VUV and X-ray Free-Electron Lasers

FEL Simulations with Genesis 1.3

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Introduction to FEL Simulations

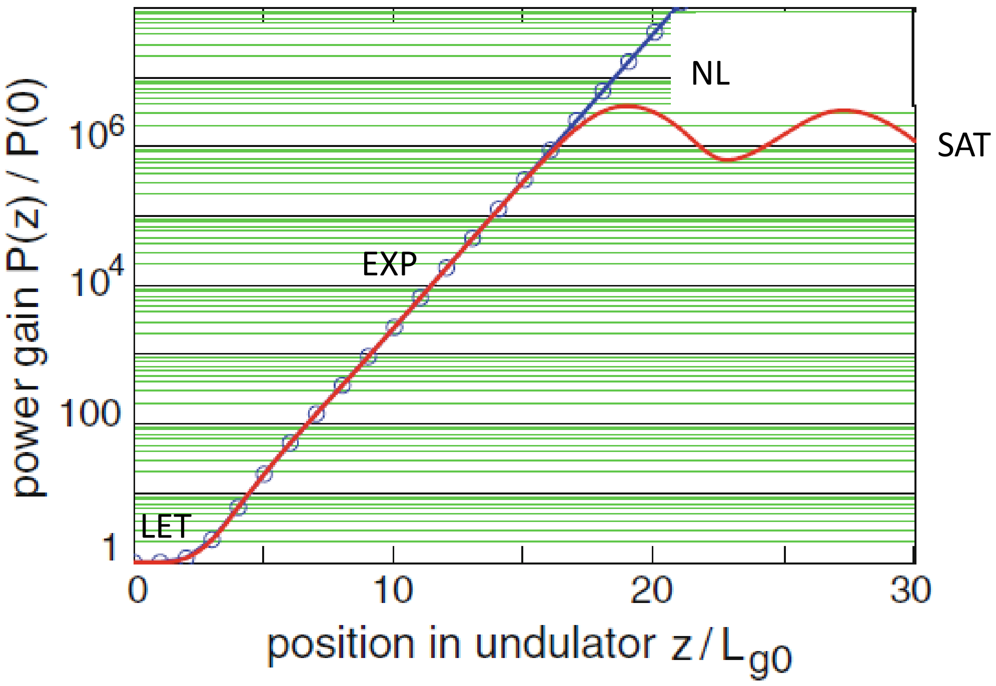
FEL Amplification in a Long Undulator

LET = Lethargy

FEL power grows very slowly as the three "modes" interfere with one another.

EXP = Exponential growth

The radiation power grows exponentially with z. On the semi-log plot of power vs z, the power growth curve is linear. This region is also known as the linear growth regime.



NL = Nonlinear regime

FEL power reaches a maximum as electrons are trapped and bunched inside the bucket

SAT = Saturation

FEL power saturates and oscillates as a function of z as particles rotate inside the bucket.

Dimensionless FEL ρ Parameter

The dimensionless FEL ρ parameter governs both the FEL gain and output power. There are a number of ways to write the expressions for ρ. Below is the correct expression.

$$\rho = \frac{1}{\gamma_r}\left(\frac{JJ\,K\,\lambda_u}{8\pi\sigma}\right)^{\frac{2}{3}}\left(\frac{I_p}{I_A}\right)^{\frac{1}{3}}$$

where
- γ_r resonant electron beam energy
- JJ difference in Bessel functions (see next slide)
- K undulator parameter
- λ_u undulator period
- σ rms electron beam radius
- I_p peak electron beam current
- I_A Alfvén current

$$I_A = 4\pi\varepsilon_o \frac{m_e c^3}{e}$$

$$I_A = 17\ kA$$

Bessel JJ Factor Explained

The figure-8 motion of the electrons in a planar undulator modulates the electrons' longitudinal velocity and reduces the electron-radiation wave interaction. The reduction is expressed in terms of the difference between the J_0 and J_1 Bessel functions of an argument ξ that depends on K. This reduction affects planar, but not helical, undulators.

For a planar undulator, JJ decreases to ~0.7 at very large K.

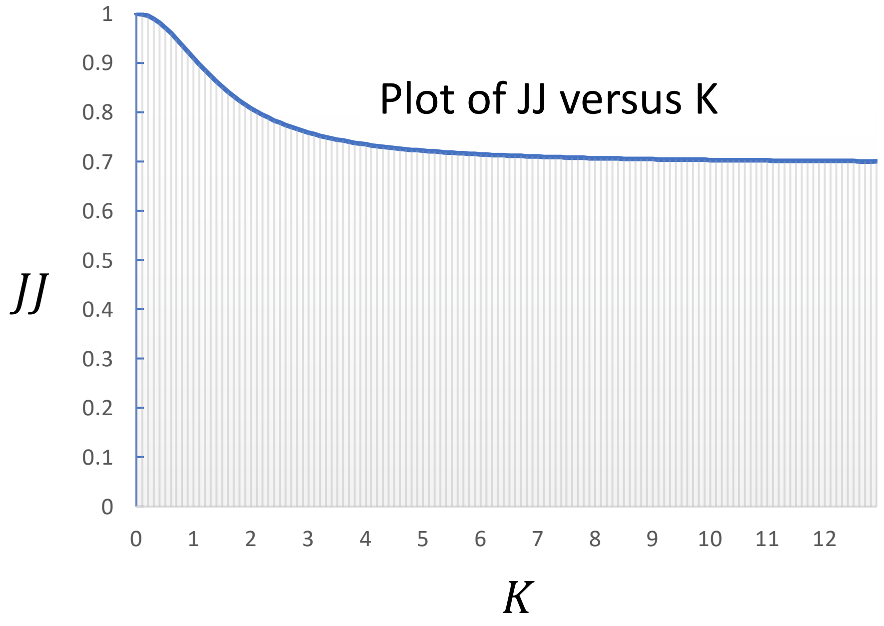
The textbook defines the modified undulator parameter, $\widehat{K}$ a product of K and the difference in Bessel functions. $\widehat{K}$ is to be used in calculations that involve the interaction strength. For wavelength calculations, one should use K.

