NT=5], which indicates that a series of 5 different scaling coefficient values follow (the data line following NT) at 5 different timings (shown on the next data line in units of the turn number)

```
'SCALING'                                ! This sets the power supplies for unchanged optics,
1 7                                         ! given a reference rigidity OBJET[BORO=2.12998742e3].
BEND
-1
2.12998742
1
MULTIPOL
-1
2.12998742
1
MULTIPOL QH*                             ! These two families, QH* and QV* (* stands for whatever suffix),
-1                                         ! control the tunes.
2.12998742      *      1.0864492
1
MULTIPOL QV*                             ! QV* quadrupole family field changes to cause tune jump in the
5                                         ! vicinity of the resonance.
2.27000043  2.81833569  2.832133844842106  3.51572336  3.81685555
0 1400 1600 3500 4503
MULTIPOL SH*                             ! Chromaticities may be controlled, via these two sextupoles
-1                                         ! families.
2.12998742
1
MULTIPOL SV*
-1
2.12998742
1
MULTIPOL DVCA*                           ! Make sure all vertical kickers are zero-ed.
-1
2.12998742
1
```

What is the new crossing speed with this tune-jump setting of the vertical quadrupoles?

Calculate the expected  $P_f$  given the resonance strength and the new crossing speed. Does this agree with the value from the simulation?

#### 14.1.1.16 Preserve Polarization Using an AC dipole

An AC dipole can preserve polarization through intrinsic resonances by driving large amplitude vertical betatron oscillations of the entire bunch. This is done with a horizontal magnetic field that oscillates in phase with the vertical motion of the particles. The amplitude of the driven oscillations,  $Y_{coh}$ , follows (Eq. 5.11)

$$Y_{coh} = \frac{B_m L}{4\pi B\rho\delta_m} \beta_y$$

where  $B_m L$  is the integrated field of the AC dipole magnet,  $\beta_y$  is the beta function at the AC dipole and  $B\rho$  is the bunch rigidity.

For simulation purposes, create a copy of superA.inc Booster superperiod (Tab. 14.4), which will be called superA2.inc in the following, in which a new half-cell is used, LA32.inc, a copy of LA3.inc (Tab. 14.6). In LA32.inc, the long drift section now includes a 1 kG, 0.5 m vertical dipole, simulated using MULTIPOL, labeled 'ACD'.

The SCALING command in superA.inc, in addition, uses the option NT=-88 for that 'MULTIPOL ACD' element of the optical sequence, so to define it as an AC dipole. The format for option NT=-88 is:

```
MULTIPOL ACD
-88
0 0.19 0.19 12.2
100 700 1300 700
```

The first line following NT=-88 specifies the AC dipole phase offset, the AC dipole tune Q1 at the start of the sweep, tune Q2 at the end of the sweep, and a scaling factor to be applied to the magnet field. The next line specifies the duration of these steps, namely,  $N_{in}$  =hold duration (field held at zero),  $N_{up}$  =ramp up,  $N_{flat}$  =plateau,  $N_{down}$  =ramp down.

Set Q1=Q2=(1-frac( $v_y$ +0.01))

Use OBJET[KOBJ=2] to create a set of 32 particles with the following coordinates to represent a bunch with RMS size [9]:

$$y = A_n C \cos(j\pi/4) \quad (14.3)$$

and

$$y' = A_n C \sin(j\pi/4) \quad (14.4)$$

for  $j \in \{0, 1, 2, \dots, 7\}$ ,  $C \in \{-0.2671, -0.94, -1.9617, -4.1589\}$  and  $A_n$  an amplitude factor.

Question 14.1.1.16-1: track particles across the  $|G\gamma| = 0 + \nu_y$  with AC dipole field scaling factor set to 0.0. Does  $P_f$  with the 32 particles equal  $P_f$  value obtained with a single particle at the RMS amplitude?

Question 14.1.1.16-2: set the scale factor to 10 G and track particles again. What is the value of  $P_f$ ?

Determine what field is needed to get spin-flip from 100% to -99%.

#### 14.1.1.17 Acceleration of a Polarized 6D Bunch

In this question, a simulation of the acceleration of a 100-particle bunch from  $G\gamma = -5.5$  to  $G\gamma = -13.5$  is set up and run.

The lattice is the same as before, lattice and acceleration parameters are taken from Tab. 14.2. Bunch parameters are taken from Tab. 14.3.

Synchrotron motion in the bunch is accounted for in this simulation: use CAVITE [IOPT=2].

Install selected polarization preservation measures based on the previous questions, in order to maximize polarization transmission through the resonances present in that energy range.

In performing the following, comment on the results obtained in regard to expectations, justify results based on theoretical expectations.

From tracking output data (logged turn-by-turn in zgoubi.fai), produce graphs of

- horizontal and vertical beam excursions,
- transverse and longitudinal phase spaces,
- a few individual spins,
- average bunch polarization,

over the acceleration range.

Produce histograms of the 6 beam coordinates at the top energy. Produce the spin component densities at the top energy. Hint: use HISTO; it is possible to plot from zgoubi.HISTO.out - this requires HISTO[PRINT]. Zpop can be used as well by reading the data from zgoubi.fai.

### 14.1.2 Electron Spin Dynamics, Synchrotron Radiation

The AGS Booster ring is utilized for the exploration of electron polarization. The optical functions of one super cell are displayed in Fig. 14.3, as described in Sec. 14.1.1.1. The electron beam energy is chosen at a relatively high energy of 10 GeV in order to have the beam reach the equilibrium in a short simulation time. The polarization is evaluated after a few damping times. There are three assignments for the electron polarization studies:

- calculation of equilibrium emittances and energy spread,
- study of spin diffusion, and
- exploration of spin matching technique.

