Beam Measurements and Diagnostics with Electron Storage Ring

Instructors:

Ying K. Wu and Jun Yan, Duke University; Hao Hao, Oak Ridge National Laboratory

Purpose and Audience

The purpose of this course is to introduce the students to essential instrumentation and diagnostics, measurement techniques, and setup and tuning procedures used in accelerator physics research, commissioning, and operation of light source storage rings. The course is intended to be mainly hands-on work using online software to monitor, measure, and control electron and photon beams in the storage ring at the High Intensity Gamma-ray Source (HIGS) facility, Triangle Universities Nuclear Laboratory. The course is appropriate for graduate students with a strong background in accelerator physics, as well as for postdocs, physicists, and engineers working in the field.

Prerequisites

Courses in classical mechanics, electrodynamics, and physical or engineering mathematics, all at entrance graduate level; and the USPAS course ‘Fundamentals of Accelerator Physics & Technology,’ or equivalent are required. USPAS graduate course "Accelerator Physics" or equivalent experience and experience with software data analysis using tools such as Matlab are strongly recommended.

It is the responsibility of the student to ensure that they meet the course prerequisites or have equivalent experience.

Objectives

The objectives of the course are:

1) Become familiar with beam instrumentation, diagnostics, and measurement techniques at the storage ring;
2) Learn about the fundamental physics principles for beam instrumentation and diagnostic systems;
3) Learn about the controls and data acquisition software for beam measurements;
4) Evaluate and analyze beam measurement data; and
5) Participate in tuning a storage ring and a storage-ring based free-electron laser (FEL).

Instructional Method

This course will focus on a set of beam monitoring and measurement experiments at the Duke University electron storage ring and storage ring FEL using already developed online controls and measurement software. The emphasis is not on software development but the use of applications in making measurements and tuning the accelerators. Short lectures will be given to introduce essential physics concepts, operational principles and techniques, and related hardware equipment and software tools used for the measurements to be conducted. In follow-on sessions, the students will work individually or in small groups to develop a plan and high-level software for the measurements that they will carry out when the accelerators are available. The students will develop hands-on experimental skills by working with the accelerators and beam measurement systems. The students will perform beam studies in a monitoring mode while the storage ring and related light sources are in the routine user mode of operation; and for some studies, the students will be divided into groups to participate in the beam manipulation and measurements, exercising full control of the operation and tuning of the storage ring. A set of planned experiments are listed below. All measurements using beam monitoring will be carried out simultaneously by all students. However, only one group can carry out a beam tuning or manipulation measurement at a time, during which the other students will be working on the data analysis and lab report.

One of the goals of the course is to allow the students to become familiar with the diagnostic instruments used for beam measurements at a light source storage ring. The students will learn about instrumentation for measuring beam current, beam orbit, transverse beam size, bunch length, and synchrotron and betatron tunes. They will use these basic measurement techniques to develop more sophisticated experiments. The students will also learn about the basic operation steps to set up a light source storage ring and carry out beam injection and basic tuning of the storage ring. They will participate in the setup, tuning, and optimization of a storage ring FEL.For homework, students will first analyze the measurement data, and then write lab reports to document the measurement procedures, describe the observations made during the experiment, and present results using figures and tables.

Course Content
Beam monitoring and beam tuning experiments

1. Electron beam orbit stability monitoring (the beam position monitor or BPM system);
2. Beam lifetime measurement (using a DCCT);
3. Transverse beam size measurement (a beam profile measurement system using synchrotron radiation);
4. Bunch length measurement (using a dissector);
5. Synchrotron tune measurement (using a beam pickup);
6. Measurement of eta functions (using BPMs);
7. Measurement of betatron tunes and chromaticity (the tune measurement system); and
8. Measurement of response matrices (s-matrices) of the storage ring optics and closed orbit correction.

Storage ring and FEL operation demonstration:

1. Storage ring injection tuning and study of injection efficiency; and
2. Storage ring FEL setup, tuning, and optimization.

Reading Requirements
The course will follow online technical notes and other publications made available by the instructors.

Credit Requirements
Students will be evaluated based on their participation and contributions during the beam measurements and tuning (40%), their lab reports (40%), and a concept quiz (20%).

Beam Measurements and Diagnostics with Electron Storage Ring

Instructors: Ying K. Wu[1], Jun Yan[1], and Hao Hao[2]

Teaching assistant: Wei Li[1]

[1]TUNL and Duke University

[2]Oak Ridge National Laboratory

February, 2022

Lecture 0: Introduction and Lab Tour

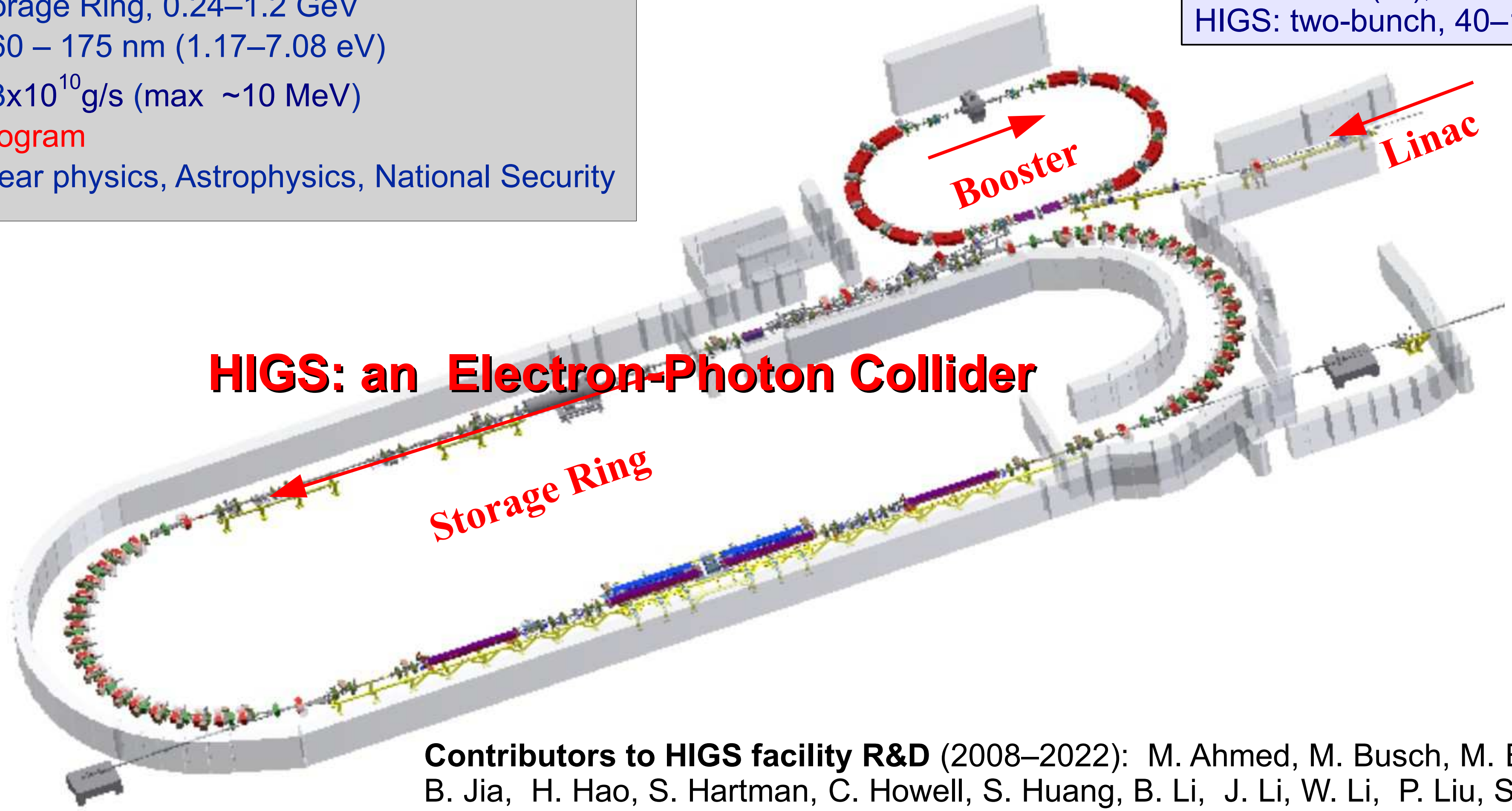
TUNL

Duke UNIVERSITY

HIGS/TUNL: Accelerator Facility

Facility/Project: High Intensity Gamma-ray Source (HIGS)
Institution: TUNL
Country: US
Energy (MeV): 1–120
Accelerator: Storage Ring, 0.24–1.2 GeV
Laser: FEL, 1060 – 175 nm (1.17–7.08 eV)
Total flux: 10^7–$3x10^{10}$ g/s (max ~10 MeV)
Status: User Program
Research: Nuclear physics, Astrophysics, National Security

Accelerator Facility
160 MeV Linac pre-injector
160 MeV–1.2 GeV Booster injector
240 MeV–1.2 GeV Storage ring
FELs: OK-4 (lin), OK-5 (cir)
HIGS: two-bunch, 40–120 mA (typ)



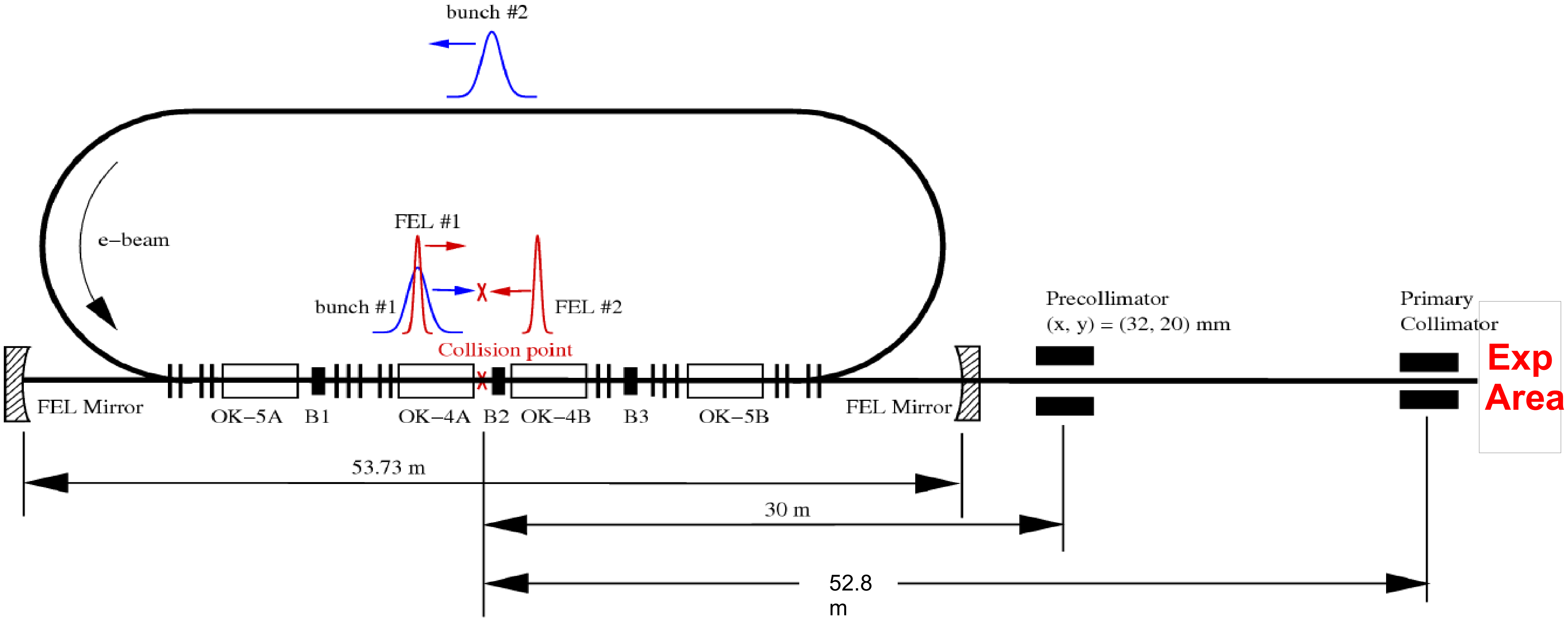
Contributors to HIGS facility R&D (2008–2022): M. Ahmed, M. Busch, M. Emamian, J. Faircloth, B. Jia, H. Hao, S. Hartman, C. Howell, S. Huang, B. Li, J. Li, W. Li, P. Liu, S. Mikhailov, M. Pentico, V. Popov, W. Tornow, C. Sun, G. Swift, B. Thomas, P. Wang, P. Wallace, W. Wu, Y.K. Wu, W. Xu, J. Yan

Virtual Tour of

HIGS Accelerator Facility

Operation Principle of HIGS

Two electron bunches + two FEL pulses



https://www.youtube.com/watch?v=JoIXGPNGEOc

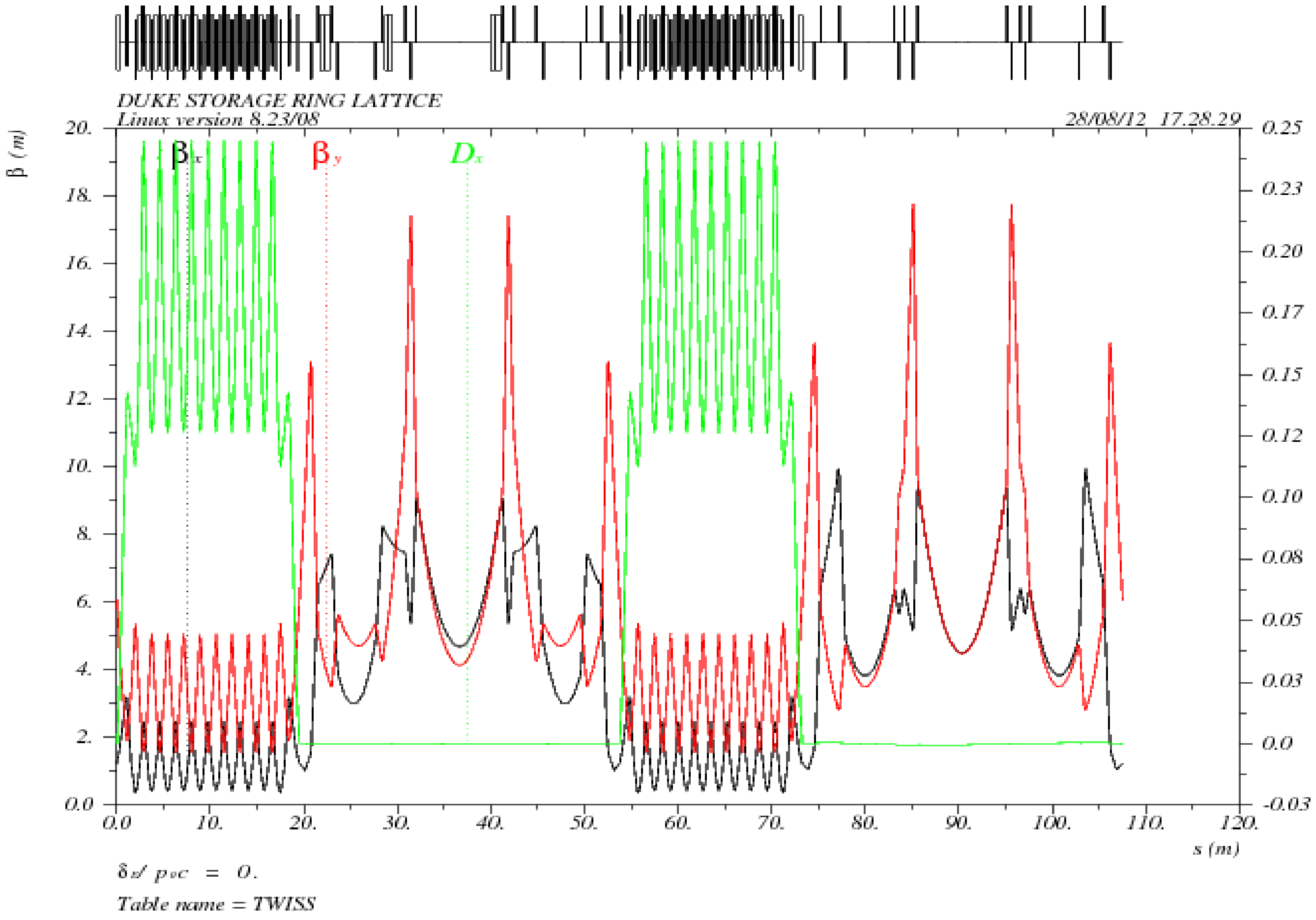
V.N. Litvinenko *et al*. PRL v. 78, n. 24, p. 4569 (1997)

Parameters of the Duke Storage Ring

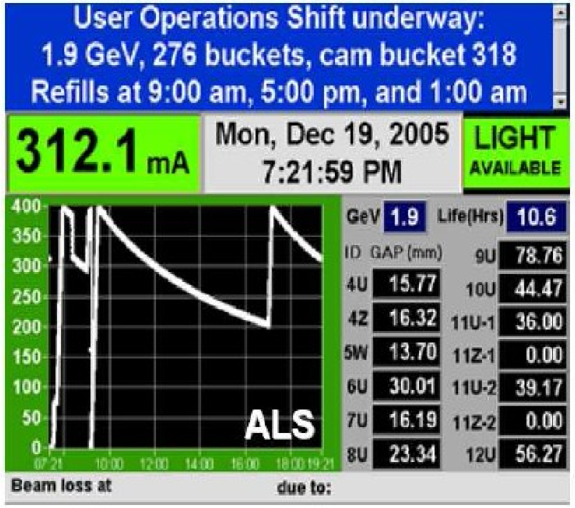
Electron beam energy	180 MeV – 1.2 GeV
Circumference	107.46 m
RF cavity frequency	178.55 MHz
Betatron tunes	9.11/4.18
Max. stored beam current	~400 mA (multi-bunch), 120 mA (two-bunch)
Momentum compaction factor	0.0086
Damping time (@1 GeV)	18.3 (H), 17.0 (V), 8.2 (energy) ms
Bunch length (natural, @1 GeV)	67 ps (rms)
Emittance (hori. @1 GeV)	18 nm-rad
Synchrotron radiation energy/turn (@1 GeV)	42 keV
FEL cavity length	53.73 m
FEL undulators	OK-4 (planar), OK-5 (helical)
FEL wavelength	2.1 microns to 168.6 nm

