Proton and Ion Linear Accelerators

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RF accelerating structures, Tasks for Homework

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Homework

Task 1. SRF 5-cell cavity designed for PIP II has the following parameters:

- Operating frequency is 650 MHz
- Acceleration voltage V is 20 MV
- R/Q is 620 Ohm
- Q_0 at operation voltage is 3e10
- The beam current I is 2 mA

Estimate for CW operation:

- The cavity loaded Q,
- The cavity time constant
- The cavity bandwidth
- Loss power in the cavity walls;
- The power transferred to the beam;
- Power required for refrigeration (take COP= 1.e3 W/W),
  i.e., in order to remove 1 W from the cavity wall one needs wall plug power of 1 kW);
- Acceleration efficiency, the beam power over the sum of the power delivered by the RF source (which is equal in this case to the beam power, because power dissipated in the cavity is small) and power required for refrigeration.

Homework

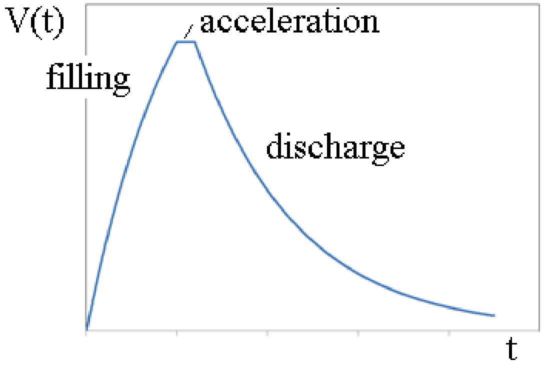
Task 2.

PIP II SRF accelerator has CW capability but will operate in the pulsed mode as an injector to the booster ring. The beam and cavity parameters are the same as for CW, Task 1. The beam pulse t_{beam} is 0.55 msec, repletion rate is 20 Hz. The beam appears when the cavity voltage reaches the operating value V, and backward wave (from the cavity to the RF source) is zero. Note that this wave is a sum of the reflection from the coupling element (which is equal to the incident wave), and the wave radiated from the cavity to the line. In the beginning of the cavity filling, the radiation is zero (the cavity is empty), and the backward wave is equal to reflection from the coupling element, and thus, to the incident wave. If there is no beam, the backward wave is again equal to the incident wave (no losses in the cavity) after the voltage reaches its maximal value, but it is again the sum of the wave reflected from the coupling element and radiated wave. It can be only if the radiated wave is two times larger than the wave reflected from the coupling element and has opposite sign. It means that the beam appears when the cavity field reaches half of the maximal value (zero backward wave, the reflected wave is equal to the radiated wave, and they compensate each other). The cavity voltage, thus, increases during the filling as $V(t) = 2V(1-exp(-t/\tau))$, τ is the time constant, $\tau = 2Q_L/\omega$. Filling is over when $V(t_{fill}) = V$, and therefore, the filling time t_{fill} is equal to $\tau \cdot ln2$. After the beam ends, the RF source is turned off, and cavity discharges as $V(t) = Vexp(-t/\tau)$.

Homework

Therefore, the cavity voltage has the following behavior:

- $V(t) = 2V(1-\exp(-t/\tau)), t < t_{fill} = \tau \cdot \ln 2$ - filling; RF is on, no beam;
- $V(t) = V$, the beam acceleration; RF is on; $0 < t < t_{beam}$, $t=0$ corresponds to the end of filling process
- $V(t) = V\exp(-t/\tau)$, cavity discharge; RF is off, no beam. $t=0$ corresponds to the end of acceleration



Estimate:

- Energy, delivered by the RF source to the beam during the pulse;
- Total energy, delivered by the RF source during the pulse;
- Total energy dissipated in the cavity wall during the pulse;
- Energy, required for refrigeration;
- Beam power /cavity (20 Hz repetition rate);
- Average RF power/cavity;
- Power necessary for refrigeration/cavity;
- Acceleration efficiency, the beam power over the sum of the power delivered by the RF source and power required for refrigeration.

Homework

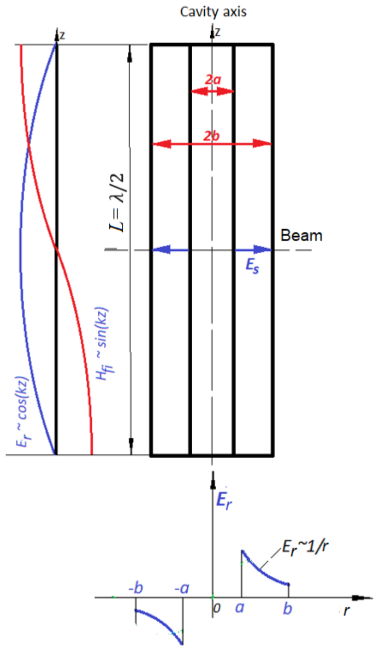
Task 3

The ideal HWR cavity for β = 0.112 operates at frequency f = 162.5 MHz. It provides the energy gain (acceleration voltage) V = 2 MeV. The inner radius is a, the outer radius is b.

Estimate:

- The cavity length L;
- Effective cavity length;
- Acceleration gradient
- The optimal ratio b/a to achieve minimal surface electric field (on the inner electrode) at fixed b, taking into account that in a coaxial line $E_r \sim 1/r$;
- b and a to get maximal energy gain;
- The voltage difference U between the inner and outer electrodes at z=0.
- The coaxial impedance Z_c.
- The energy stored in the cavity;
- R/Q;
- The Ohmic loss in the cavity taking R_s=3 nOhm.
- Unloaded quality factor Q_0 and G-factor
- Maximal surface electric and magnetic fields and field enhancement factors K_B and K_E.

Use online Si calculator (for example, https://www.wolframalpha.com/input/?i=Si%28x%29)



Homework, bonus problem

Calculate the maximum temperature rise δT and maximum surface magnetic field H_b in the cavity, when cavity has thermal breakdown. For estimations take the following parameters:

- *Operating frequency:* $f=3.9$ *GHz*
- *Helium temperature:* $T_0=2^oK$
- *Niobium thickness:* $d=3$ *mm*
- *Thermal conductivity:* $k(T_0)=30\ W/(m\bullet K)$
- *Kapitza resistance:* $h(T_0)=10^4\ W/(m^2\bullet K)$
- *BCS surface resistance for Nb :*

$$R_s(T)=R_0\cdot\left[\frac{f\,(GHz)}{1.3}\right]^2\cdot\left(\frac{T_c}{T}\right)\cdot e^{-\Delta\cdot\frac{T_c}{T}}\ ,\ T<<T_c$$

where: $R_0=10^{-5}\,[\Omega]$; $\Delta=1.8$; $T_c=9.2^oK$

Use assumption, that temperature rise is small compared to T_0 and $k(T_s)=k(T_m)=k(T_0)$.

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