**Table 14.13** Left: Zgoubi code producing a table of the optical functions zgoubi.TWISS.out. Right: the scaling\_10GeVVel.inc INCLUDE file, also used in subsequent exercises. The superA.inc INCLUDE file, which defines the Booster ring optical sequence is given in Tab. 14.4, it includes one of the six booster super-cells

<pre> Twiss parameters 'OBJET' 33.3564095089e3          ! 10GeV E_k+M e- or e+. 5 .01 .001 .01 .001 0. .001 0. 0. 0. 0. 0. 1.  'PARTICUL' ! e+, not e-, is stated here, as the optics POSITRON ! was initially set for positive charge (3He2+).  'SPNTRK' 3  'SRLOSS' 0          ! SR loss on or off (1/0). BEND      ! BENDs only are concerned. 1 123456  'INCLUDE' 1 scaling_10GeVVel.inc[SCALING_S:SCALING_E]  'INCLUDE' 1 6* superA.inc[superA_S:superA_E]  'TWISS' 2 1. 1.  'SYSTEM' 1 gnuplot &lt;./ gnuplot_TWISS.gnu  'END' </pre>	<pre> ! scaling_10GeVVel.inc INCLUDE file. 'MARKER' SCALING_S  'SCALING' 1 6 BEND -1 33.3564095089 1 MULTIPOL -1 33.3564095089* 1. 1 MULTIPOL QH* ! Quadrupoles set for Qx=4.73, Qy=4.82. -1 33.3564095089* 1.0864799 ! FIT variable #12. 1 MULTIPOL QV* -1 33.3564095089* 1.0657626 ! FIT variable #16. 1 MULTIPOL SH* -1 33.3564095089 1 MULTIPOL SV* -1 33.3564095089 1  'MARKER' SCALING_E  'END' </pre>
---	--

In this study, we will be mainly using somewhat modified versions of the input files [10] that were introduced in Section 14.1.1.2 and subsequent sections. Note that,

- moving from helions to electrons simulation is essentially a matter of changing the reference rigidity (OBJET[BORO] or MCOBJET[BORO]) and the nature of the particle (PARTICUL[ELECTRON], from PARTICUL[HELION]). To avoid having to change the polarities of the magnetic fields in the AGS Booster that are designed for the positively charged particles and thus simplify the set-up of the simulation, we use PARTICUL[POSITRON];

**Table 14.14** Gnuplot script for plotting the optics and orbit results from zgoubi.TWISS.out

```

set title "Optical functions, from zgoubi.TWISS.out" font "roman,16"

set xlabel "s [m]" font "roman,16"
set ylabel "{/Symbol b}_x, {/Symbol b}_y [m]" font "roman,13"
set y2label "{/Symbol h}_x, {/Symbol h}_y" font "roman,13"

set xtics font "roman,12" nomirror
set x2tics font "roman,12" mirror
set ytics font "roman,12" nomirror
set y2tics font "roman,12" nomirror

set key t c maxrows 1 width 4
set key font "roman, 14" sample 1

set xrange []
set x2range []
set yrange []
set y2range []

plot \
  "zgoubi.TWISS.out" u ($13):($2) axes xly1 w l lt 1 lc rgb "red" lw 1. tit "{/Symbol b}_x" ,\
  "zgoubi.TWISS.out" u ($13):($4) axes xly1 w l lt 1 lc rgb "blue" lw 1. tit "{/Symbol b}_y" ,\
  "zgoubi.TWISS.out" u ($13):($7) axes xly2 w l lt 1 lc rgb "black" lw 1. tit "{/Symbol h}_x" ,\
  "zgoubi.TWISS.out" u ($13):($9) axes xly2 w l lt 2 lc rgb "green" lw 1. tit "{/Symbol h}_y"

set samples 10000
set terminal postscript eps blacktext color enh size 8.3cm,5cm "Times-Roman" 12
set output "gnuplot_TWISS_btxy.eps"
replot
set terminal X11
unset output

pause 2

set title "Orbit, from zgoubi.TWISS.out" font "roman,16"

set ylabel "x, y [m]" font "roman,13"
set y2label "x, y [m]" font "roman,13"

unset x2tics
set xtics font "roman,12" mirror
unset y2tics
set ytics font "roman,12" mirror

plot \
  "zgoubi.TWISS.out" u ($13):($15) axes xly1 w l lt 1 lc rgb "red" tit "x" ,\
  "zgoubi.TWISS.out" u ($13):($17) axes xly1 w l lt 1 lc rgb "blue" tit "y"

set samples 10000
set terminal postscript eps blacktext color enh size 9.3cm,6cm "Times-Roman" 12
set output "gnuplot_TWISS_xy.eps"
replot
set terminal X11
unset output

pause 2

exit

```

- synchrotron radiation from all the dipole magnets is introduced by SRLOSS. With the OPTIONS[WRITE ON] (default option), one can check the expected theoretical synchrotron radiation loss after each BEND in zgoubi.res file. Synchrotron radiation statistics can be logged in zgoubi.res using SRPRNT;
- RF voltage and phase need to be set correctly to compensate the energy loss due to the synchrotron radiation.