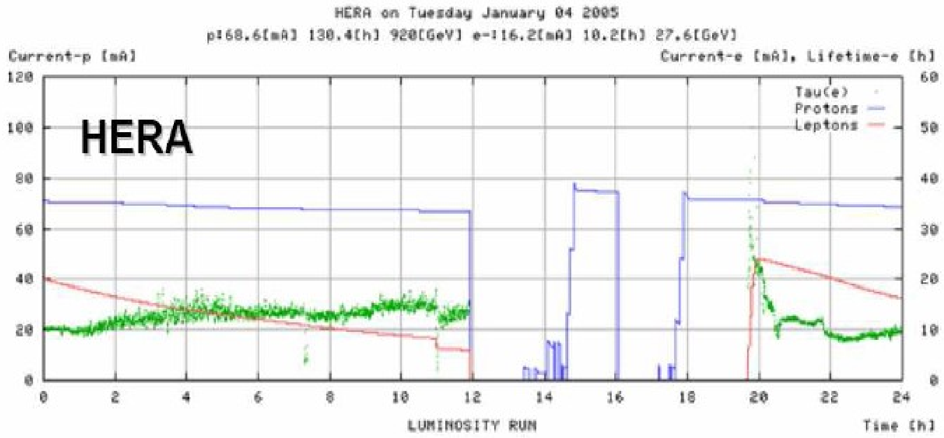
TUNL Duke UNIVERSITY

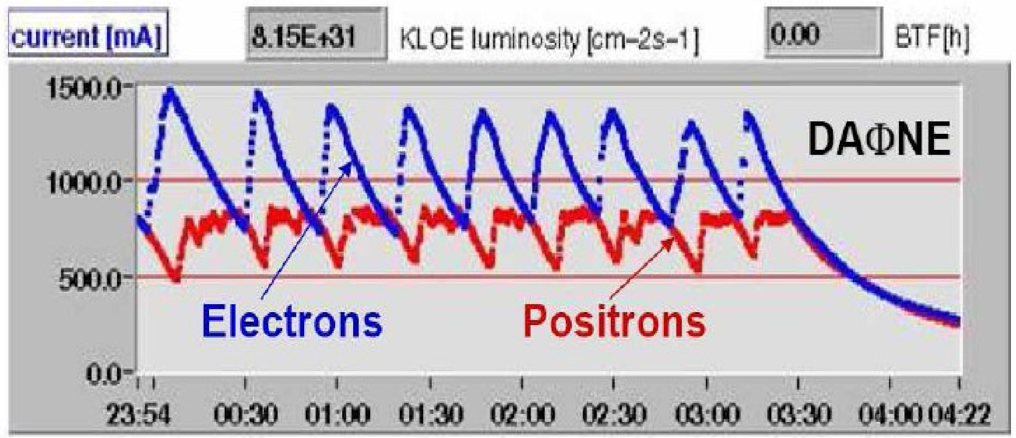
Beta-function and Eta-function of Duke Storage Ring

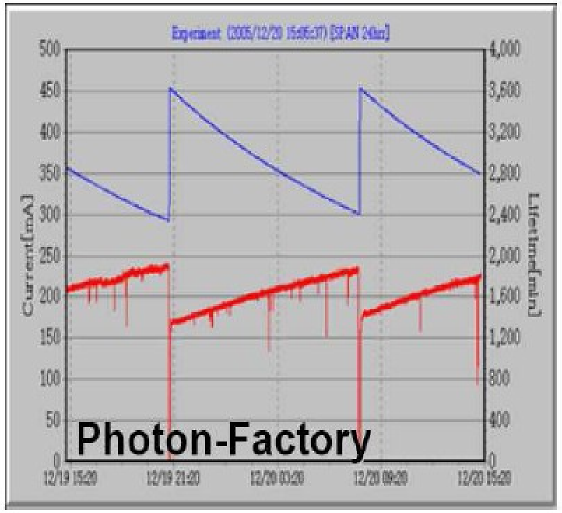


Lab 1: Beam Lifetime Measurements





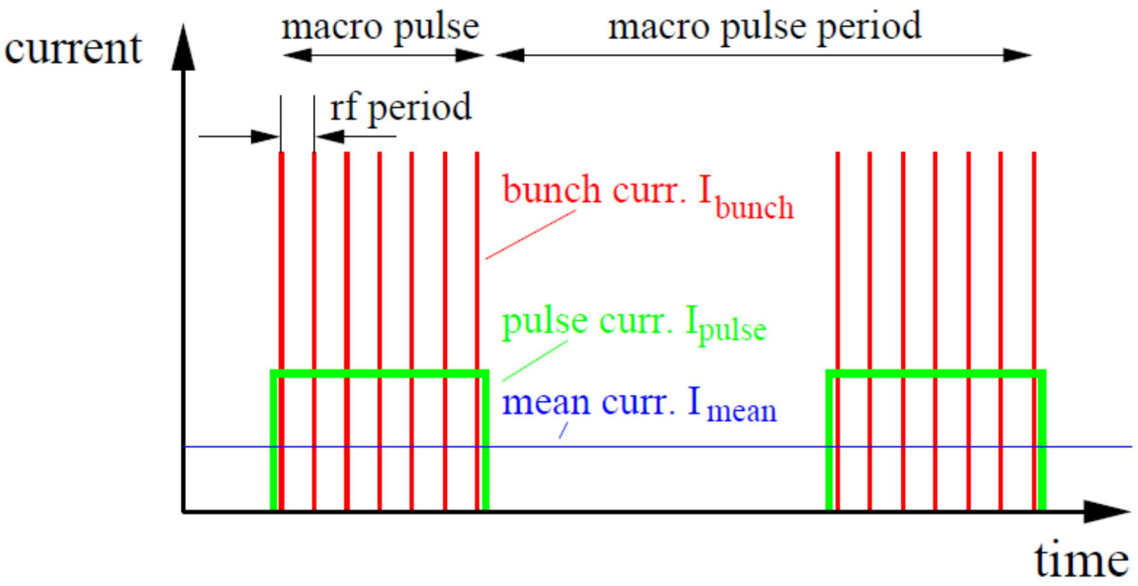


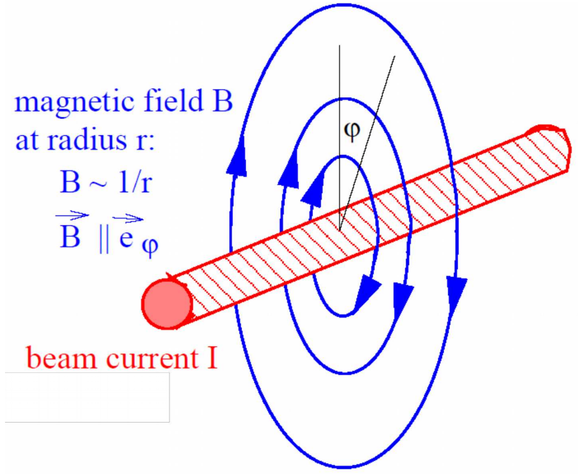


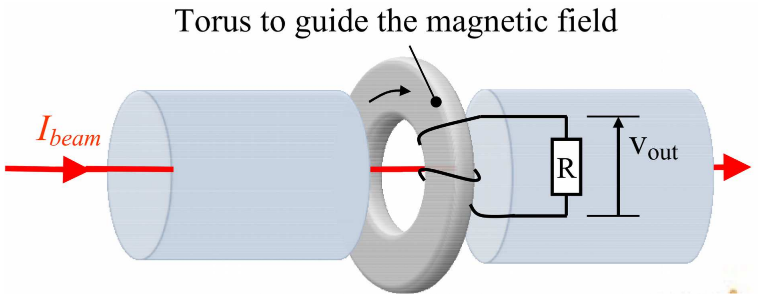
Courtesy of F. Sannibale

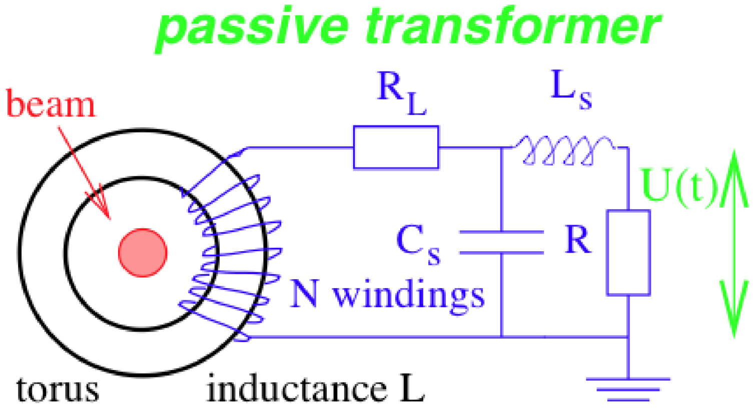
Fast Current Transformer (FCT), Passive Transformer

- Pulsed Linac beams





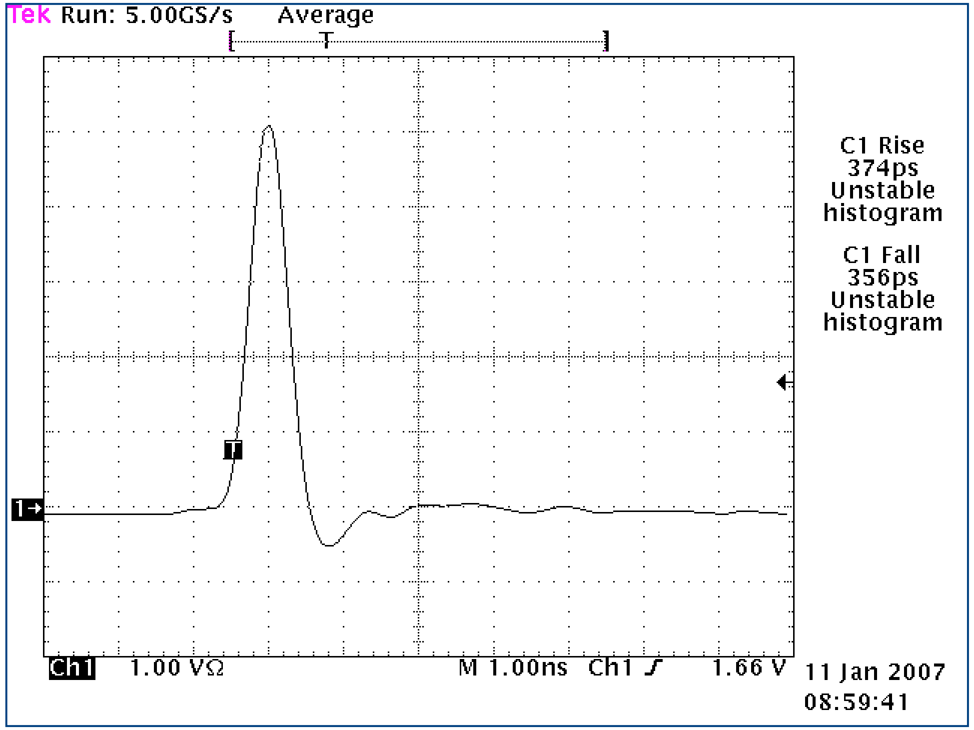




P. Forck, "Beam Current Measurement", Joint University Accelerator School,, Archamps (2011)

http://www-bd.gsi.de/conf/juas/current_trans.pdf

Bergoz Fast Current Transformer



Bandwidth up to 1.75 GHz

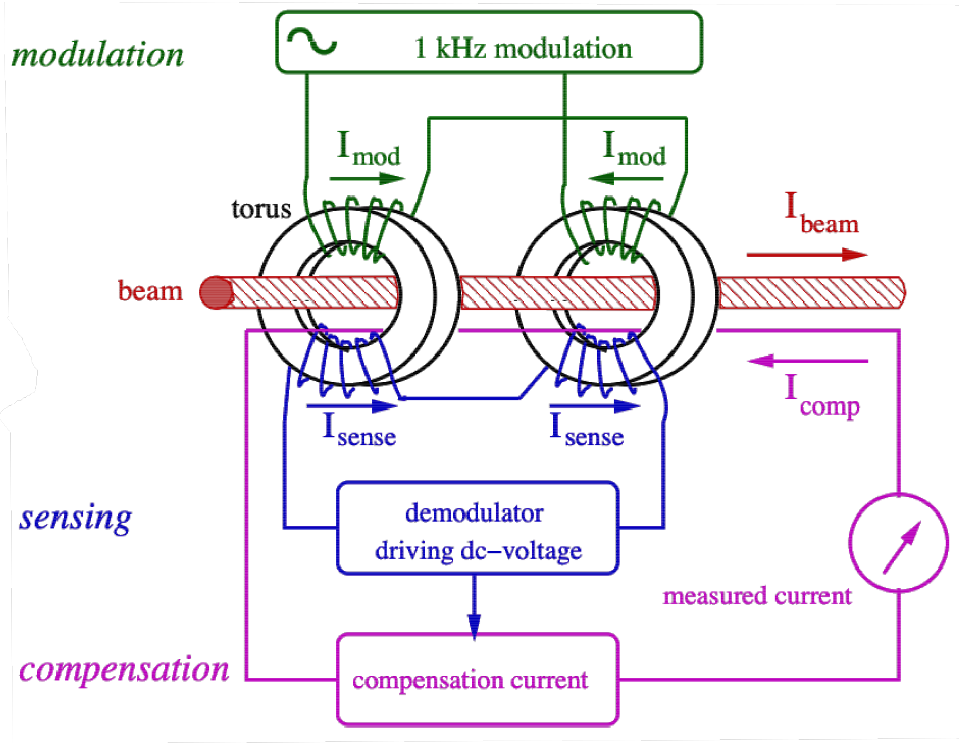
200 ps risetime

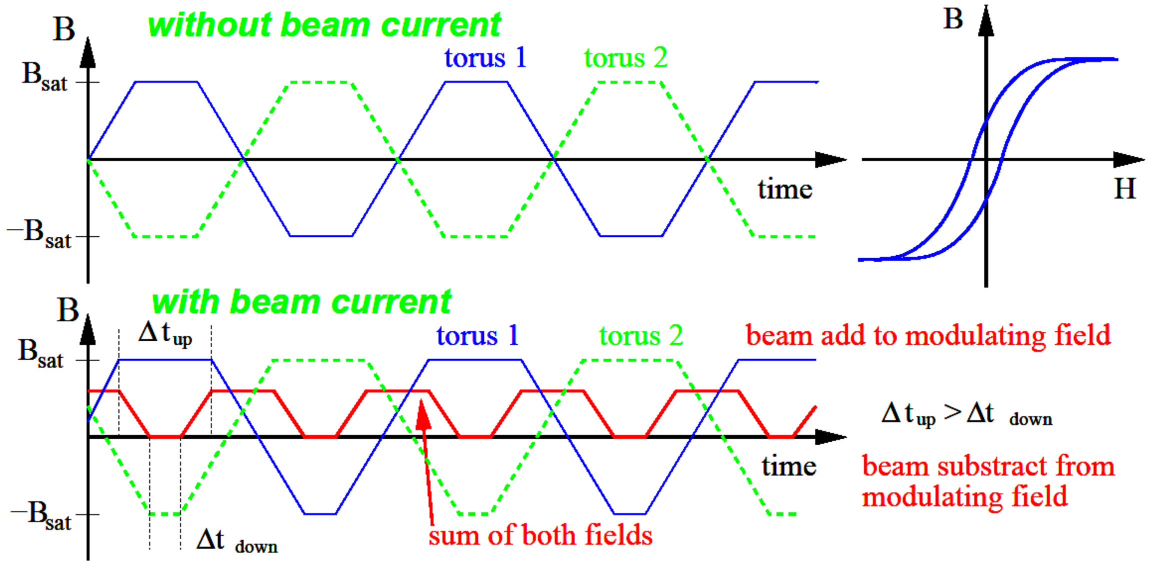
Sensitivity up to 5V/A

DC Current Transformer (DCCT)

Method: using magnetic saturation of two tori

- Modulation of the primary windings forces both tori into saturation twice per cycle.
- Sense windings measure the modulation signal and cancel each other.
- With the beam, the saturation is shifted and the sense current is not zero, I_{sense} != 0
- Compensation current is adjusted until the sense current is zero again, I_{sense} = 0





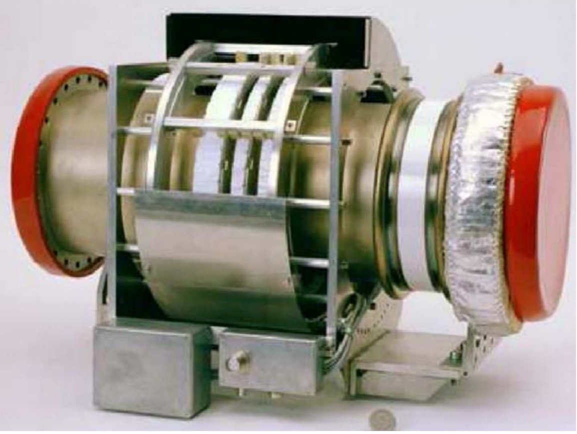
P. Forck, “Beam Current Measurement”, Joint University Accelerator School,, Archamps (2011)

http://www-bd.gsi.de/conf/juas/current_trans.pdf

DC Current Transformer (DCCT)

DCCT at GSI Synchrotron (designed in 1990)

Core radii	$r_i = 135$ mm, $r_o = 145$mm
Core thickness	10 mm
Core material	Vitrovac 6025: $(CoFe)_{70\%}(MoSiB)_{30\%}$
Core permeability	$\mu_r \simeq 10^5$
Saturation B_{sat}	$\simeq 0.6$ T
Isolating cap	Al_2O_3
Number of windings	16 for modulation and sensing
	12 for feedback
Ranges for beam current	300 μA to 1 A
Resolution	2 μA
Bandwidth	dc to 20 kHz
rise time	20 μs
Offset compensation	± 2.5 μA in auto mode
	< 15 μA/day in free run
temperature coeff.	1.5 μA/^{o}C



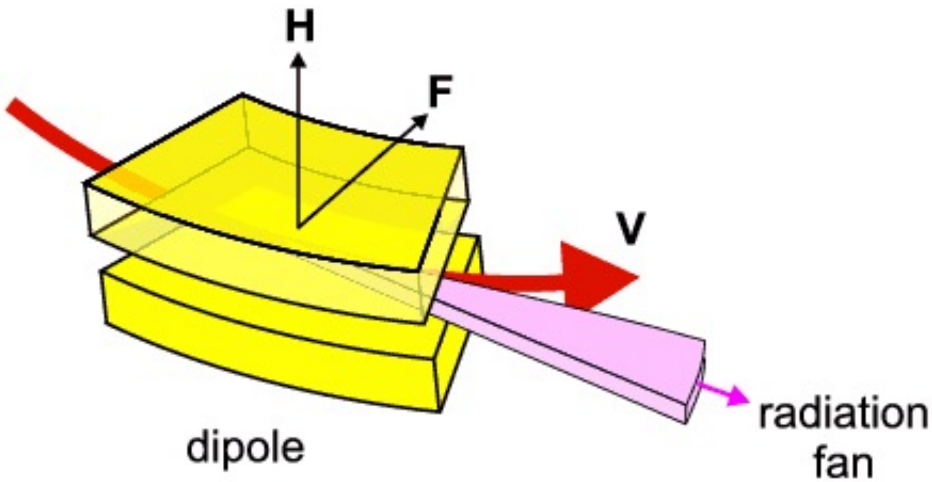
Duke Storage Ring DCCT

P. Forck, "Beam Current Measurement", Joint University Accelerator School,, Archamps (2011)
http://www-bd.gsi.de/conf/juas/current_trans.pdf

Beam Lifetime

- Measure the beam current as a function of time
- Fit to find the lifetime: $I(t) = I_0 \exp(-t/\tau)$
- Many operation factors can affect the beam lifetime in a storage ring:
  - How good the vacuum is?
  - Beam energy, beam current, bunch pattern
  - Beam orbit, insertion devices, etc.
  - Beam sizes and bunch length which can be influenced by weak instability, FEL operation, differetn operation modes, etc.
  - More ...