#### 14.1.2.1 Electron Equilibrium Emittances and Energy Spread



**Table 14.15** Electron beam parameters. Table to be completed as part of the exercises

Parameter	Units	Value
E	[GeV]	10
$\alpha\gamma$		
Energy loss per turn	[MeV]	
RF voltage	[MV]	
RF phase	[rad]	2.618
Harmonic number		100
RF frequency	[MHz]	
Horizontal damping time	[s]	
Vertical damping time	[s]	
Longitudinal damping time	[s]	
Horizontal emittance	[m-rad]	
Energy spread		

When injected in a storage ring, an electron bunch, if unmatched, will eventually reach equilibrium emittances, under the effect of synchrotron radiation (SR). This is the effect addressed in these preliminary simulation exercises.

The electron equilibrium emittances and damping rates can be calculated analytically, as discussed in Chap. 6, using the Twiss and dispersion parameters of the linear optics design. These damping parameters in a circular accelerator or in a storage ring can also be obtained from a particle tracking simulation.

Question 14.1.2.1-1: run the Zgoubi code in Tab. 14.13 to generate a table of the optical functions of the entire AGS Booster ring, zgoubi.TWISS.out. Generate graphs of the optics and orbit using the Gnuplot script in Tab. 14.14. Apply the expressions given in Chap. 6 to calculate the damped equilibrium emittances, energy spread and damping times of 10 GeV electrons for the optics tabulated in zgoubi.TWISS.out. Fill out Tab. 14.15 with your results.

Question 14.1.2.1-2: run the code in Tab. 14.16. Obtain the energy loss from zgoubi.res for the electron beam energy at 10 GeV and compare it with the analytical calculation in Tab. 14.15. Use the energy loss to obtain the RF voltage for the RF in Tab. 14.15. Compare your result with the CAVITE element setting in Tab. 14.17.

Question 14.1.2.1-3: examine the initial beam setup in Tab. 14.17. Check whether the initial beam distribution is matched transversely by comparing the beam setup parameters with the periodic Twiss functions in zgoubi.TWISS.out you obtained earlier. Run the code in Tab. 14.17 with 100 particles up to 1000 turns with synchrotron radiation enabled. Use the Gnuplot script in Tab. 14.18 (or a code of your own) to obtain the rms vertical beam size  $\sigma_y$  as a function of the turn number from the Zgoubi output zgoubi.fai file. Using the Gnuplot script in Tab. 14.19, calculate the vertical rms emittance from the vertical rms beam size and optics parameters, plot evolution of the vertical emittance, and extract the vertical damping time by fitting the data to an exponential. Compare the obtained vertical damping time to the theoretical value in Tab. 14.15.

**Table 14.17** Zgoubi code for simulating the beam dynamics with synchrotron radiation**Table 14.16** Zgoubi code for calculating the energy loss per turn. Recall: the scaling\_10GeVVel.inc file is given in Tab. 14.13

```

Energy loss per turn.
'OBJET'
33.3564095089e3 10GeV E_k+M electron.
1
10000 1 1 1 1 1
0.0 0.0 0.0 0.0 0.00 0.0
0.000 0.000 0.000 0.000 0.00 1.

'PARTICUL'
POSITRON

'SPNTRK'
3

'SRLOSS'
1
BEND
1 123456

'FAISTORE'
zgoubi.fai
1

'INCLUDE'
1
scaling_10GeVVel.inc[SCALING_S:SCALING_E]

'INCLUDE'
1
6* superA.inc[superA_S:superA_E]

'FAISCEAU'
'SRPRNT'

'END'

```

```

Beam dynamics with synchrotron radiation.
'OBJET'
33.3564095089e3 10GeV E_k+M electron.
8
0 100 0
0.0006 -0.00065 0. 0. 0.00 1.00315000
0.98253425 5.4881122 0
-1.5447545 9.6995264 1e-5
1. 1. 0.
'PARTICUL'
POSITRON
'SPNTRK'
3

'SRLOSS'
1
BEND
1 123456

'FAISTORE'
zgoubi.fai
10 ! Log to zgoubi.fai every 10 turn.

'OPTIONS'
1 1 ! Inhibit writes to zgoubi.res:
WRITE OFF ! this saves on CPU time.

'INCLUDE'
1
scaling_10GeVVel.inc[SCALING_S:SCALING_E]
'INCLUDE'
1
6* superA.inc[superA_S:superA_E]

'CAVITE'
2
201.780049 100 ! orbit length, h.
122345.25e3 2.61799387799 ! volts, phi_s rad.

'REBELOTE'
999 0.1 99 ! 999 additional passes.

'OPTIONS'
0 1
WRITE ON ! Re-establish writes to zgoubi.res.
'FAISCEAU' ! Print out particle coordinates.
'SPNPRT' ! Print out spin coordinates.
'SRPRNT' ! Print out SR data.
'END'

```

**Table 14.18** Gnuplot script for calculating the rms beam parameters

```

set fit logfile '/dev/null'

fname="zgoubi.fai"

f(x)=ave

set print "ave_sig.txt"

do for [z=10:1000:10] {

    FIT_STDFIT=0
    fit f(x) fname u 38:((($38==z)?($1==1?$9:1/0):1/0) via ave
    d_ave=ave
    d_sig=FIT_STDFIT

    FIT_STDFIT=0
    fit f(x) fname u 38:((($38==z)?($1==1?$10:1/0):1/0) via ave
    x_ave=ave
    x_sig=FIT_STDFIT

    FIT_STDFIT=0
    fit f(x) fname u 38:((($38==z)?($1==1?$11:1/0):1/0) via ave
    xp_ave=ave
    xp_sig=FIT_STDFIT

    FIT_STDFIT=0
    fit f(x) fname u 38:((($38==z)?($1==1?$12:1/0):1/0) via ave
    y_ave=ave
    y_sig=FIT_STDFIT

    FIT_STDFIT=0
    fit f(x) fname u 38:((($38==z)?($1==1?$13:1/0):1/0) via ave
    yp_ave=ave
    yp_sig=FIT_STDFIT

    print z,d_ave,d_sig,x_ave,x_sig,xp_ave,xp_sig,y_ave,y_sig,yp_ave,yp_sig

}

unset print

exit

```

**Table 14.19** Gnuplot script for plotting and fitting the vertical emittance data

```

set term postscript eps enhanced color size 9.3cm,6cm "Times-Roman" 12
set output "damping_time_y.eps"
set grid
set size 1.0,1.0
set xlabel "Turns [x10^{3}]"
set ylabel "{/Symbol e}_{y} [{/Symbol m}m]"
unset key

A = 1
B = 0.175
C = 0

f(x)=A*exp(-x/B)

fit f(x) "/ave_sig.txt" u ($1/1000):($1>0?($9**2*0.01**2/9.6995264*1e6):1/0) via A,B

plot "/ave_sig.txt" u ($1/1000):($1>0?($9**2*0.01**2/9.6995264*1e6):1/0) w l lc rgb 'red' lw 1.5,\
f(x) lc rgb 'blue' lw 1.5

exit

```

### 14.1.2.2 Spin Diffusion Studies

Synchrotron radiation causes spin-flip through the Sokolov-Ternov effect, and spin diffusion. These effects determine the evolution of polarization, and polarization life time (Secs. 6.3, 6.5). The evaluation of spin diffusion in general requires numerical simulations, which allow deriving the polarization life-time.

Spin diffusion, *i.e.* depolarization, of polarized electrons can be suppressed and mitigated through the design of an accelerator and proper correction schemes. Alternatively, spin diffusion can also be enhanced if the design or the correction schemes are not done properly. In this exercise, we explore how and how fast spin diffusion happens when  $\mathbf{n}_0$  changes. We will also demonstrate how spin diffusion can be suppressed by adjustment of the magnet layout and beam optics.

**Table 14.20** Zgoubi code for obtaining  $\mathbf{n}_0$  at the start point of the ideal lattice

```
$\vec{v}[n]_0$ at the start point of the ideal lattice.
'OBJET'
33.3564095089e3 10GeV E_k+M electron.
1
3 1 1 1 1 1
0. 0. 0. 0. 0.00 0.
0.00 0.00 0.00 0.00 0.00 1.

'PARTICUL'
POSITRON

'SPNTRK'
4
1. 0. 0.
0. 1. 0.
0. 0. 1.

'SRLOSS'
0
BEND
1 123456

'OPTIONS'
1 1
WRITE OFF

'INCLUDE'
1
scaling_10GeV1.inc[SCALING_S:SCALING_E]

'INCLUDE'
1
6* superA.inc[superA_S:superA_E]

'OPTIONS'
1 1
WRITE ON

'SPNPRT' MATRIX

'END'
```

**Table 14.21** Zgoubi code for tracking electron polarization with synchrotron radiation enabled

```
Electron polarization with synchrotron radiation
'OBJET'
33.3564095089e3 10GeV E_k+M electron.
1
100 1 1 1 1 1
0. 0. 0. 0. 0.00 0.
0.00 0.00 0.00 0.00 0.00 1.

'PARTICUL'
POSITRON

'SPNTRK'
3
! All initial spins vertical.

'SRLOSS'
1
BEND
1 123456

'FAISTORE'
zgoubi.fai
10
! Log to zgoubi.fai every 10 turn.

'OPTIONS'
1 1
WRITE OFF

'INCLUDE'
1
scaling_10GeV1.inc[SCALING_S:SCALING_E]
'INCLUDE'
1
6* superA.inc[superA_S:superA_E]

'OPTIONS'
0 1
WRITE ON

'CAVITE'
2
201.780049 100 ! orbit length, h.
122345.25e3 2.61799387799 ! volts, phi_s rad.

'REBELOTE'
999 0.1 99

'END'
```

### 14.1.2.3 Spin Diffusion

Question 14.1.2.2-1: run the code in Tab. 14.20 to obtain  $\mathbf{n}_0$  at the start point of the ideal lattice. This is done by tracking three electrons with spins aligning along three orthogonal directions with synchrotron radiation disabled. Find the spin transfer matrix and precession axis (i.e.  $\mathbf{n}_0$ ) in zgoubi.res file.

**Table 14.22** Gnuplot script for calculating the polarization

```
set fit logfile '/dev/null'

fname="zgoubi.fai"

f(x)=ave

set print "ave_pol_align.txt"

do for [z=1:1:1] {

    FIT_STDFIT=0
    fit f(x) fname u 38:($38==z?($1==1?$20:1/0):1/0) via ave
    px_ave=ave
    px_sig=FIT_STDFIT

    FIT_STDFIT=0
    fit f(x) fname u 38:($38==z?($1==1?$21:1/0):1/0) via ave
    py_ave=ave
    py_sig=FIT_STDFIT

    FIT_STDFIT=0
    fit f(x) fname u 38:($38==z?($1==1?$22:1/0):1/0) via ave
    pz_ave=ave
    pz_sig=FIT_STDFIT

    print z,px_ave,px_sig,py_ave,py_sig,pz_ave,pz_sig

}

do for [z=10:1000:10] {

    FIT_STDFIT=0
    fit f(x) fname u 38:($38==z?($1==1?$20:1/0):1/0) via ave
    px_ave=ave
    px_sig=FIT_STDFIT

    FIT_STDFIT=0
    fit f(x) fname u 38:($38==z?($1==1?$21:1/0):1/0) via ave
    py_ave=ave
    py_sig=FIT_STDFIT

    FIT_STDFIT=0
    fit f(x) fname u 38:($38==z?($1==1?$22:1/0):1/0) via ave
    pz_ave=ave
    pz_sig=FIT_STDFIT

    print z,px_ave,px_sig,py_ave,py_sig,pz_ave,pz_sig

}

unset print

exit
```