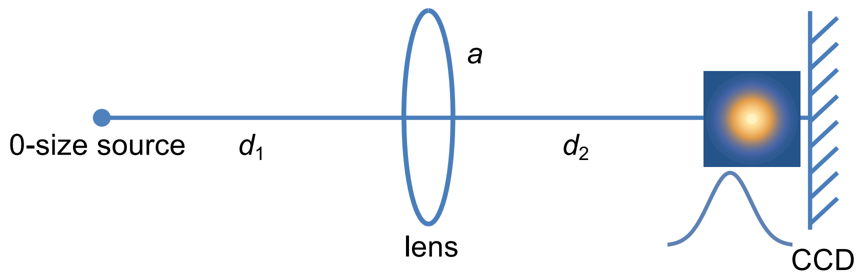
Lab 2: Transverse Beam Profile Measurement

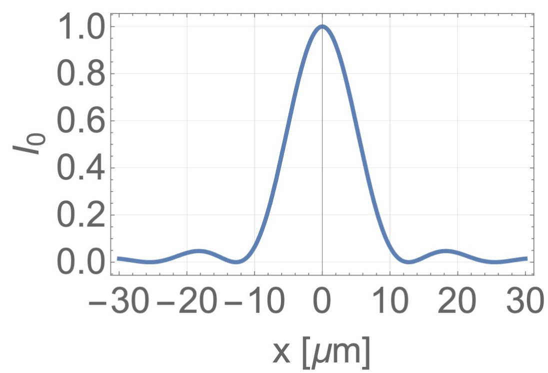


What does the synchrotron radiation look like at the view port?

http://pd.chem.ucl.ac.uk/pdnn/inst2/prop.htm

Transverse Beam Profile Measurement





$$a = 2 \text{ cm}$$
$$d_2 = 1.5 \text{ m}$$
$$\lambda = 340 \text{ nm}$$

- Smallest (on-focus) photon beam distribution:

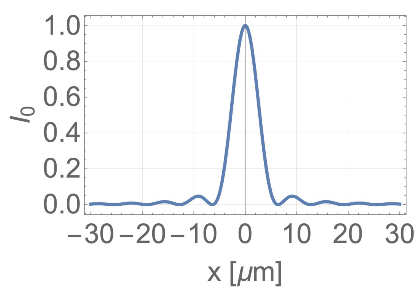
$$I_0(x) = I(0)\,\text{sinc}\left(2\pi \frac{a}{d_2 \lambda} x\right)^2$$

- Large opening angle (a/d_2): high resolution, small tolerance of out-of-focus error

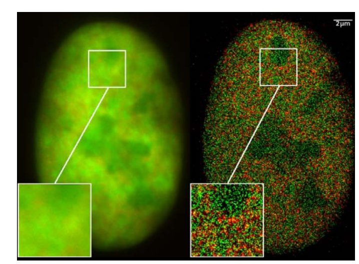
$$\epsilon = \frac{1}{d_1} + \frac{1}{d_2} - \frac{1}{f}$$

- Resolution: match CCD size

$$\frac{\lambda d_2}{a} < \text{CCD pixel size}$$



$$a = 4 \text{ cm}$$



https://en.wikipedia.org/wiki/Super-resolution_microscopy

Transverse Beam Profile Measurement

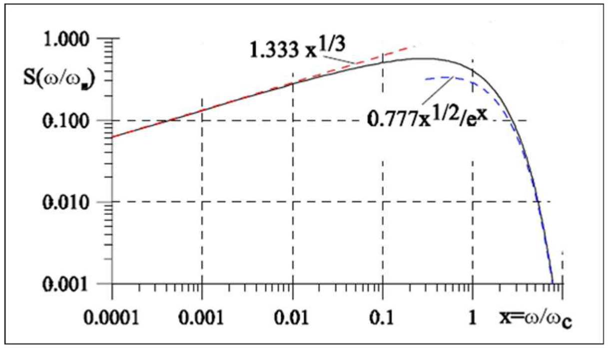
- Emittance of photon beam from a single electron:

$$\varepsilon_{\mathrm{ph}} = \frac{\lambda_{\mathrm{L}}}{4\pi}$$

shorter wavelength, a smaller single electron "beam size"

- When there are multiple electron, e.g., 10^{10}, the photon beam distribution is the convolution of electron beam distribution and the single electron's photon beam distribution:

$$\varepsilon_{\mathrm{ph}} = \varepsilon_{\mathrm{e}} + \frac{\lambda_{\mathrm{L}}}{4\pi}$$



- A meaningful measurement:

$$\varepsilon_{\mathrm{e}} \gg \frac{\lambda_{\mathrm{L}}}{4\pi}$$

- If electron beam size itself is smaller than single electron "beam size", we call this ring as diffraction limited storage ring. Note: vertical beam size can easily enter the diffraction limit zone.

- $\varepsilon_c = 0.665 E^2 B = 0.665 \times 0.3^2 \times 0.45 = 27 \text{ eV} \rightarrow \lambda_c = 50 \text{ nm}$

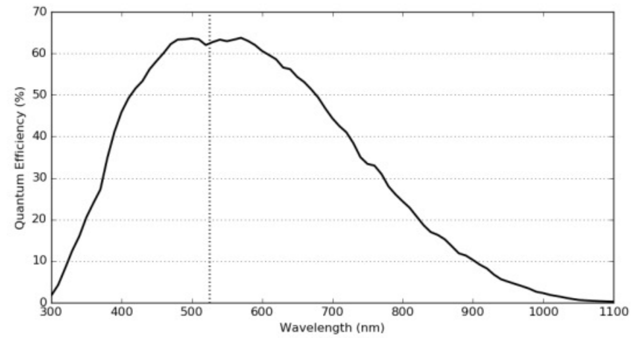
  $\varepsilon_c = 0.665 E^2 B = 0.665 \times 1.2^2 \times 1.8 = 1.72 \text{ eV} \rightarrow \lambda_c = 0.7 \text{ nm}$

Transverse Beam Profile Measurement

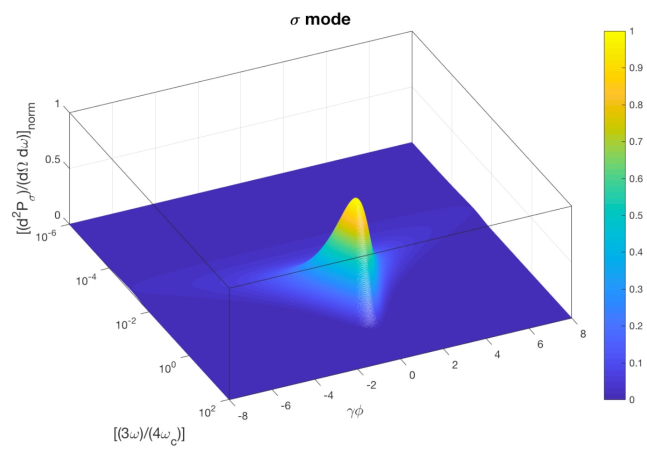
- Synchrotron radiation polarization
- Sensor spectral response
- CCD ADC bits: 8-bit, 12-bit, or 16-bit
- Exposure time
- Pixel size
- Where to measure? In the bending magnet.

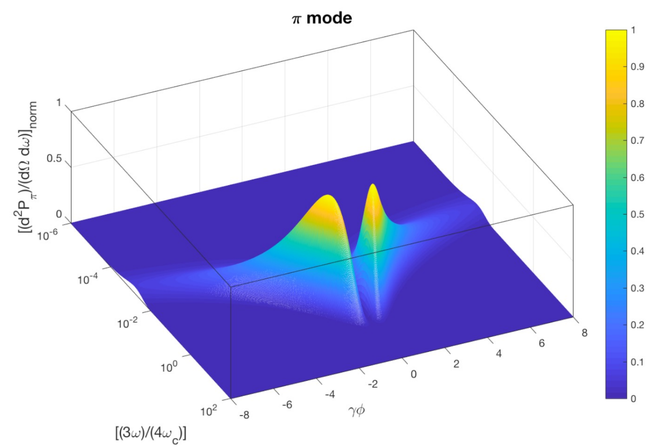
$$x = x_\beta + \eta\delta \qquad \sigma_x = \sigma_{x,\beta} + \eta\sigma_\delta$$

η as small as possible

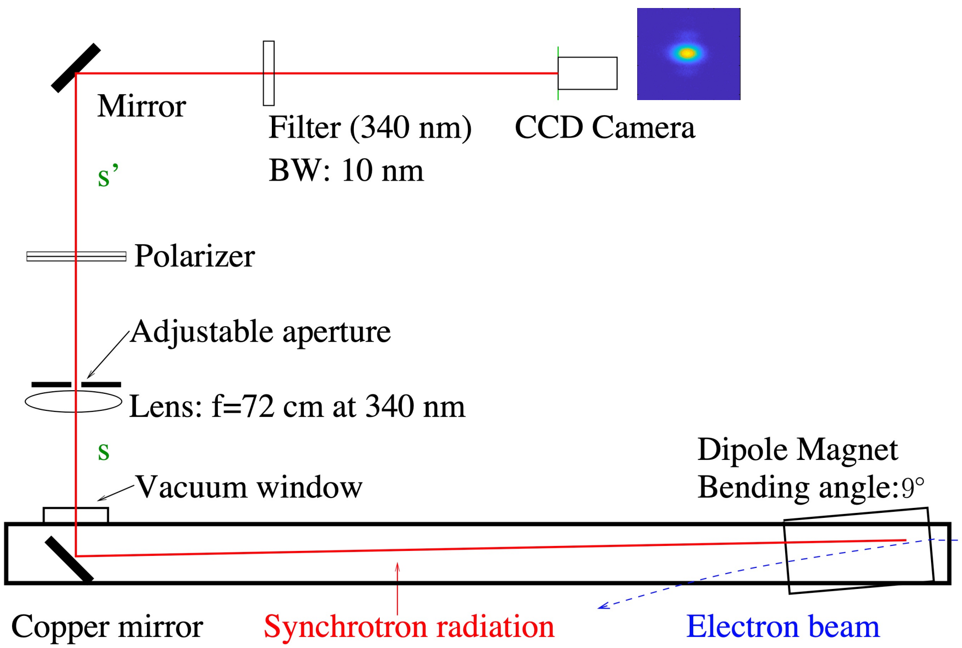


http://softwareservices.flir.com/BFS-U3-04S2/latest/EMVA/EMVA-Local.html





Transverse Beam Profile Measurement



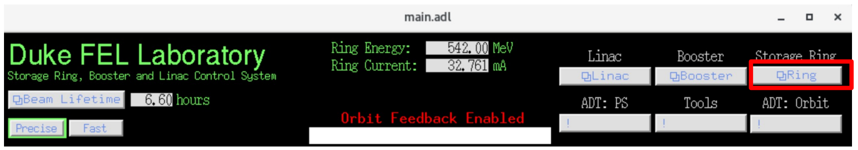
- CCD pixel size: 5.3 micron x 5.3 micron
- Wavelength: 320 nm ~ 340 nm

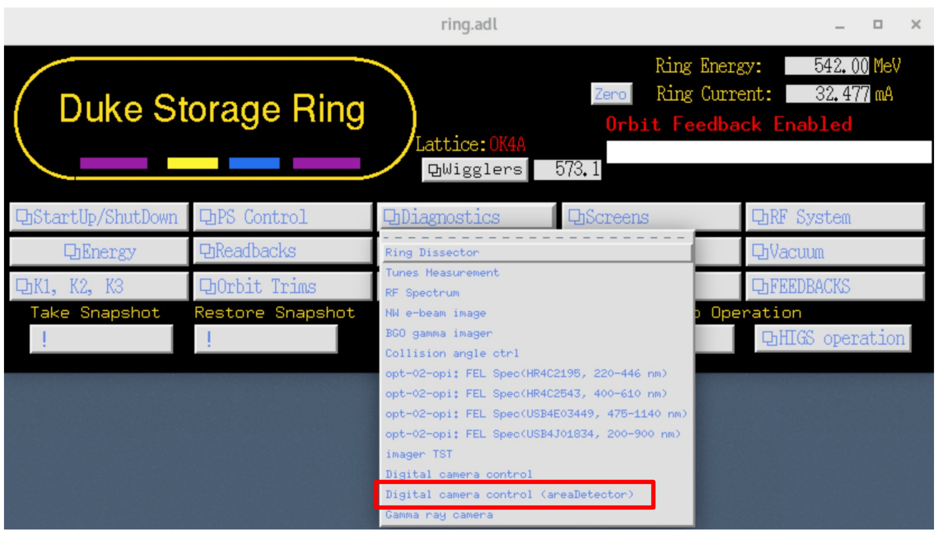
Parameters	β_x (m)	η_x (mm)	δ	ϵ_x (nm·rad
Value	1.32	3.04×10^{-3}	5.9×10^{-4} at 1 GeV	18 at 1 GeV

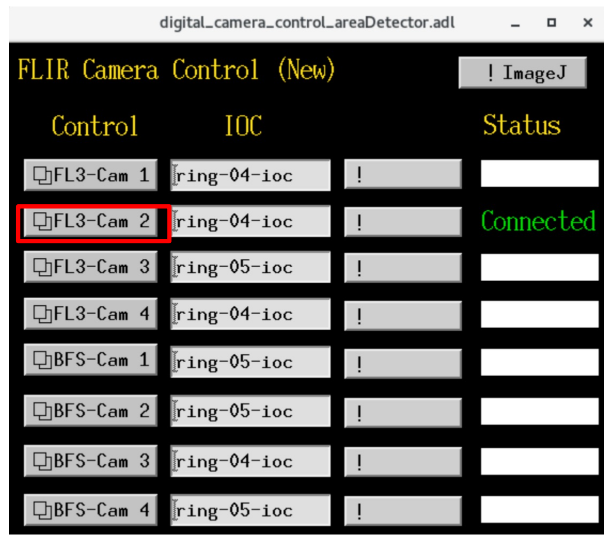
Transverse Beam Profile Measurement

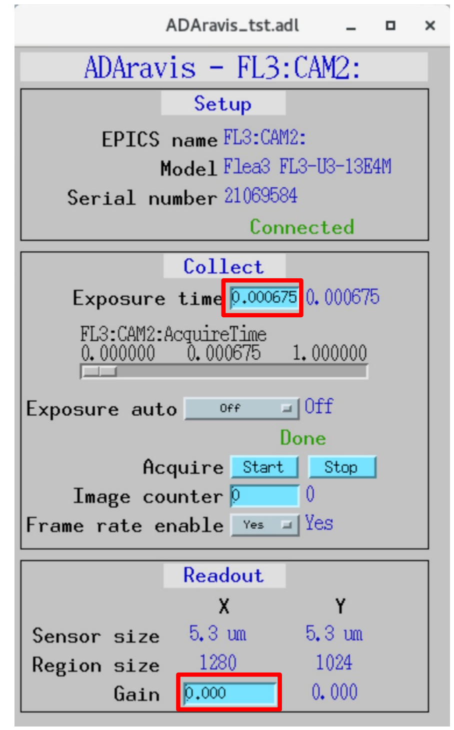
- Measured beam size is determined by:
  - Linear and nonlinear beam optics: eta-function, coupling …
  - Wavelength
  - Beam current: intra-beam scattering
  - Beam energy: intra-beam scattering
  - Other factors: orbit stability, degree of out-of-focus …

Transverse Beam Profile Measurement









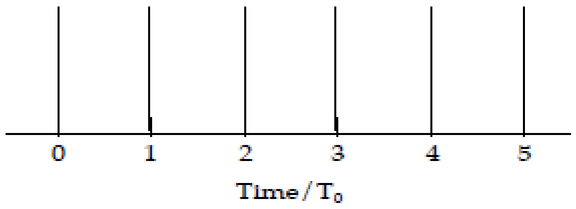
Transverse Beam Profile Measurement

- EPICS PV names
  - `RNG:ENG` electron storage ring energy
  - `RNG:BEAM:CURR` electron beam current in storage ring
  - `FL3:CAM2:AcquireTime` camera exposure time
  - `FL3:CAM2:Gain` camera gain
- MATLAB API
  - `takeImage()`, takes an image from FL3:CAM2 camera, returns a struct that contains exposure time, gain, and the image data.