**Table 14.23** Gnuplot script for plotting the polarization

```
set term postscript eps enhanced \
    color size 9.3cm,6cm "Times-Roman" 12
set output "pol_align.eps"
set grid
set size 1.0,1.0
set xlabel "Turns [x10^3]"
set ylabel "P_{z}"
set xtics 0.2
set ytics 0.0005
unset key

A = 1
B = 1000000

f(x)=A*exp(-x/B)

fit f(x) "./ave_pol_align.txt" u ($1/1000):($6) via A,B

plot [][0.999:1.001] "./ave_pol_align.txt" \
    u ($1/1000):($6) w l lc rgb 'red' lw 1.5 , \
    f(x) lc rgb 'blue' lw 1.5

exit
```

Question 14.1.2.2-2: set the initial spins of 100 electrons to be aligned with the  $\mathbf{n}_0$  axis of the ideal lattice. Enable synchrotron radiation. Track the particles by running the code in Tab. 14.21. Calculate the average spin, or polarization, of the electrons using the script in Tab. 14.22. Plot the polarization as a function of the turn number and extract the spin diffusion rate by fitting the data. Refer to Tab. 14.23.

Question 14.1.2.2-3: offset the first “QVA1” quadrupole from the start of the lattice by 0.1 cm as shown in Tab. 14.24 and find the resulting closed orbit using the “FIT2” procedure give in Tab. 14.25. Make sure that SRLOSS is disabled. The output is saved in the zgoubi.FIT.out.dat. Obtain the  $\mathbf{n}_0$  axis by specifying the closed orbit offset at the start of the lattice. Launch 100 electrons along the perturbed closed orbit following the initial beam setup provided in Tab. 14.26 and track them for 1000 turns with synchrotron radiation enabled. Find the average polarization as a function of the turn number, plot it, and extract the spin diffusion rate as in Question 14.1.2.2-2.

**Table 14.24** Setup of the “QVA1” quadrupole with 1 mm vertical offset

```
'MULTIPOL' QVA1      QUAD
0 .Quad
0.493916E+02 10.0000  0.0000 -0.5472896982 0. 0. 0. 0. 0. 0.
0. 0. 6.00  3.00 1.00 0.00 0.00 0.00 0.00 0.00 0. 0. 0.
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
0. 0. 6.00  3.00 1.00 0.00 0.00 0.00 0.00 0.00 0. 0. 0.
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
2.00 ! cm MultQVA1
4 0. 0. 0. 0.1 0 ! KPOS=4 allows specifying ZS=0.1cm.
```

**Table 14.25** Setup of the “FIT2” procedure for finding the closed orbit

```
'FIT2'
4 noSYSout ! final (NOT nofinal) is mandatory: causes store of actual c.o. in zgoubi.SVD.out
1 30 0 [-5.9,5.9] ! Vary Y0
1 31 0 [-9.9,9.9] ! Vary T0
1 32 0 [-5.9,5.9] ! Vary Z0
1 33 0 [-9.9,9.9] ! Vary P0
4 le-10
3.1 1 2 #End 0. 1. 0 ! Yfinal = Y0
3.1 1 3 #End 0. 1. 0 ! Tfinal = T0
3.1 1 4 #End 0. 1. 0 ! Zfinal = Z0
3.1 1 5 #End 0. 1. 0 ! Pfinal = P0
```

**Table 14.26** Setup of the “OBJET” element for launching 100 electrons along the perturbed closed orbit

```
'OBJET'
33.3564095089e3 10GeV E_k+M electron.
1
100 1 1 1 1 1
0.E+00 0.E+00 0.E+00 0.E+00 0.00 0.E+00
-5.78398841E-06 -5.02369232E-06 -2.40168827E-01 -5.80016582E-01 0.00 1.
```

Question 14.1.2.2-4: repeat the calculation in Question 14.1.2.2-3 and obtain the  $\mathbf{n}_0$  vectors and the spin diffusion rates with the quadrupole “QVA1” offset by 0.2 and 0.5 cm. Explain your results.

### 14.1.2.4 Suppression of Spin Diffusion

We will next illustrate how the spin diffusion can be mitigated by changing the polarities of magnetic fields.

Question 14.1.2.2-5: compare the magnet layouts in Tabs. 14.27 and 14.28 and describe spin rotations in the two layouts assuming vertical initial spin direction.

**Table 14.27** Zgoubi file of a spin rotator lattice where the spin is not longitudinally matched

```
Spin rotator, version 1
'OBJET'
33.3564095089e3 10GeV E_k+M electron.
1
1 1 1 1 1 1
0.E+00 0.E+00 0.E+00 0.E+00 0.00 0.E+00
0. 0. 0. 0. 1.

'PARTICUL'
POSITRON

'SPNTRK'
4.1
0. 0. 1.

'SCALING'
1 1
BEND
-1
1
1

'DRIFT' DRIF
10.
'SOLENOID' SOLE
2 .sole
1000.0 2.0 52.3354354
25 25
1. cm
1 0. 0. 0.
'DRIFT' DRIF
10.
'BEND' DHA1T
2 .Bend
100. 0.0E+00 23.08831973 ! 69.2 mrad bend
0.00 0.00 0.00
4 .2401 1.8639 -.5572 .3904 0. 0. 0.
0.00 0.00 0.00
4 .2401 1.8639 -.5572 .3904 0. 0. 0.
1.0000E+00 cm Bend
3 0. 0. 0.
'DRIFT' DRIF
10.
'DRIFT' DRIF
10.
'BEND' DHA1T
2 .Bend
100. 0.0E+00 23.08831973
0.00 0.00 0.00
4 .2401 1.8639 -.5572 .3904 0. 0. 0.
0.00 0.00 0.00
4 .2401 1.8639 -.5572 .3904 0. 0. 0.
1.0000E+00 cm Bend
3 0. 0. 0.
'DRIFT' DRIF
10.
'SOLENOID' SOLE
2 .sole
1000.0 2.0 52.3354354
25 25
1. cm
1 0. 0. 0.
'DRIFT' DRIFEND
10.

'END'
```

**Table 14.28** Zgoubi file of a spin rotator lattice where the spin is longitudinally matched

```
Spin rotator, version 2
'OBJET'
33.3564095089e3 10GeV E_k+M electron.
1
1 1 1 1 1 1
0.E+00 0.E+00 0.E+00 0.E+00 0.00 0.E+00
0. 0. 0. 0. 1.

'PARTICUL'
POSITRON

'SPNTRK'
4.1
0. 0. 1.

'SCALING'
1 1
BEND
-1
1
1

'DRIFT' DRIF
10.
'SOLENOID' SOLE
2 .sole
1000.0 2.0 52.3354354
25 25
1. cm
1 0. 0. 0.
'DRIFT' DRIF
10.
'BEND' DHA1T
2 .Bend
100. 0.0E+00 23.08831973
0.00 0.00 0.00
4 .2401 1.8639 -.5572 .3904 0. 0. 0.
0.00 0.00 0.00
4 .2401 1.8639 -.5572 .3904 0. 0. 0.
1.0000E+00 cm Bend
3 0. 0. 0.
'DRIFT' DRIF
10.
'DRIFT' DRIF
10.
'BEND' DHA1T
2 .Bend
100. 0.0E+00 -23.08831973
0.00 0.00 0.00
4 .2401 1.8639 -.5572 .3904 0. 0. 0.
0.00 0.00 0.00
4 .2401 1.8639 -.5572 .3904 0. 0. 0.
1.0000E+00 cm Bend
3 0. 0. 0.
'DRIFT' DRIF
10.
'SOLENOID' SOLE
2 .sole
1000.0 2.0 -52.3354354
25 25
1. cm
1 0. 0. 0.
'DRIFT' DRIFEND
10.

'END'
```

Question 14.1.2.2-6: run the codes in Tabs. 14.27 and 14.28 and plot the magnetic fields and three spin components along the trajectory using the zgoubi.plt output file (particle and field data logging to zgoubi.plt results from  $IL=2$  in all optical elements; the data format of zgoubi.plt is detailed in [1, Sec. 8.3]). See the plotting examples in Tabs. 14.29 and 14.30 and use analogous files for the second rotator design. Compare the results with your description in the previous exercise.

**Table 14.29** Gnuplot script for plotting the field components

```
set term postscript eps enhanced color size 9.3cm,6cm "Times-Roman" 12
set output "spin_rotator_fields.eps"
set grid
set size 1.0,1.0
set xlabel "s [m]"
set ylabel "B (T)"
set xtics 5
set ytics 2
set key bottom left

plot [-1:][0.1:] \
"/zgoubi.plt" u ($14/100):($23/10) w l lc rgb 'red' lw 1.5 title "B_{x}", \
"/zgoubi.plt" u ($14/100):($24/10) w l lc rgb 'green' lw 1.5 title "B_{y}", \
"/zgoubi.plt" u ($14/100):($25/10) w l lc rgb 'blue' lw 1.5 title "B_{z}"

exit
```

**Table 14.30** Gnuplot script for plotting the spin components

```
set term postscript eps enhanced color size 9.3cm,6cm "Times-Roman" 12
set output "spin_rotator_spin.eps"
set grid
set size 1.0,1.0
set xlabel "s [m]"
set ylabel "S"
set xtics 5
set ytics 0.5
set key bottom left

plot [-1:][-1.1:1.1] \
"/zgoubi.plt" u ($14/100):($33) w l lc rgb 'red' lw 1.5 title "S_{x}", \
"/zgoubi.plt" u ($14/100):($34) w l lc rgb 'green' lw 1.5 title "S_{y}", \
"/zgoubi.plt" u ($14/100):($35) w l lc rgb 'blue' lw 1.5 title "S_{z}"

exit
```

Question 14.1.2.2-7: track the spins of particles with different relative momentum offsets (-0.04, -0.03, -0.02, -0.01, 0, 0.01, 0.02, 0.03, 0.04) in the two spin rotator schematics using the code in Tab. 14.31 for the first design version and an analogous code for the second one. Explore how the final spin depends on the momentum deviation by plotting the final spin versus the momentum offset in the two cases using the script in Tab. 14.32. Discuss which case you expect to have a lower spin diffusion rate and why. Note that this is the first order spin matching in the longitudinal direction.