Lab 3-1: Synchrotron Tune Measurement

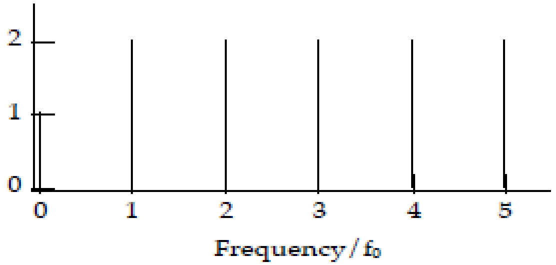
At a fixed point in the ring,

$$I(t) \sim \sum_n \delta(t - nT_0)$$



FT

$$I(\omega) \sim \sum_k \delta(\omega - k\omega_0)$$



w/ synchrotron oscillation,

$$I(t) \sim \sum_n \delta[t - nT_0 + \tau_s \cos(\omega_s t)]$$

phase modulation

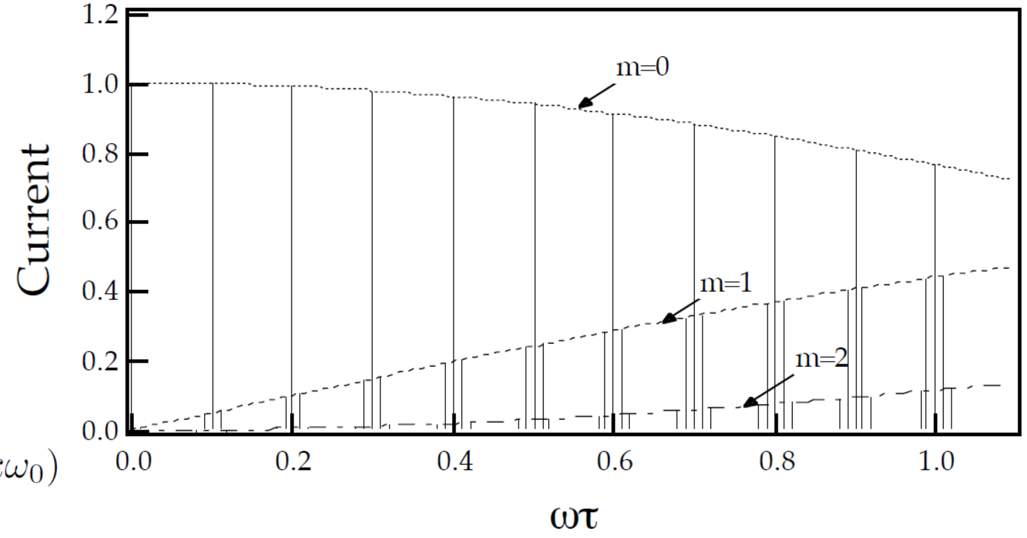
FT

$$I(\omega) \sim \omega_0 \sum_m j^{-m} J_m(\omega\tau_s) \sum_k \delta(\omega + m\omega_s - k\omega_0)$$

sideband order

harmonic #

$$\boxed{\textbf{Synchrotron tune} \quad \nu_s = \frac{\omega_s}{\omega_0}}$$



Measure at higher harmonics of the revolution frequency

Larger sideband signal

Materials and figures from Derun Li *et al.* "Microwave Measurements and Beam Instrumentation Laboratory", USPAS2020 Winter Session, UC San Diego, January 2020

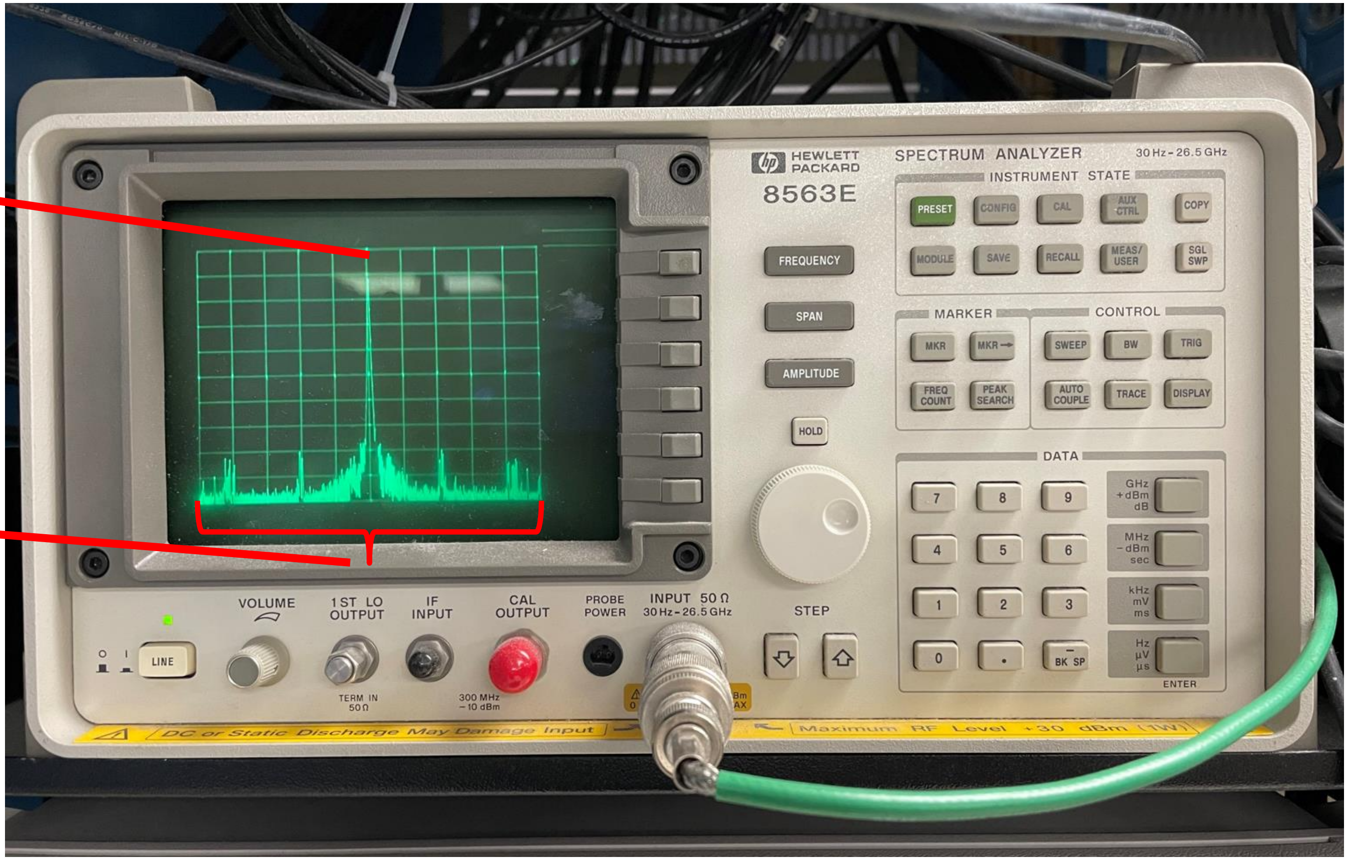
Lab 3-1: Synchrotron Tune Measurement

- Synchrotron tunes can be measured using a spectrum analyzer connected to a beam pickup device (e.g. the sum signal)
- For longitudinally unstable/semi-unstable beam excitation is usually unnecessary

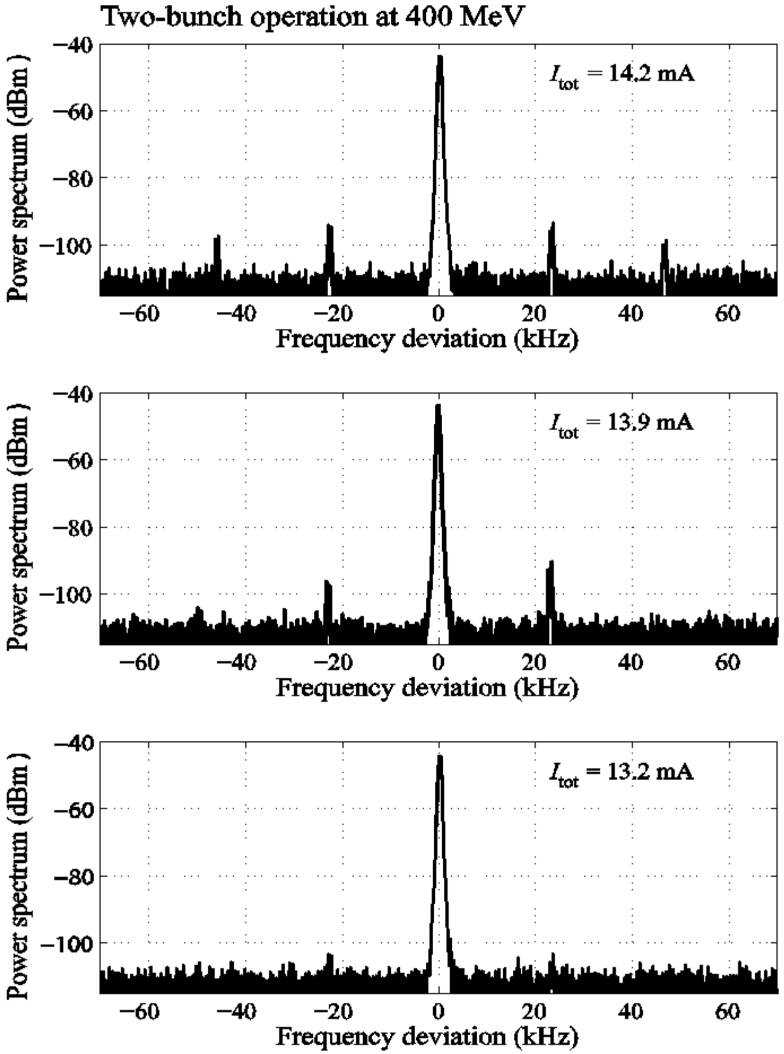
1. Central frequency: 178.5 MHz
   (storage ring rf freq,
   64th harmonic of revolution freq)

2. Freq span (sweeping range): 100 kHz

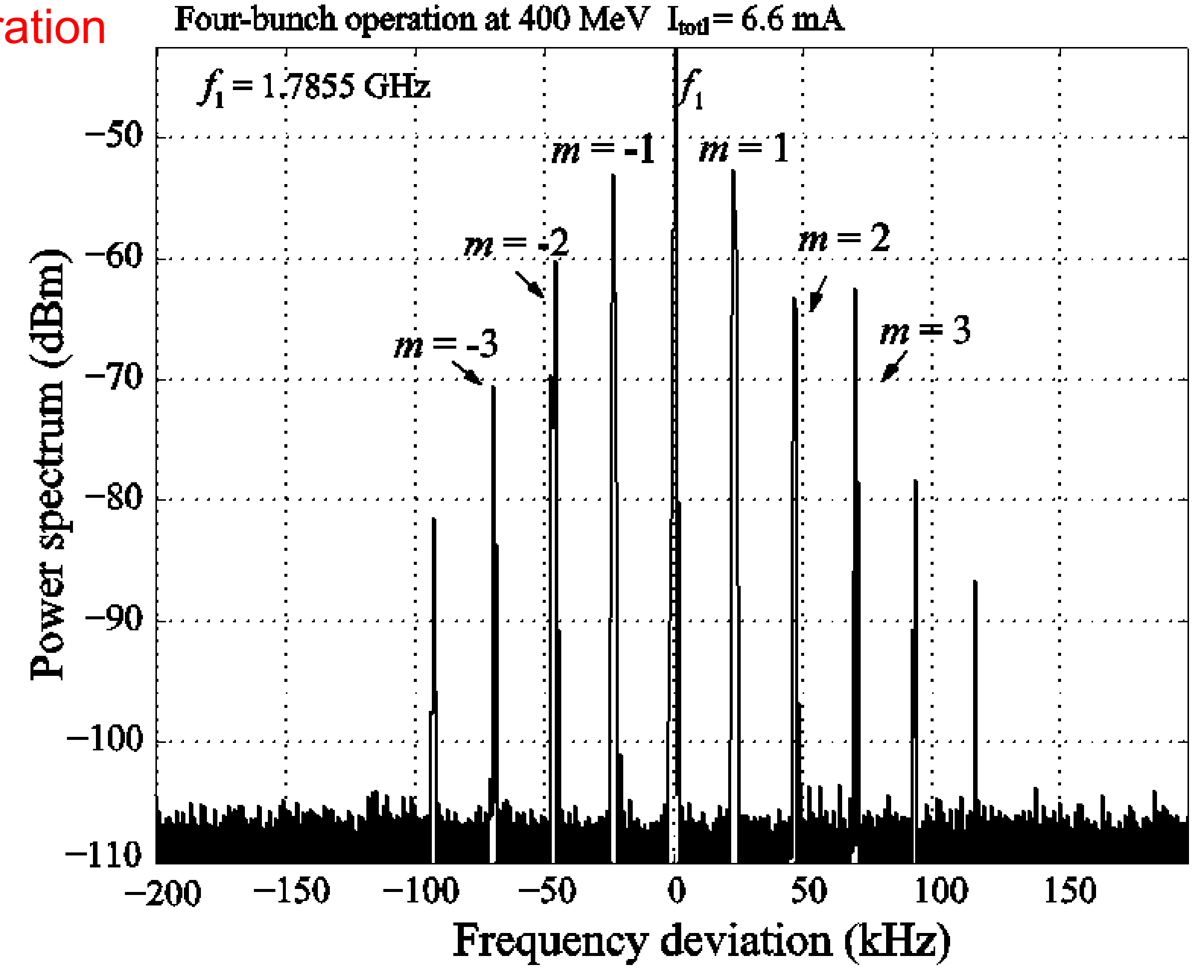
3. Resolution bandwidth
(better resolution -> longer sweeping time)



Lab 3-1: Synchrotron Tune Measurement



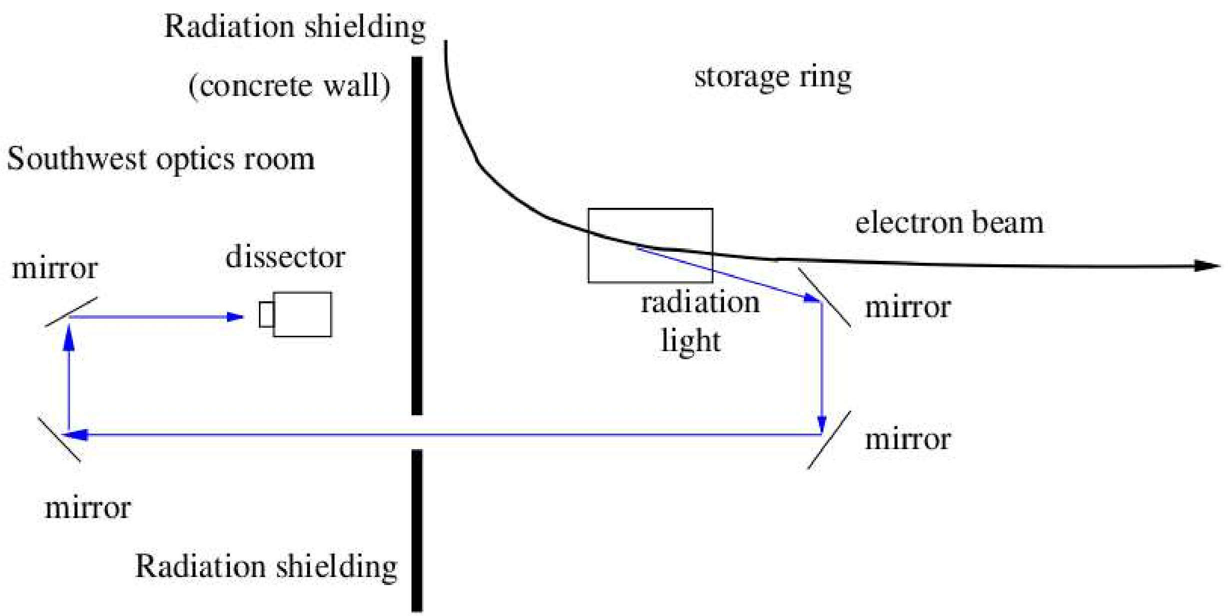
Duke Storage Ring:
2-bunch, 4-bunch Operation

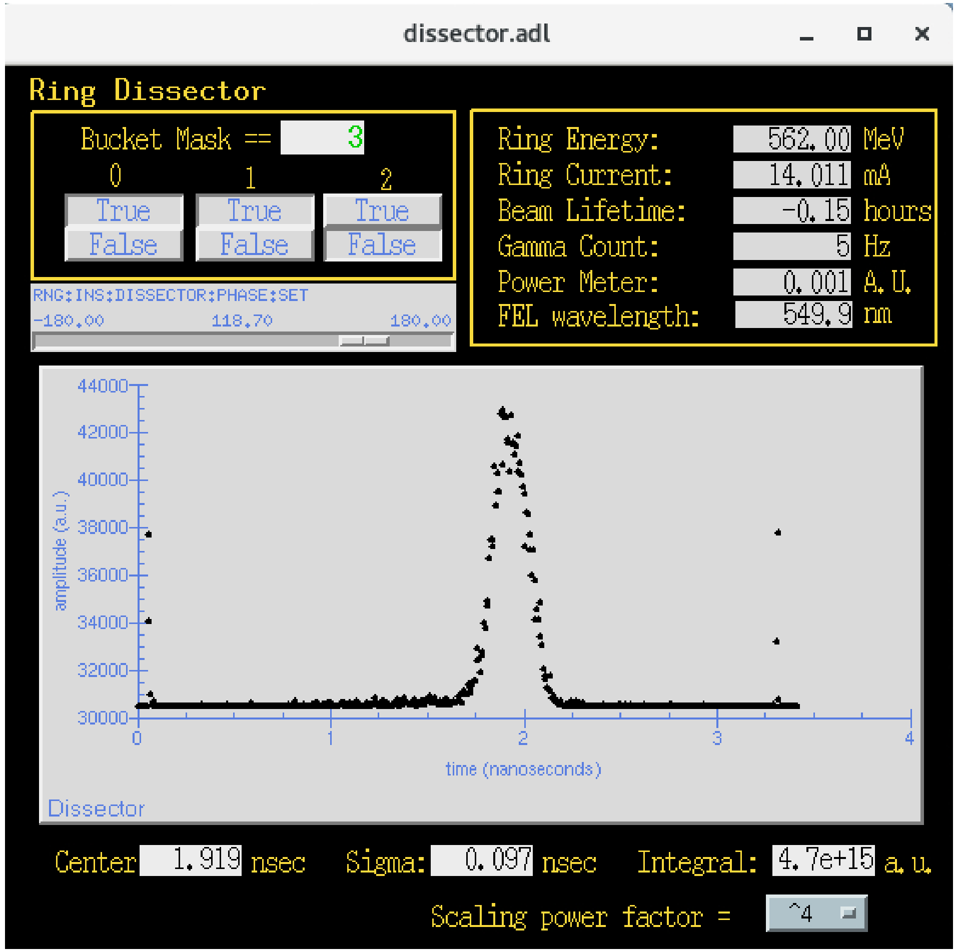


Lab 3-2: Bunch Length Measurement

Common technique to measure the beam longitudinal profile in a storage ring:

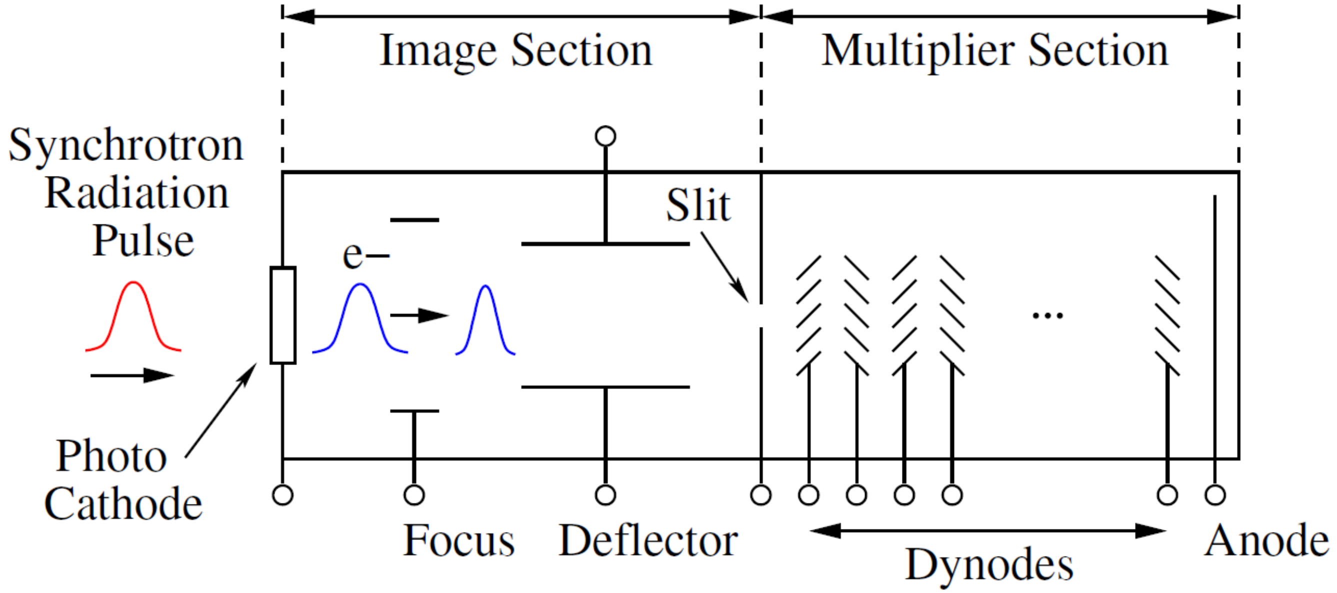
Dissector

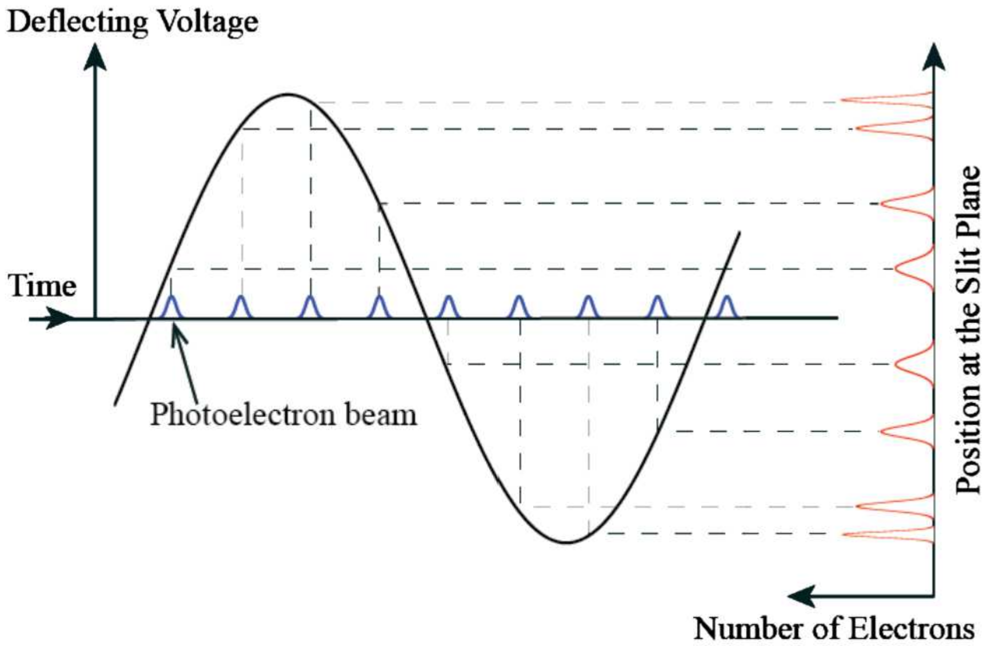




Lab 3-2: Bunch Length Measurement

Dissector





- **The photocathode produces electrons when illuminated by the synchrotron radiation pulse.**
- **Electrons are swept by the high frequency deflecting field, part of them could pass through the slit and get multiplication.**
- **The resolution of the dissector can reach tens of picoseconds.**

Lab 3-2: Bunch Length Measurement

- **The bunch length has a dependency on RF voltage (beyond microwave instability threshold):**

$$\sigma_{b,\ \mathrm{real}} \propto \frac{1}{\sqrt{V_{rf}}},$$

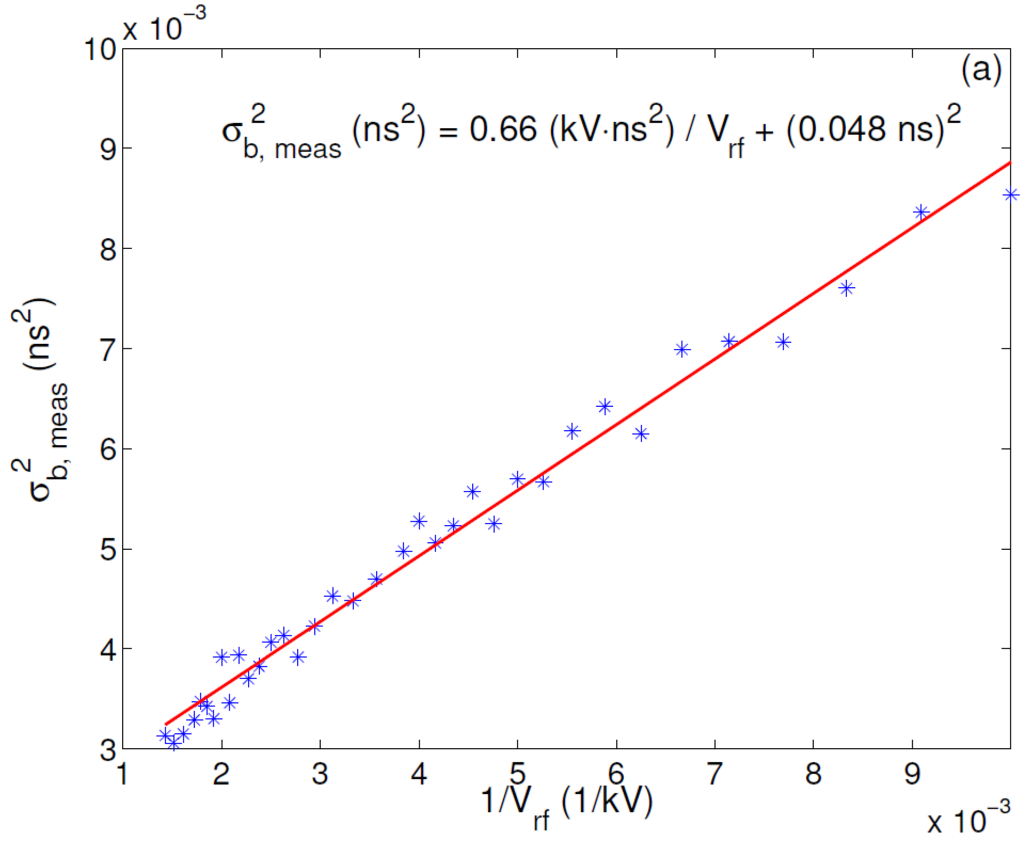
thus

$$\sigma^2_{b,\ \mathrm{meas}} = \frac{A}{V_{rf}} + \sigma^2_{b,\ \mathrm{res}},$$

Dissector resolution

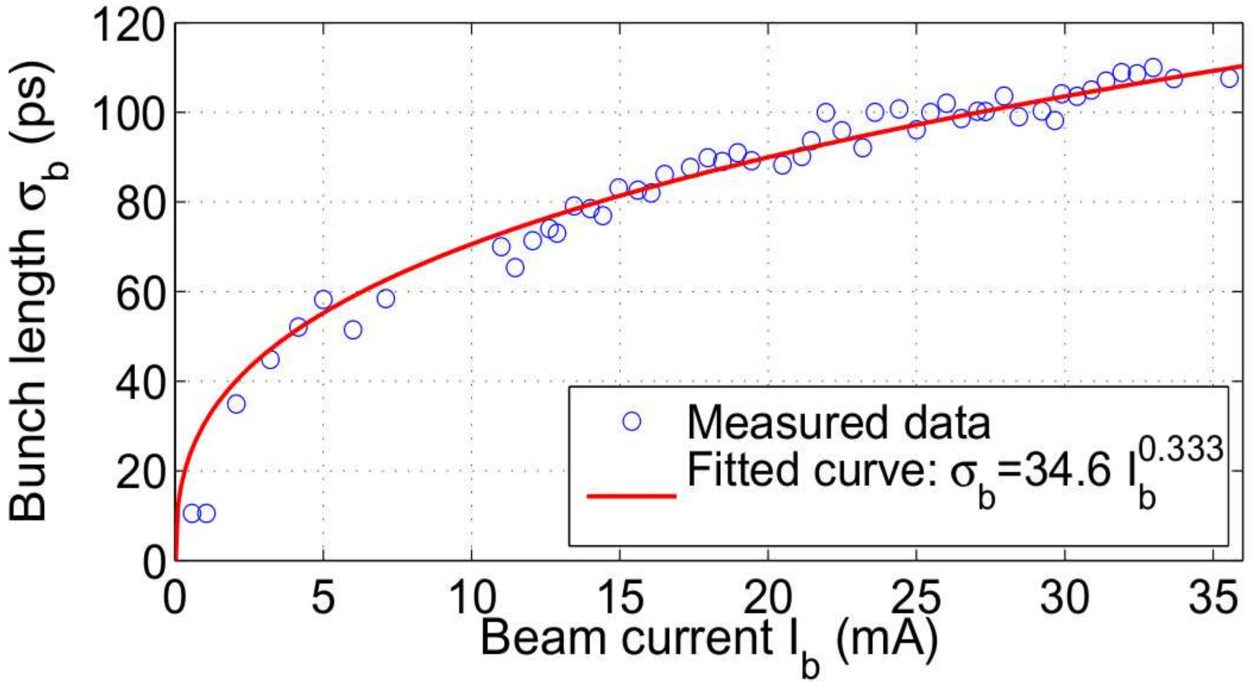
- **The measured bunch length is the convolution of the actual bunch length and the dissector resolution.**

$$\sigma^2_{b,\ \mathrm{meas}} = \sigma^2_{b,\ \mathrm{real}} + \sigma^2_{b,\ \mathrm{res}},$$



Lab 3-2: Bunch Length Measurement

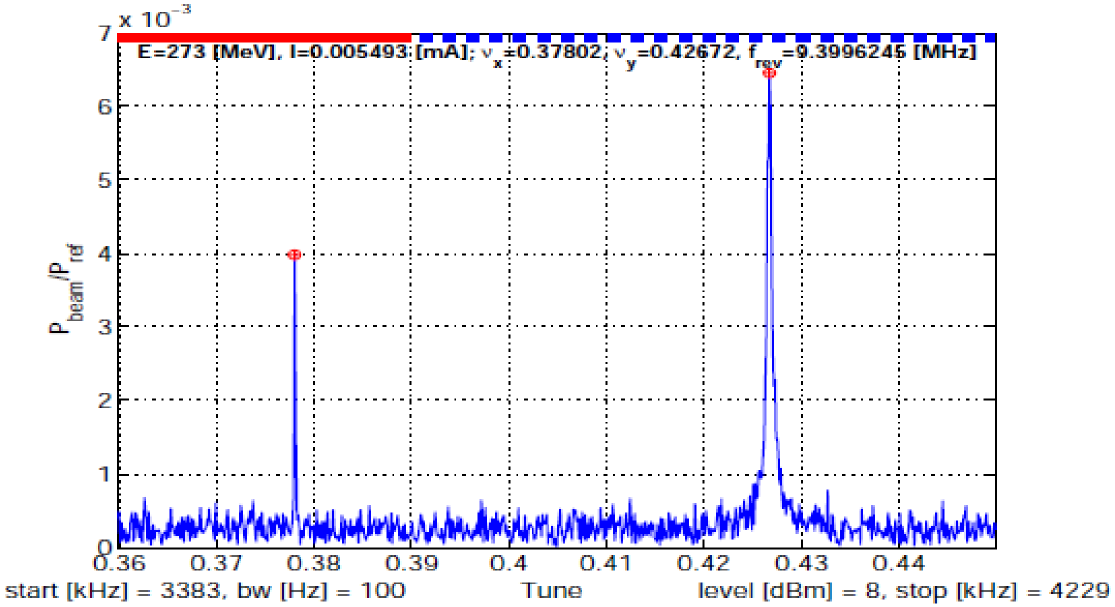
Bunch length vs Beam current

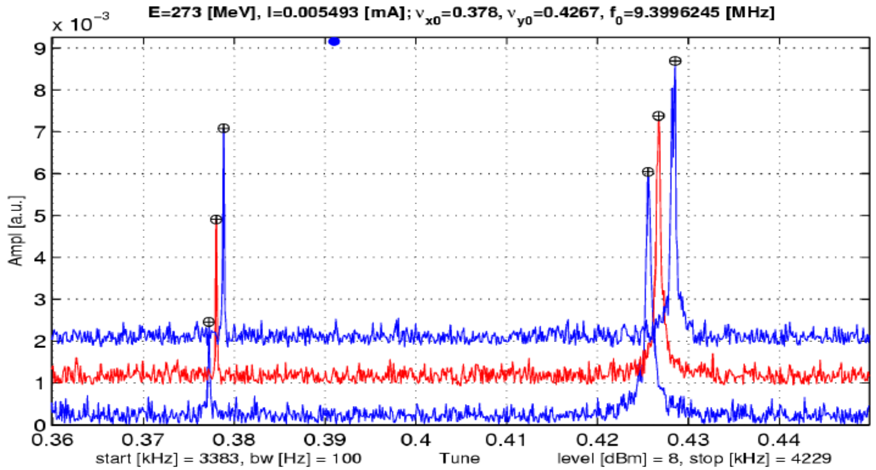


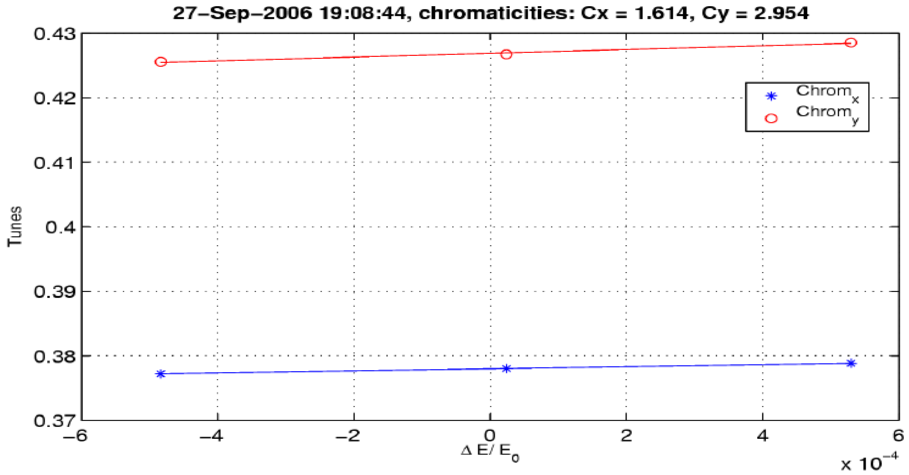
$$\sigma_b \propto I_b^{1/3}$$

- **With the increase of the beam current, the electromagnetic environment is significantly changed w.r.t. the 0-current case. The potential well for longitudinal motion is also distorted.**
- **One dominate bunch lengthening mechanism is the microwave instability.**

Lab 4: Betatron Tune and Chromaticity Measurements







- Damped driven harmonic oscillator

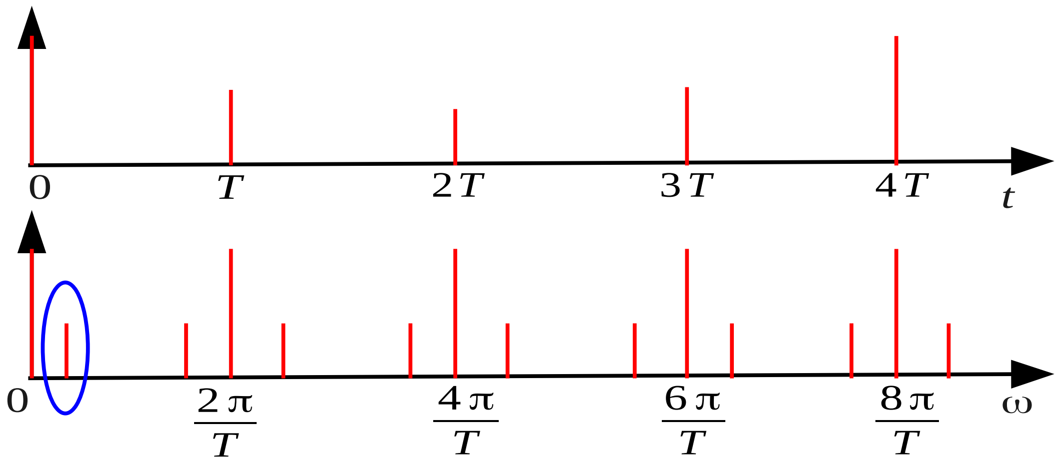
$$\frac{d^2x}{dt^2}+\beta\frac{dx}{dt}+\omega_0^2x=f\sin(\omega t)$$

damping natural freq external drive

Steady state solution after transient $$x(t)=\frac{f}{\sqrt{(\omega^2-\omega_0^2)^2+\beta^2\omega^2}}\sin(\omega t+\phi)$$

At resonance, $\omega=\omega_0$, maximum power coupling into the system: $\approx A_{max}$

- Tune measurement: drive the betatron motion, sweep excitation frequency, monitor for peak response
- In beam spectrum, betatron resonance shows up as sidebands of the revolution signal

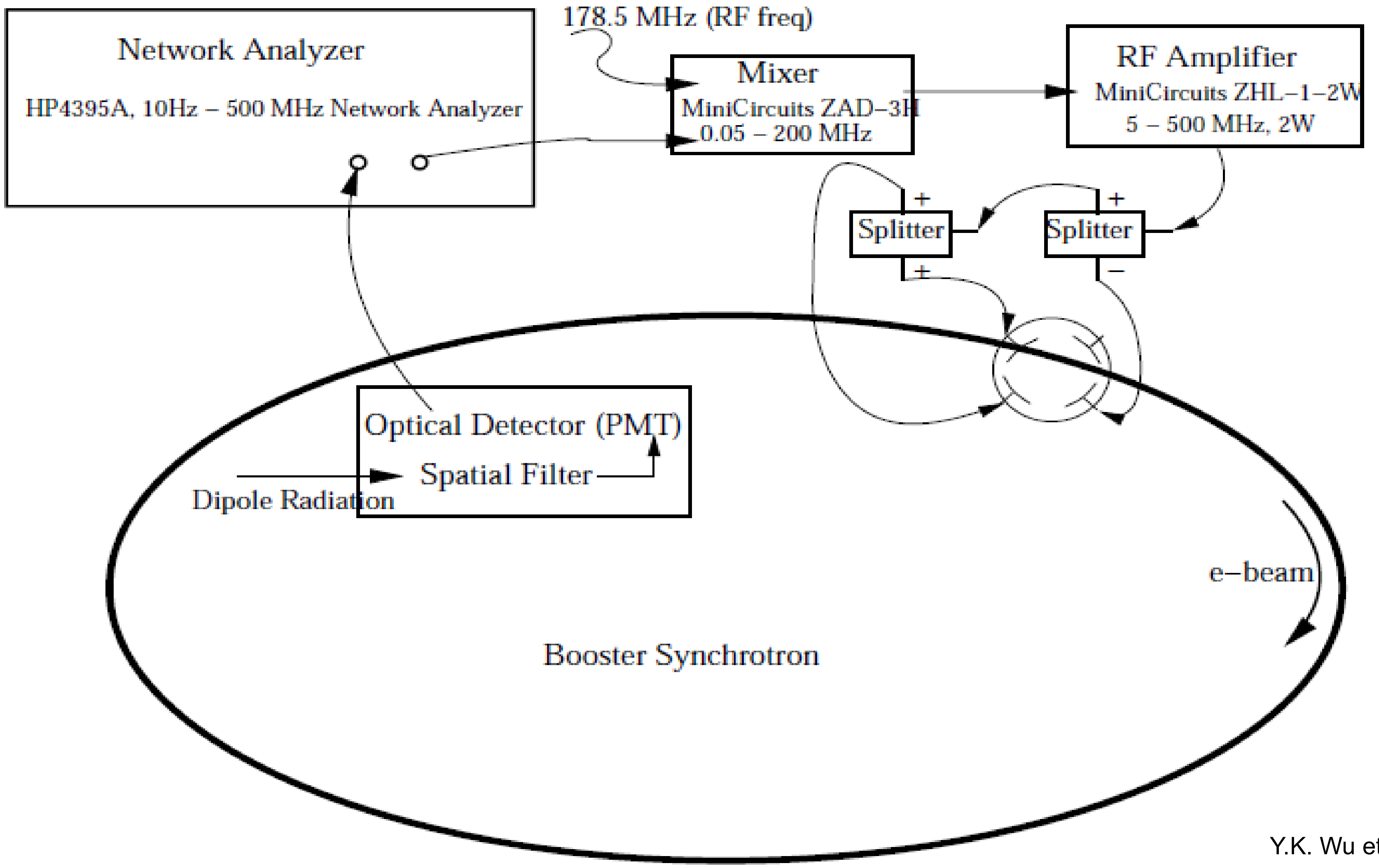


$$n\frac{2\pi}{T}\pm\omega_0$$

Y.K. Wu et. al, PAC07, p.4063 (2007).