### 14.1.2.5 Spin Matching



**Table 14.31** Zgoubi code for studying the momentum dependence of the spin rotation using the unmatched lattice

```

Momentum dependence of spin rotation - unmatched lattice
'OBJET'
33.3564095089e3 10GeV E_k+M electron.
1
1 1 1 1 1 9
0.E+00 0.E+00 0.E+00 0.E+00 0.00 0.010000E+00
0. 0. 0. 0. 0. 1. 'o'

'PARTICUL'
POSITRON

'SPNTRK'
4.1
0. 0. 1.

'FAISTORE' ! Hint: use zgoubi_matched.fai for 2nd case.
zgoubi_unmatched.fai DRIFEND
1

'SCALING'
1 1
BEND
-1
1
1

'DRIFT' DRIF
10.
'SOLENOID' SOLE
0 .sole
1000.0 2.0 52.3354354
25 25
1. cm
1 0. 0. 0.
'DRIFT' DRIF
10.
'BEND' DHAIT
0 .Bend
100. 0.0E+00 23.08831973
0.00 0.00 0.00
4 .2401 1.8639 -.5572 .3904 0. 0. 0.
0.00 0.00 0.00
4 .2401 1.8639 -.5572 .3904 0. 0. 0.
1.0000E+00 cm Bend
3 0. 0. 0.
'DRIFT' DRIF
10.
'DRIFT' DRIF
10.
'BEND' DHAIT
0 .Bend
100. 0.0E+00 23.08831973
0.00 0.00 0.00
4 .2401 1.8639 -.5572 .3904 0. 0. 0.
0.00 0.00 0.00
4 .2401 1.8639 -.5572 .3904 0. 0. 0.
1.0000E+00 cm Bend
3 0. 0. 0.
'DRIFT' DRIF
10.
'SOLENOID' SOLE
0 .sole
1000.0 2.0 52.3354354
25 25
1. cm
1 0. 0. 0.
'DRIFT' DRIFEND
10.

'FAISCEAU'

'END'

```

**Table 14.32** Gnuplot script for plotting the final vertical spin component as a function of the particle's momentum offset for the two rotator schemes

```

set term postscript eps enhanced color size 9.3cm,6cm "Times-Roman" 12
set output "final_spin.eps"
set grid
set size 1.0,1.0
set xlabel "{/Symbol D}p/p"
set ylabel "S_{z}"
set xtics 0.02
set ytics 0.002
set key bottom left

plot [] \
"< sort -nk2 zgoubi_1.fai" u 2:22 w l lc rgb 'red' lw 1.5 title "Scheme 1", \
"< sort -nk2 zgoubi_2.fai" u 2:22 w l lc rgb 'blue' lw 1.5 title "Scheme 2"

exit

```

This section studies the electron spin dynamics at 5 GeV in the AGS Booster in the presence of a solenoidal snake. We consider the cases of spin-matched and spin-mismatched snake configurations. The spin-matched snake lattice is given by the include file listed in Tab. 14.33. The snake consists of two solenoids with six quadrupoles between them. The quadrupoles are used to compensate betatron coupling from the solenoids and to satisfy the spin matching conditions.

Question 14.1.2.5-1: use the snake include file in Tab. 14.33 to insert the snake at the end of the AGS Booster lattice as shown in Tab. 14.34. Examine the resulting periodic optics using the Gnuplot script in Tab. 14.14.

Question 14.1.2.5-2: obtain the  $\mathbf{n}_0$  axis at the start point of the lattice by running the code in Tab. 14.35 and examining its output “zgoubi.res” file.

Question 14.1.2.5-3: determine the spin diffusion rate of this lattice. Track 100 electrons for  $10^4$  turns with synchrotron radiation enabled. Start the electrons on the design trajectory with their initial spins aligned with the  $\mathbf{n}_0$  axis. Refer to Tab. 14.36 for the corresponding Zgoubi code. Calculate the polarization components using a Gnuplot script similar to that in Tab. 14.22 and plot the total transverse polarization as a function of the turn number following the example of Tab. 14.23.

Question 14.1.2.5-4: reverse the polarity of all quadrupoles (“HQ1” through “HQ6”) between the snake solenoids in Tab. 14.33. This change keeps betatron coupling compensated but results in violation of the spin matching conditions. Complete tracking through the AGS Booster with the modified snake lattice and analyze the results as in Question 14.1.2.5-3. Compare the spin diffusion rates obtained in the spin matched and unmatched cases.

**Table 14.33** Contents of the “snake\_matched.inc” include file providing the lattice of a spin-matched solenoidal snake

```

'MARKER'    snake_sol_s
'DRIFT'     DRIF      LDH1SN
11.0
'SOLENOID'  SOLE      SN_SOL
0 .sole
40.0 2.0 39.26987500004383
40. 40.
0.1 cm
1 0.0 0.0 0.0
'DRIFT'     DRIF      LSN1
19.5
'MULTIPOL'  HQ1      QUAD
0 .Quad
1.0 10.0 0.0 -9.012691897 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 6.00 3.00 1.00 0.00 0.00 0.00 0.00 0.00 0.0
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
0.0 0.0 6.00 3.00 1.00 0.00 0.00 0.00 0.00 0.00 0.0
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
1. cm
1 0.0 0.0 0.0
'DRIFT'     DRIF      LSN2
19.0
'MULTIPOL'  HQ2      QUAD
0 .Quad
1.0 10.0 0.0 503.4519374 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 6.00 3.00 1.00 0.00 0.00 0.00 0.00 0.00 0.0
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
0.0 0.0 6.00 3.00 1.00 0.00 0.00 0.00 0.00 0.00 0.0
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
1. cm
1 0.0 0.0 0.0
'DRIFT'     DRIF      LSN2
19.0
'MULTIPOL'  HQ3      QUAD
0 .Quad
1.0 10.0 0.0 -535.3938472 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 6.00 3.00 1.00 0.00 0.00 0.00 0.00 0.00 0.0
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
0.0 0.0 6.00 3.00 1.00 0.00 0.00 0.00 0.00 0.00 0.0
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
1. cm
1 0.0 0.0 0.0
'DRIFT'     DRIF      LSN2
19.0
'MULTIPOL'  HQ4      QUAD
0 .Quad
1.0 10.0 0.0 775.4968481 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 6.00 3.00 1.00 0.00 0.00 0.00 0.00 0.00 0.0
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
0.0 0.0 6.00 3.00 1.00 0.00 0.00 0.00 0.00 0.00 0.0
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
1. cm
1 0.0 0.0 0.0
'DRIFT'     DRIF      LSN2
19.0
'MULTIPOL'  HQ5      QUAD
0 .Quad
1.0 10.0 0.0 -437.5504952 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 6.00 3.00 1.00 0.00 0.00 0.00 0.00 0.00 0.0
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
0.0 0.0 6.00 3.00 1.00 0.00 0.00 0.00 0.00 0.00 0.0
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
1. cm
1 0.0 0.0 0.0
'DRIFT'     DRIF      LSN2
19.0
'MULTIPOL'  HQ6      QUAD
0 .Quad
1.0 10.0 0.0 406.0280703 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 6.00 3.00 1.00 0.00 0.00 0.00 0.00 0.00 0.0
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
0.0 0.0 6.00 3.00 1.00 0.00 0.00 0.00 0.00 0.00 0.0
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
1. cm
1 0.0 0.0 0.0
'DRIFT'     DRIF      LSN1
19.5
'SOLENOID'  SOLE      SN_SOL
0 .sole
40.0 2.0 39.26987500004383
40. 40.
0.1 cm
1 0.0 0.0 0.0
'DRIFT'     DRIF      LDH4SN
11.0
'MARKER'    snake_sol_e

```

**Table 14.34** Left: Zgoubi file of the AGS Booster lattice with the spin-matched snake for Twiss calculation. Right: the scaling\_5GeVl.inc INCLUDE file, also used in subsequent exercises

```

AGS Booster lattice with the spin-matched snake.
'OBJET'
16.67820475445e3 ! reference rigidity -> p = 5 GeV/c
5
.001 .0001 .001 .0001 0. .0001
0. 0. 0. 0. 0. 1.

'PARTICUL'
0.51099892 1.60217653e-19 1.15965218076e-3 0. 0.
'SRLOSS'
0 ! .srloss
BEND
1 123456
'SPWTRK'
3

'INCLUDE'
1
scaling_5GeVl.inc[SCALING_S:SCALING_E]

'INCLUDE'
1
6* superA.inc[superA_S:superA_E]

'INCLUDE'
1
snake_matched.inc[snake_sol_s:snake_sol_e]

'TWISS'
2 1. 1.

'END'
! scaling_5GeVl.inc INCLUDE file.
'MARKER' SCALING_S

'SCALING'
1 7
BEND
-1
16.67820475445
1
MULTIPOL
-1
16.67820475445* 1.
1
SOLENOID
-1
16.67820475445
1
MULTIPOL QH* ! Quadrupoles set for Qx=4.73, Qy=4.82.
-1
16.67820475445* 1.0864799 ! FIT variable #12.
1
MULTIPOL QV*
-1
16.67820475445* 1.0657626 ! FIT variable #16.
1
MULTIPOL SH*
-1
16.67820475445
1
MULTIPOL SV*
-1
16.67820475445
1
'MARKER' SCALING_E

'END'

```

**Table 14.35** Zgoubi file of the AGS Booster lattice with the spin-matched snake for  $n_0$  calculation

```

AGS Booster lattice with spin-matched snake for n_0 calculation
'OBJET'
16.67820475445e3      ! reference rigidity -> p = 5 GeV/c
1
3 1 1 1 1 1
0.E+00 0.E+00 0.E+00 0.E+00 0.00 0.E+00
0.00E+00 0.00E+00 0.00E+00 0.00E+00 0.00 1.

'PARTICUL'
0.51099892 1.60217653e-19 1.15965218076e-3 0. 0.

'SRLOSS'
0 ! .srloss
BEND
1 123456

'SPNTRK'
4
1. 0. 0.
0. 1. 0.
0. 0. 1.

'INCLUDE'
1
scaling_5GeV1.inc[SCALING_S:SCALING_E]

'INCLUDE'
1
6* superA.inc[superA_S:superA_E]

'INCLUDE'
1
snake_mismatched.inc[snake_sol_s:snake_sol_e]

'SPNPRT'  MATRIX

'END'

```

**Table 14.36** Zgoubi file for tracking 100 electrons for  $10^4$  turns through the AGS Booster lattice with the spin-matched snake

```

Tracking 100 electrons for 10^4 turns through Booster
'OBJET'
16.67820475445e3      ! reference rigidity -> p = 5 GeV/c
1
100 1 1 1 1 1
0.E+00 0.E+00 0.E+00 0.E+00 0.00 0.E+00
0.00E+00 0.00E+00 0.00E+00 0.00E+00 0.00 1.

'PARTICUL'
0.51099892 1.60217653e-19 1.15965218076e-3 0. 0.

'SRLOSS'
1 ! .srloss
BEND
1 123456

'SPNTRK'
4.1
0.4626 0.8866 0.0

'FAISTORE'
zgoubi.fai
100

'INCLUDE'
1
scaling_5GeV1.inc[SCALING_S:SCALING_E]

'INCLUDE'
1
6* superA.inc[superA_S:superA_E]

'INCLUDE'
1
snake_mismatched.inc[snake_sol_s:snake_sol_e]

'CAVITE'
2
204.2000486 100      ! orbit length, h
7974784.27279453 2.61799387799 ! volts, phi_s rad, 5 GeV

'REBELOTE'
9999 0.1 99

'END'

```