Tune Measurements (Betatron Tunes)

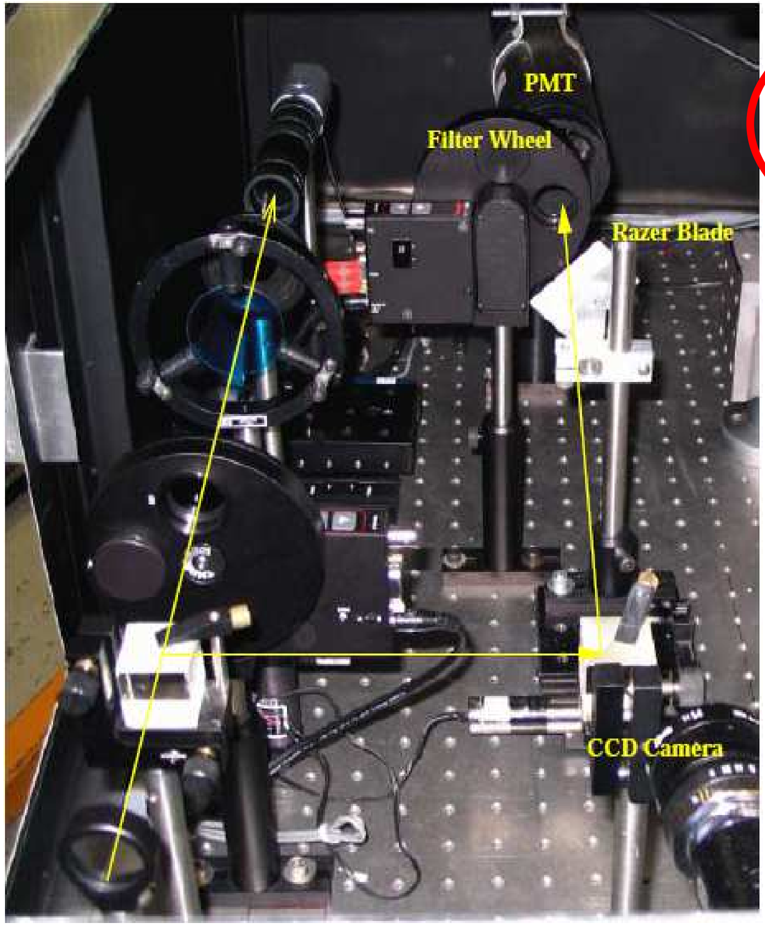
- Betatron tunes can be measured using a network analyzer which drives/excites the betatron motion and detects the beam response at a pickup

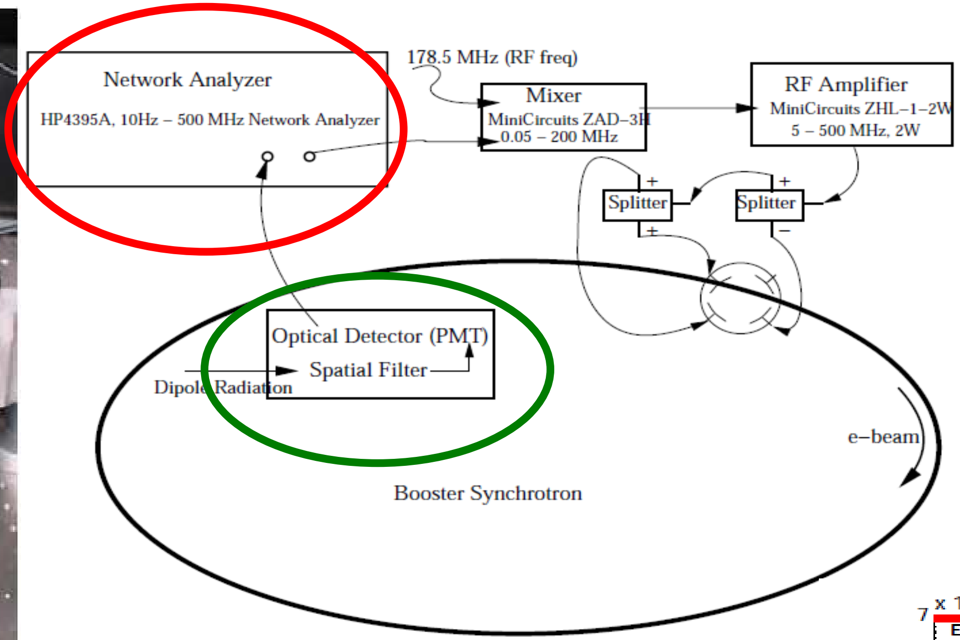


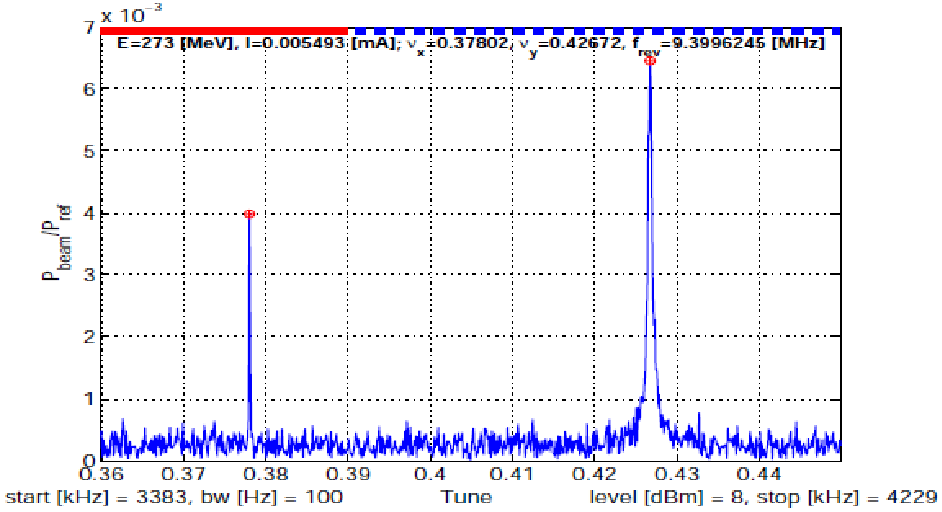
Y.K. Wu et. al, PAC07, p.4063 (2007).

Tune Measurements (Betatron Tunes)

Tune Measurement System for
Duke Booster Injector





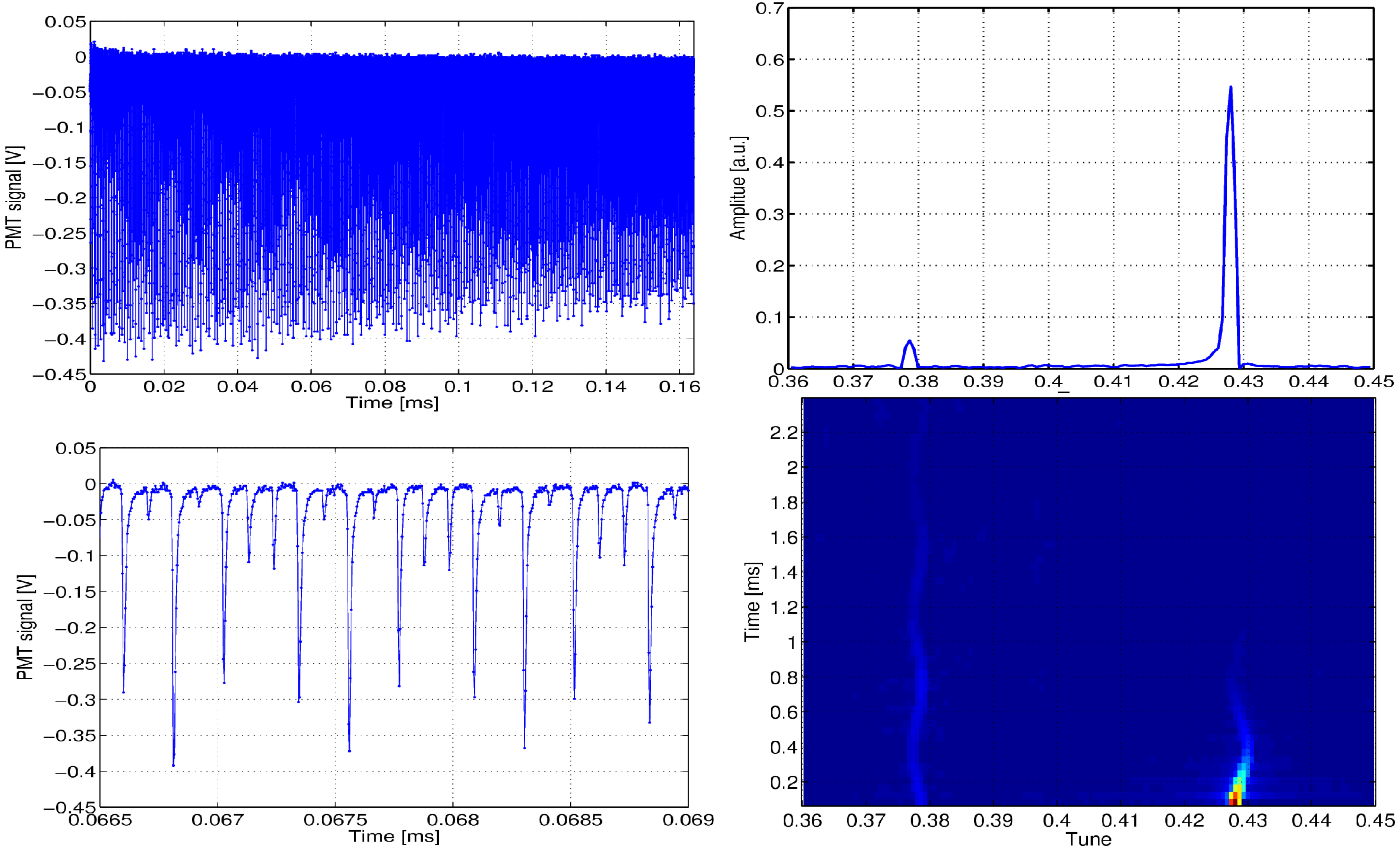


Only fractional tune can be measured

Y.K. Wu et. al, PAC07, p.4063 (2007).

Tune Measurements

- Betatron tunes can also be measured using turn-by-turn data



Duke Booster Synchrotron: 273 MeV, Vertical Kicker 1 kV, 250 MHz Sampling, PMT signal

Tune and Chromaticity Measurements

- Betatron tune: $$\nu_{x,y} = \frac{f_{x,y}}{f_0}$$
- Design tune values for Duke storage ring: $\nu_x = 9.11, \nu_y = 4.18$
- Chromaticity: $$\xi_{x,y} = \frac{d\nu_{x,y}}{d\delta}$$
- Fine-adjustment of electron beam energy in the storage ring:

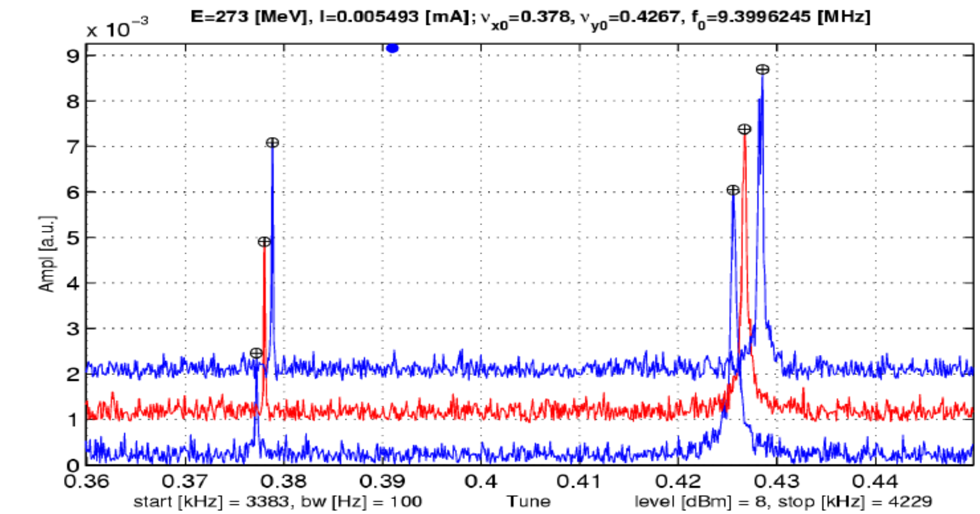
$$\frac{\Delta C}{C} \approx -\frac{\Delta f}{f} \approx \alpha_c \underbrace{\frac{\Delta E}{E}}_{\delta}$$

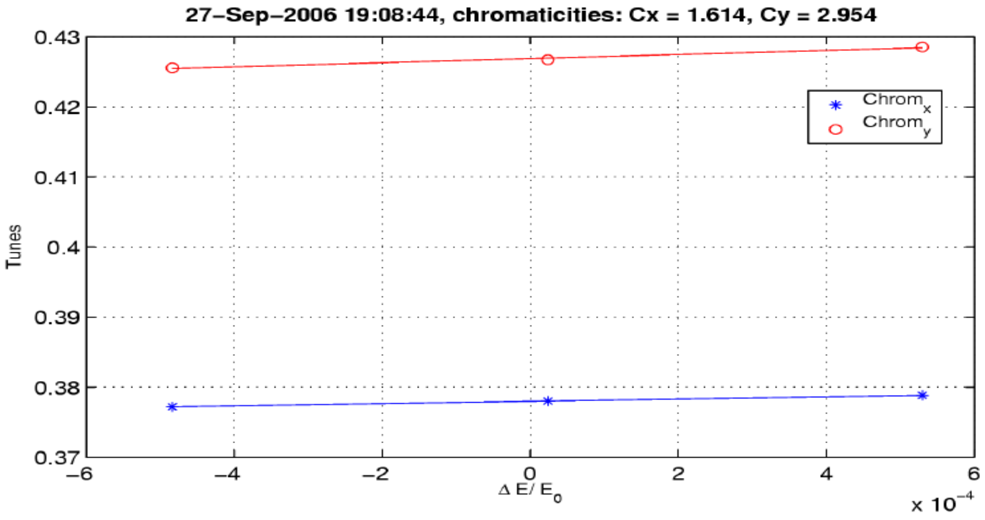
momentum compaction factor $\alpha_c = 0.0086$

ChromatICITY Measurements

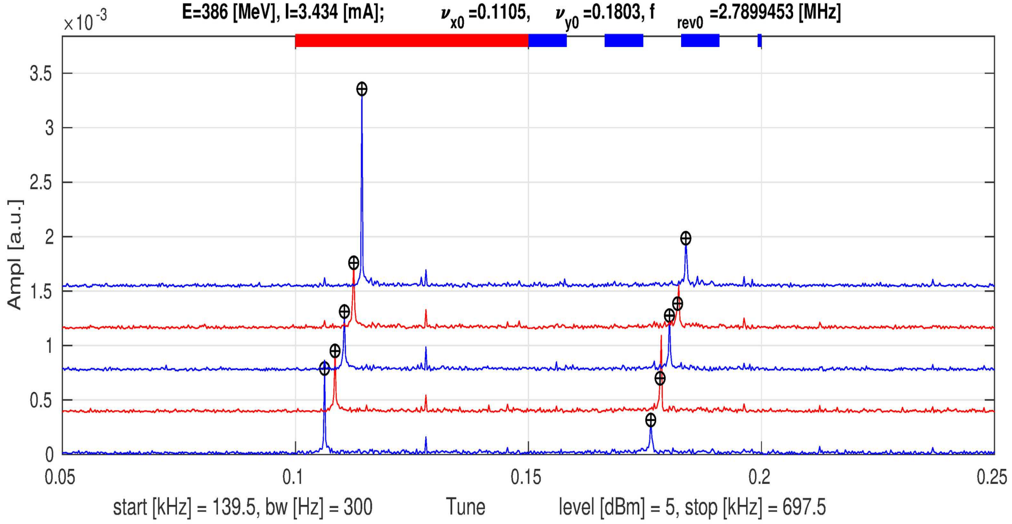
- Chromaticity is determined by measuring betatron tunes as a function beam energy

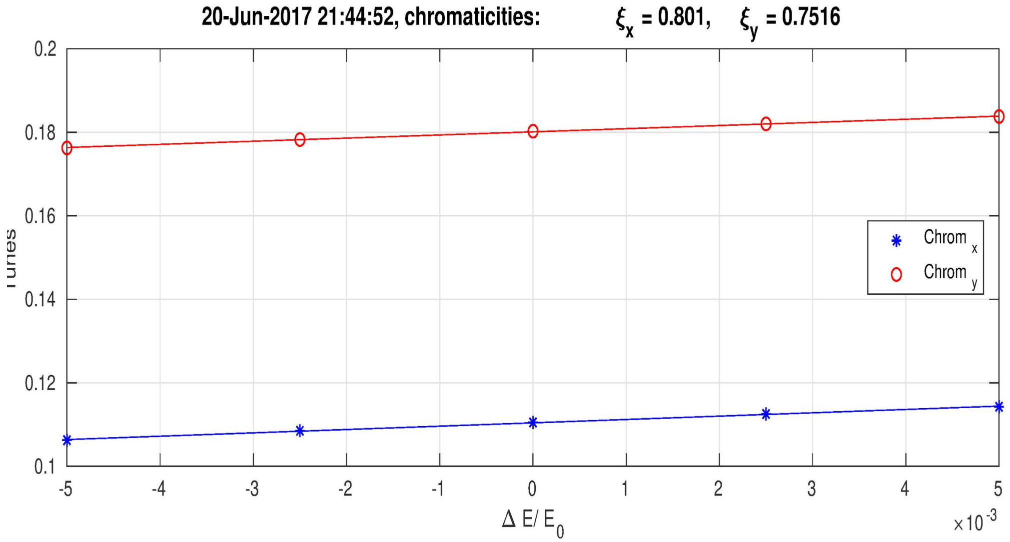
Duke Booster Synchrotron





Duke Storage Ring





Lab 5: Beam Orbit and Eta Function Measurements

- The image charge in a short electrode is proportional to the beam current and the opening angle to the electrode
- Consider a small displacement in y-direction

$$V_{\mathrm{A}} = GI\alpha, \quad V_{\mathrm{B}} = GI\beta$$

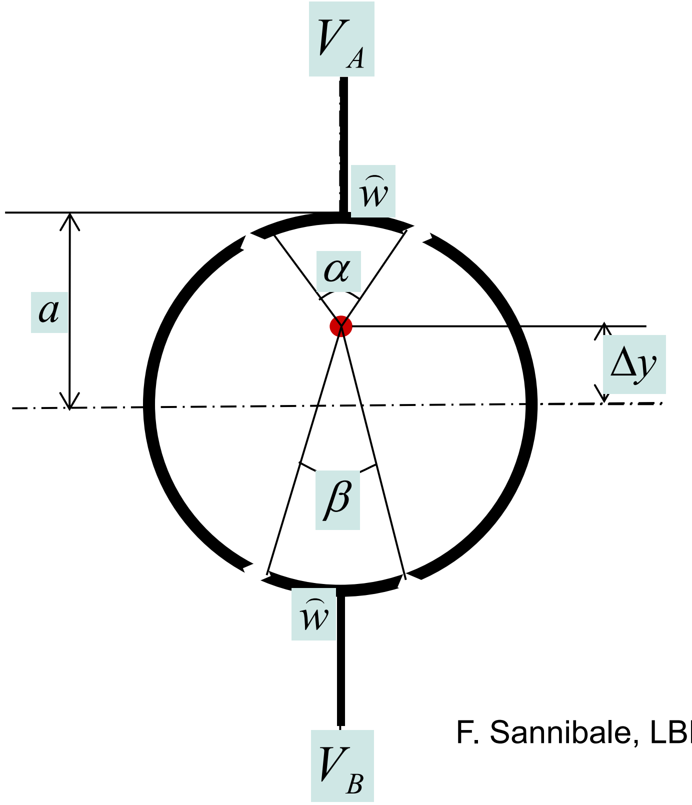
$$w = (a - \Delta y)\,\alpha = (a + \Delta y)\,\beta$$

Eliminate $\alpha, \ \beta$

$$V_{\mathrm{A}} = \frac{GIw}{a - \Delta y}, \quad V_{\mathrm{B}} = \frac{GIw}{a + \Delta y}$$

Eliminate GIw

$$\boxed{\frac{V_{\mathrm{A}} - V_{\mathrm{B}}}{V_{\mathrm{A}} + V_{\mathrm{B}}} = \frac{\Delta y}{a}}$$



F. Sannibale, LBNL

Orbit

- Trajectory of a single electron is composed of 3 parts:

$$x = x_{\mathrm{c}} + x_\beta + x_\eta$$

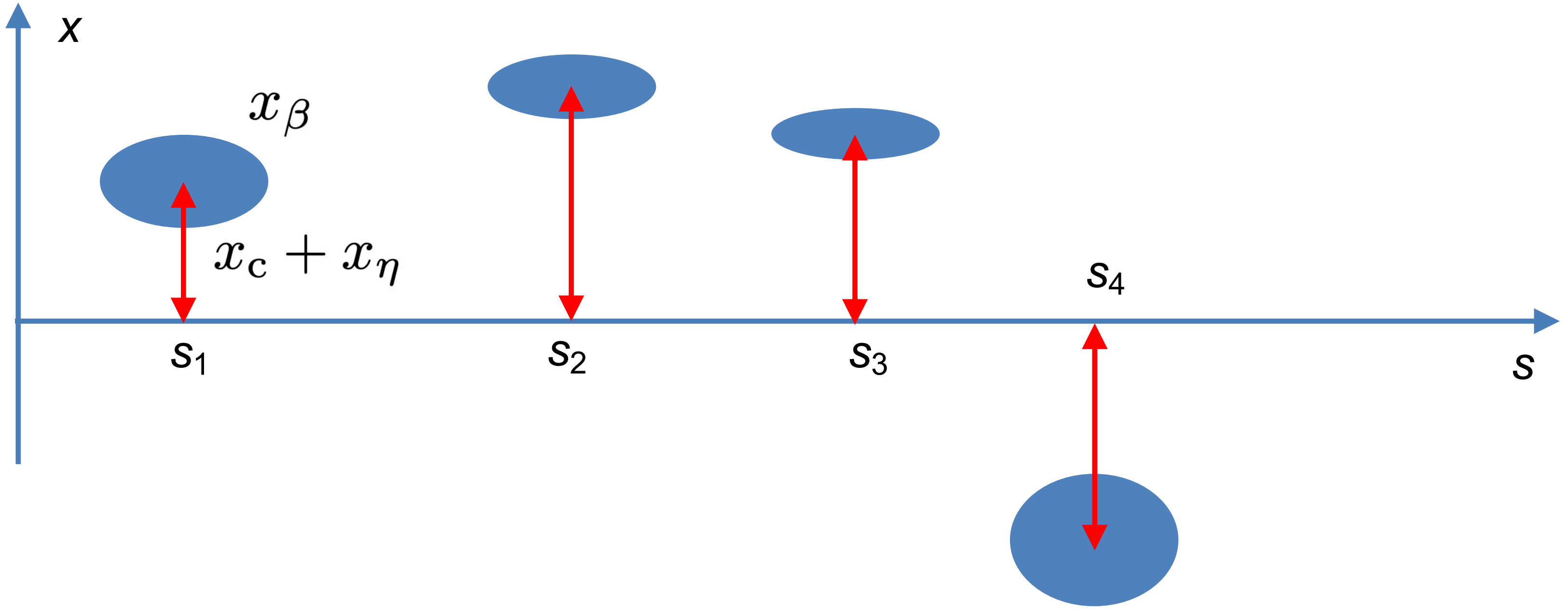
$x_{\mathrm{c}}(s) = \text{closed orbit}$ caused by storage ring imperfections, e.g. misaligned quadrupole

$x_\beta(s) = \sqrt{\epsilon\beta(s)}\cos\left(\int_0^s \frac{\mathrm{d}s'}{\beta(s')} + \phi_0\right)$ betatron oscillation

$x_\eta(s) = \eta(s)\,\delta$ caused by longitudinal-transverse “cross-talk”

cannot be measured by BPM

measurable



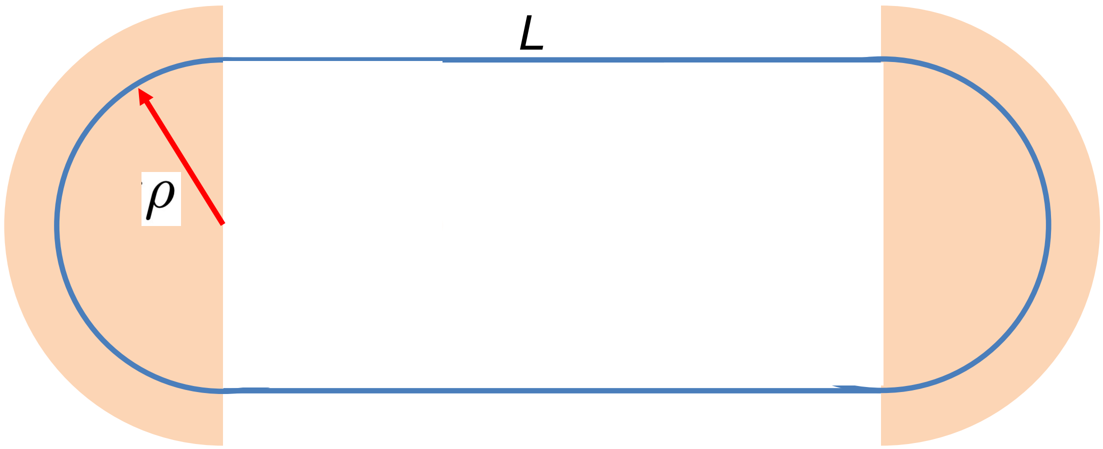
Closed Orbit

- If there is a kick to the beam, there is an orbit distortion:

$$\Delta x = \frac{\sqrt{\beta_1 \beta_2}}{2 \sin(\mu/2)} \cos\left(\Delta\phi - \frac{\mu}{2}\right) \Delta\theta$$

- This kicks are unwanted and unknown, but unavoidable:
  - magnet error
  - magnet power supply stability
  - temperature
  - ground vibration
  - …
- How to correct the orbit in the blind? We will talk about it in Experiment #6: Response matrix measurement.

Eta Function Measurement



$$C = 2\pi\rho + 2L = 2\pi\frac{E}{0.3B} + 2L$$

$$C = cT = \frac{c}{f}$$

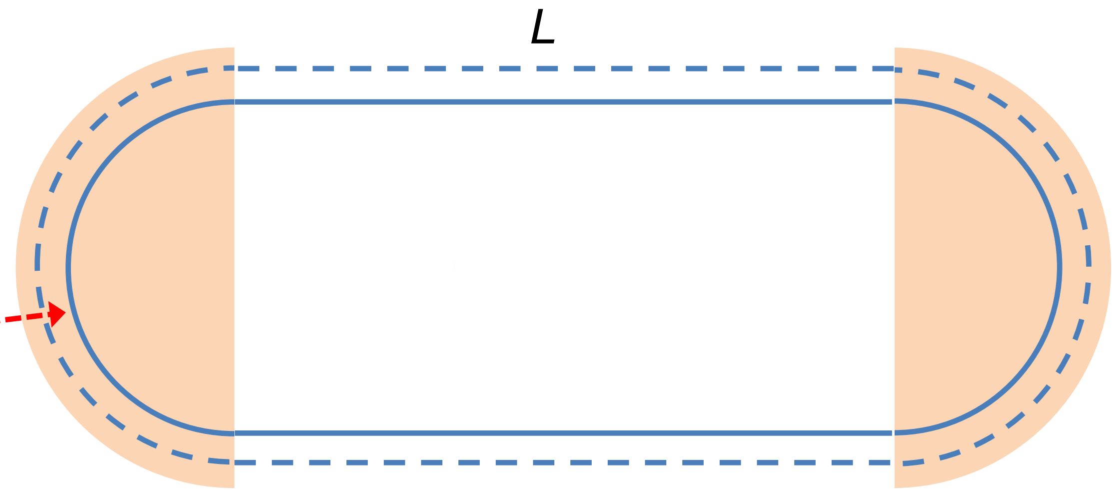
$$2\pi\frac{E}{0.3B} + 2L = \frac{c}{f}$$

I. $$2\pi\frac{E\uparrow}{0.3B\uparrow} + 2L = \frac{c}{f}$$

ramping: all the magnets are changed accordingly, dipoles, quadrupoles, sextupoles, correctors, and more.

II. $$2\pi\frac{E\uparrow}{0.3B} + 2L = \frac{c}{f\downarrow}$$

$$\eta\frac{\Delta E}{E} = \Delta x$$



Eta Function Measurement

- eta function describes the correlation between the electron energy deviation and its transverse offset

$$x_\eta = \eta \frac{\Delta E}{E}$$

measurable quantity

- Change the electron beam energy:

$$\frac{\Delta C}{C} = \alpha_c \frac{\Delta E}{E} \qquad \frac{\Delta C}{C} = -\frac{\Delta f_{\mathrm{rev}}}{f_{\mathrm{rev}}}$$

$$\frac{\Delta E}{E} = -\frac{1}{\alpha_c}\frac{\Delta f_{\mathrm{rev}}}{f_{\mathrm{rev}}}$$

- The electron beam will find the new stationary orbit by itself. On the new orbit, the overall bending angle is always 360 degree after one turn.

$$x_\eta = \eta \frac{\Delta E}{E} = -\frac{\eta}{\alpha_c}\frac{\Delta f_{\mathrm{rev}}}{f_{\mathrm{rev}}} \longrightarrow \boxed{\eta = -\alpha_c \frac{f_{\mathrm{rev}}}{\Delta f_{\mathrm{rev}}} \Delta x}$$

Beam Orbit and Eta Function Measurements

- At Duke Storage Ring:

$$\boxed{\begin{aligned} \alpha_c &= 0.0086 \\ \eta_{\max} &= 24.5\ \text{cm} \\ f_{\text{rev}} &= 2.79\ \text{MHz} \end{aligned}}$$

- For example, to create a maximum 0.1 mm orbit change, we need

$$\begin{aligned} \Delta f_{\text{rev}} &= \frac{\alpha_c}{\eta} f_{\text{rev}} \times \Delta x \\ &= \frac{0.0086}{0.245} \times 2.79 \times 10^6 \times 10^{-4} \\ &= 9.8\ \text{Hz} \end{aligned}$$

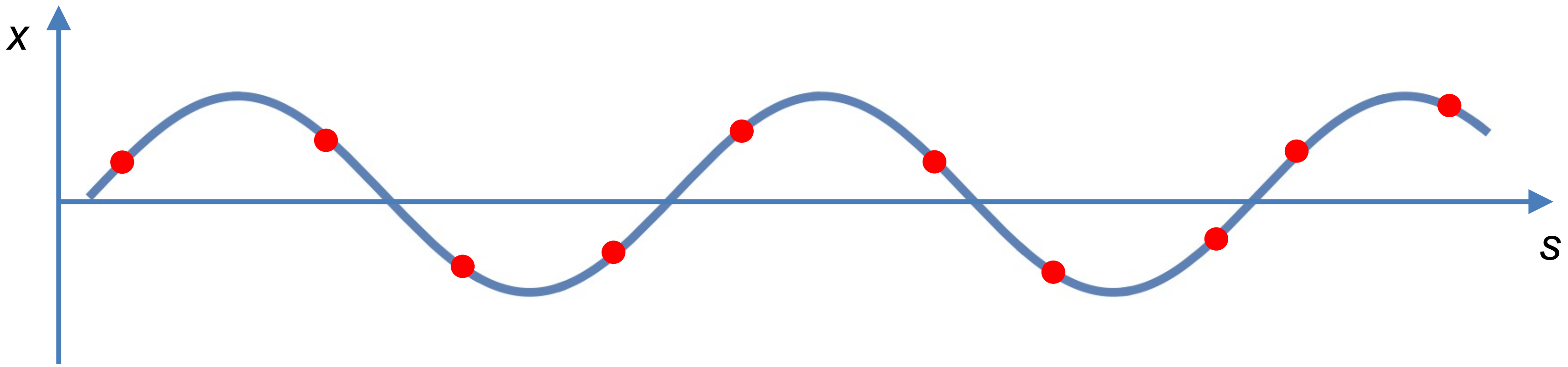
within capability of RF cavity tuning range
(a few hundred Hz)

Measure the Orbit

- Closed orbit wiggles around the storage ring:

$$x(s) = \sqrt{A\beta(s)}\cos\phi(s)$$

- Number of “wiggles” of the beam orbit = betatron tune
- We should have enough BPMs to obtain the orbit information:



- Nyquist frequency: the sampling rate should be twice of the frequency.
- In practice, we need 4 BPMs per 2-pi period. At Duke Storage Ring, 34 BPMs vs betatron tunes (9.11, 4.18).

Reference Orbit

- Where is the origin of the coordinate system?
  - Ground
  - Vacuum chamber center
  - BPM center
  - Dipole center
  - Quadrupole center
  - Other places
- At the quadrupole **magnetic** center

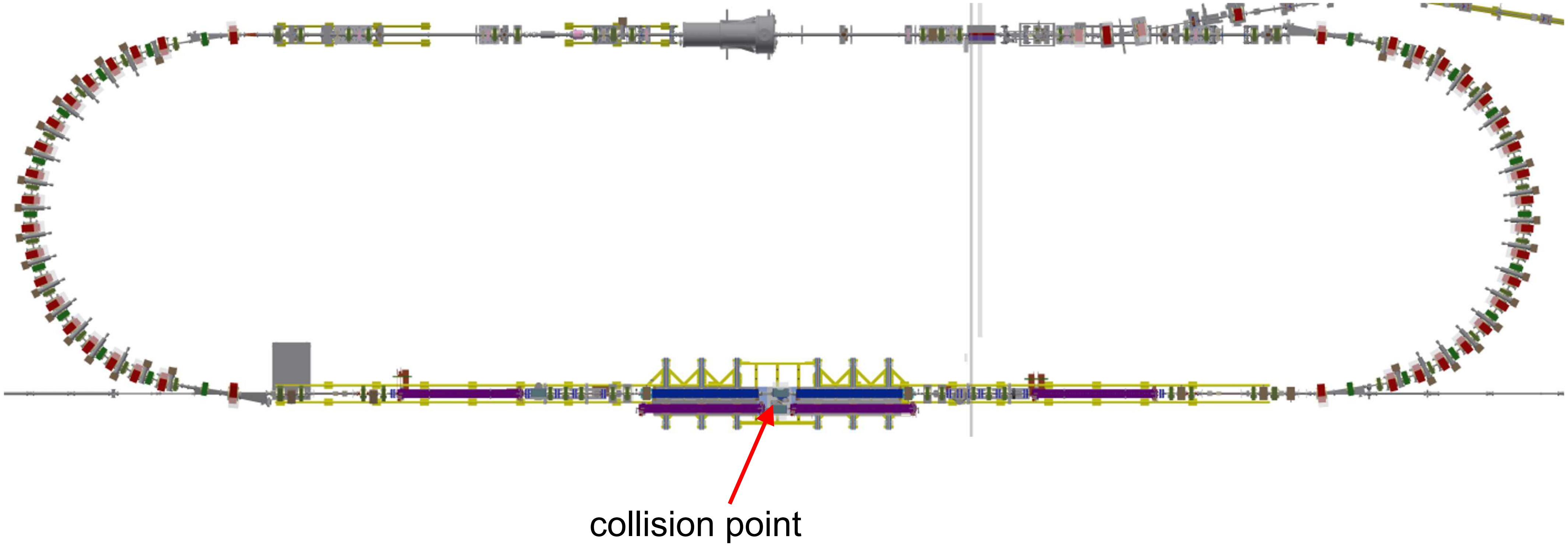
$$B_y = kx = 0\ \mathrm{T}$$
$$B_x = ky = 0\ \mathrm{T}$$

No kick to the particle.

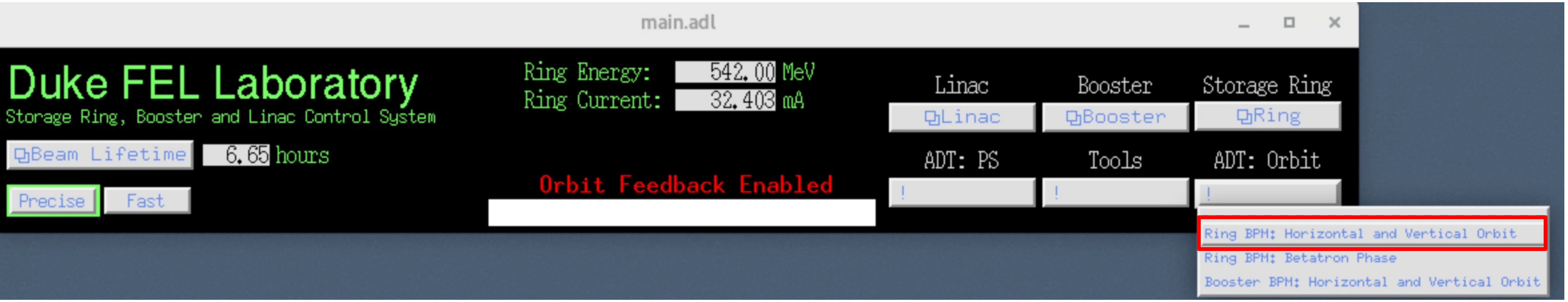
- And there is a way to find this center using beam-based method. This orbit, called “golden orbit”, could be the reference orbit for the accelerator.

Duke Storage Ring

- 34 BPMs
- $\nu_x = 9.11, \quad \nu_y = 4.18$
- Momentum compaction factor: $\boxed{\alpha_c = 0.0086}$



Beam Orbit and Eta Function Measurements



Matlab APIs

- `getx, gety` returns an array that contains all BPM horizontal/vertical readings with respect to golden orbit
- `RNG:RF:REVOL:FREQ:READ` EPICS PV of electron beam revolution frequency readback

Lab 6: Response Matrix Measurement

- The response matrix connects the correctors and the beam orbit:

$$\begin{pmatrix} \Delta x_1 \\ \Delta x_2 \\ \cdots \\ \Delta x_m \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ S_{m1} & S_{m2} & \cdots & S_{mn} \end{pmatrix} \begin{pmatrix} \Delta \theta_1 \\ \Delta \theta_2 \\ \cdots \\ \Delta \theta_n \end{pmatrix}$$

- In theory:

$$S_{ij} = \frac{\sqrt{\beta_i \beta_j}}{2 \sin(\pi \nu_x)} \cos(\phi_i - \phi_j - \pi \nu_x)$$

difficult to measure

Measure the Response Matrix

- Change one corrector:

$$\begin{pmatrix} \Delta\theta_1 \\ \Delta\theta_2 \\ \cdots \\ \Delta\theta_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \Delta\theta_j \\ 0 \end{pmatrix}$$

- Measure orbit variation:

$$\begin{pmatrix} \Delta x_1 \\ \Delta x_2 \\ \cdots \\ \Delta x_m \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ S_{m1} & S_{m2} & \cdots & S_{mn} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \Delta\theta_j \\ 0 \end{pmatrix} = \begin{pmatrix} S_{1j} \\ S_{2j} \\ \cdots \\ S_{mj} \end{pmatrix} \Delta\theta_j$$

$$\boxed{S_{ij} = \frac{\Delta x_i}{\Delta\theta_j}}$$

Corrector Setup

- Corrector strength needed:

~ 4 m

[-1, +1]

$$\Delta x_i = \frac{\sqrt{\beta_i \beta_j}}{2\sin(\pi\nu_x)} \cos(\phi_i - \phi_j - \pi\nu_x)\,\Delta\theta_j \quad \Rightarrow \quad \Delta\theta_j \sim 0.2 \text{ mrad}$$

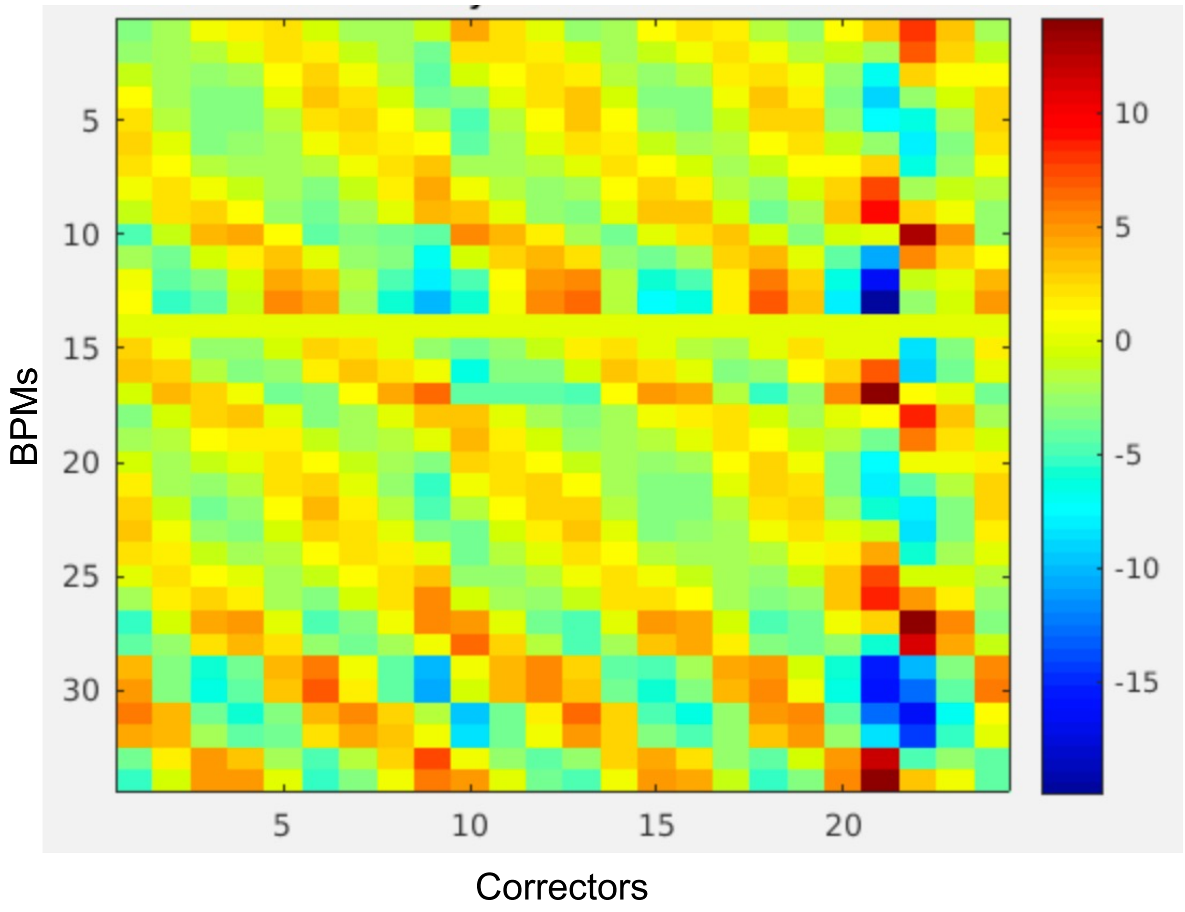
0.5 mm

$$\sin(\pi \times 9.11) = -0.34$$
$$\sin(\pi \times 4.18) = 0.54$$

Response Matrix

$$\nu_y = 4.18$$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \\ y_{16} \\ y_{17} \\ y_{18} \\ y_{19} \\ y_{20} \\ y_{21} \\ y_{22} \\ y_{23} \\ y_{24} \\ y_{25} \\ y_{26} \\ y_{27} \\ y_{28} \\ y_{29} \\ y_{30} \\ y_{31} \\ y_{32} \\ y_{33} \\ y_{34} \end{pmatrix} =$$



$$\begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \\ \theta_7 \\ \theta_8 \\ \theta_9 \\ \theta_{10} \\ \theta_{11} \\ \theta_{12} \\ \theta_{13} \\ \theta_{14} \\ \theta_{15} \\ \theta_{16} \\ \theta_{17} \\ \theta_{18} \\ \theta_{19} \\ \theta_{20} \\ \theta_{21} \\ \theta_{22} \\ \theta_{23} \\ \theta_{24} \end{pmatrix}$$

Application of Response Matrix

- The errors are unknown and distributed around the storage ring. We can use a set of correctors which are evenly distributed to correct these unknown errors.
- We set a goal for the orbit, e.g. the golden orbit, or a well-established operational orbit, $\boldsymbol{X}_0$, and measure the real orbit $\boldsymbol{X}$. Then we can apply the correctors so that the orbit deviation can be eliminated:

$$\Delta\boldsymbol{\Theta} = -S^{-1}(\boldsymbol{X} - \boldsymbol{X}_0)$$

- Response matrix may not be a square matrix. We can use mathematical methods to obtain the pseudo-inverse of a matrix, e.g. SVD decomposition,

$$S = U\Sigma V^{\mathrm{T}}$$

$$S : m \times n$$

$$U : m \times m, UU^{\mathrm{T}} = I$$

$\Sigma : m \times n$ diagonal matrix

$$V : n \times n, VV^{\mathrm{T}} = I$$

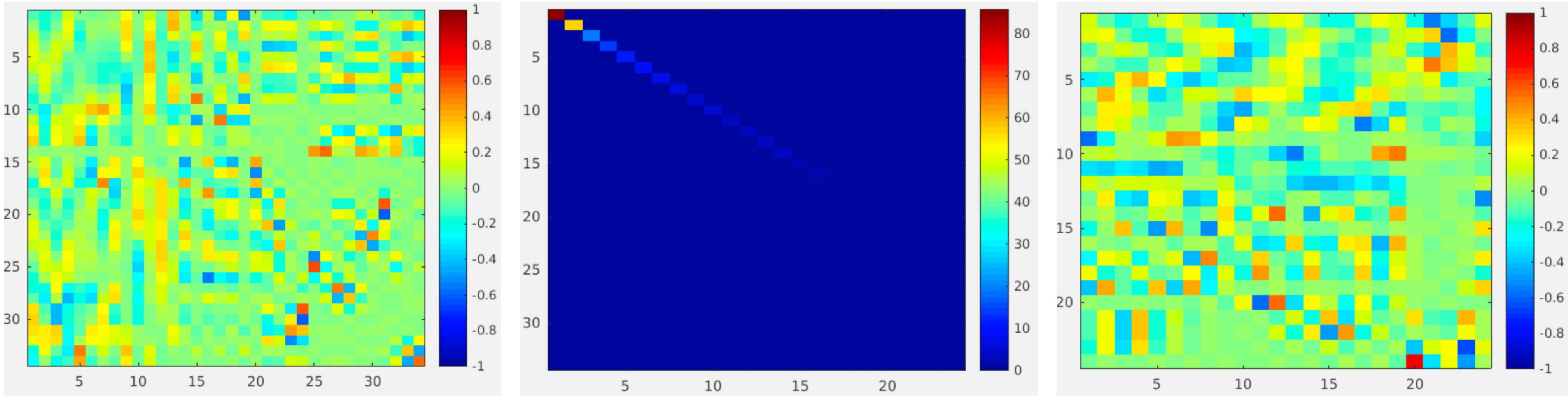
non-zero elements are inversed

$$S^{-1} = V\Sigma^{-1}U^{\mathrm{T}}$$

pseudo inverse

Response Matrix

$$
\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \\ y_{16} \\ y_{17} \\ y_{18} \\ y_{19} \\ y_{20} \\ y_{21} \\ y_{22} \\ y_{23} \\ y_{24} \\ y_{25} \\ y_{26} \\ y_{27} \\ y_{28} \\ y_{29} \\ y_{30} \\ y_{31} \\ y_{32} \\ y_{33} \\ y_{34} \end{pmatrix} =
$$

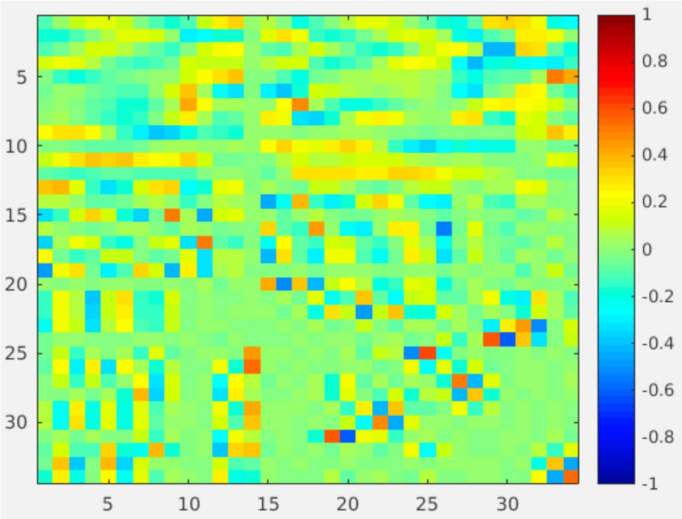


$$
\begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \\ \theta_7 \\ \theta_8 \\ \theta_9 \\ \theta_{10} \\ \theta_{11} \\ \theta_{12} \\ \theta_{13} \\ \theta_{14} \\ \theta_{15} \\ \theta_{16} \\ \theta_{17} \\ \theta_{18} \\ \theta_{19} \\ \theta_{20} \\ \theta_{21} \\ \theta_{22} \\ \theta_{23} \\ \theta_{24} \end{pmatrix}
$$

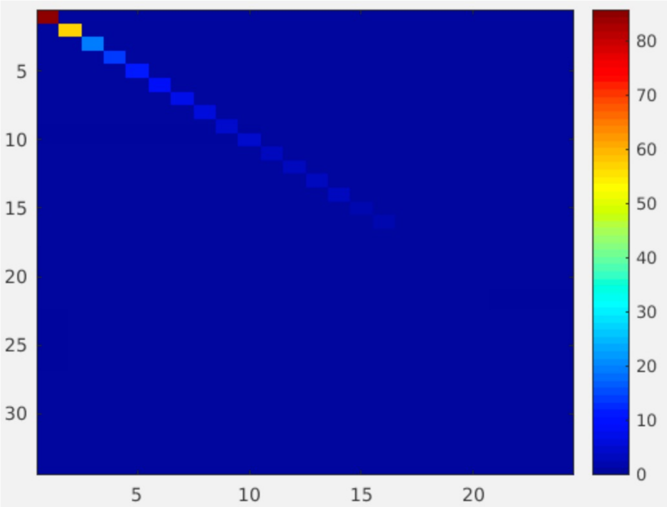
34 equations, 24 unknown variables → an over-constrained problem

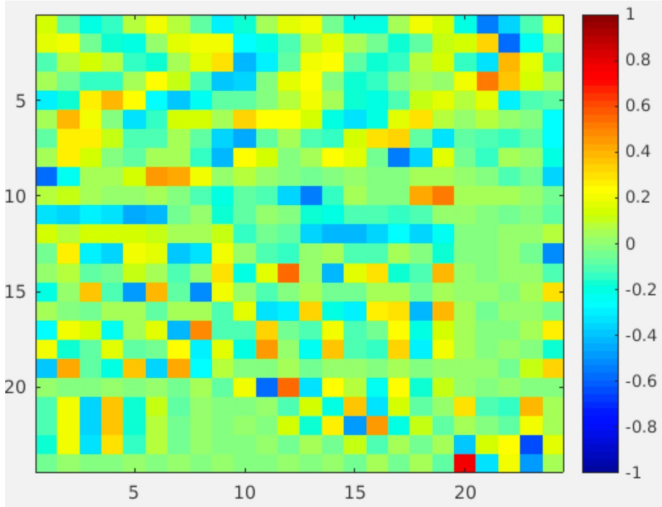
Response Matrix

$$\left(U^{\mathrm{T}}\boldsymbol{X}\right) = \Sigma\left(V^{\mathrm{T}}\boldsymbol{\Theta}\right)$$



$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \\ y_{16} \\ y_{17} \\ y_{18} \\ y_{19} \\ y_{20} \\ y_{21} \\ y_{22} \\ y_{23} \\ y_{24} \\ y_{25} \\ y_{26} \\ y_{27} \\ y_{28} \\ y_{29} \\ y_{30} \\ y_{31} \\ y_{32} \\ y_{33} \\ y_{34} \end{pmatrix} =$$





$$\begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \\ \theta_7 \\ \theta_8 \\ \theta_9 \\ \theta_{10} \\ \theta_{11} \\ \theta_{12} \\ \theta_{13} \\ \theta_{14} \\ \theta_{15} \\ \theta_{16} \\ \theta_{17} \\ \theta_{18} \\ \theta_{19} \\ \theta_{20} \\ \theta_{21} \\ \theta_{22} \\ \theta_{23} \\ \theta_{24} \end{pmatrix}$$

Measure the Orbit

- **getx, gety** returns the horizontal and vertical BPM readings as an array
- **lcaramp(pvName, value, steps)** ramp the PV **to** the value in many steps, the MATAB pauses 0.01 second after each step.
- For example, **lcaramp('RNG:ORB:E09QD:Y:TUNE', 0.1, 100)** ramps the corrector E09QD:Y strength to 0.1 mrad in 100 steps (which takes about 1 second).
- **ycorrectors.m** the vertical corrector PV names, unit is mrad