$$\widehat{K} = K \cdot JJ$$

JJ is unity for helical undulators (no correction).

$$JJ(\xi) = J_0(\xi) - J_1(\xi)$$

$$\xi = \frac{K^2}{4 + 2K^2}$$



1D Gain Length in Exponential Regime

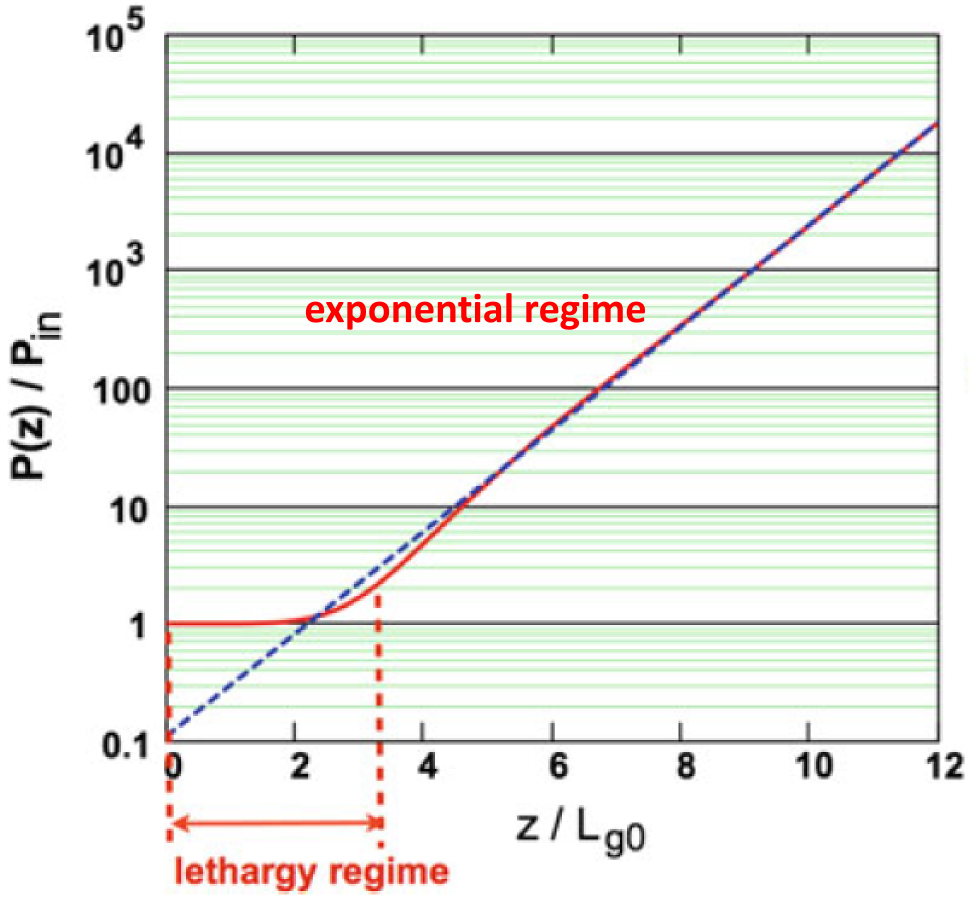
FEL power stays relatively constant in the lethargy regime and then grows exponentially with z in the exponential regime with a characteristic "power gain length," the length over which FEL power grows by one e-folding.

$$P(z) = \frac{P_0}{9} e^{\frac{z}{L_G}}$$

1D power gain length

$$L_{g0} = \frac{\lambda_u}{4\pi\sqrt{3}\rho}$$

FEL power saturates in about 20 power gain lengths.



FEL Variables

ψ_n : Phase of the n^{th} electron with respect to the FEL resonant radiation wave

η_n : Energy detuning of the n^{th} electron from the FEL resonant dimensionless energy

$$\eta_n = \frac{\gamma_n - \gamma_r}{\gamma_r}$$

γ_r : Resonant dimensionless energy

$$\gamma_r = \sqrt{\frac{k_r}{2k_u}\left[1 + \frac{K^2}{2}\right]} \qquad k_r = \frac{2\pi}{\lambda_r}$$

$\hat{K}$: Undulator parameter corrected for the difference in Bessel functions

Coupled FEL Equations

Evolution of the n^{th} electron phase

$$\frac{d\psi_n}{dz} = 2k_u\eta_n$$

Radiation field amplitude grows with the first harmonic current density

$$\frac{d\tilde{E}_x}{dz} = -\frac{\mu_0 c\hat{K}}{4\gamma_R}\tilde{j}_1$$

First harmonic current density

$$\tilde{j}_1 = j_0\frac{2\pi}{N}\sum_{n=1}^{N}\exp(-i\psi_n)$$

Evolution of the n^{th} electron energy deviation

$$\frac{d\eta_n}{dz} = -\frac{e}{m_0c^2\gamma_R}\text{Re}\left\{\left[\frac{\hat{K}\tilde{E}_x}{2\gamma_R} - \tilde{E}_z\right]\exp(i\psi_n)\right\}$$

Radiation-electron interaction

Electron-electron interaction (space charge)

Space charge effects are negligible for FELs operating in the Compton regime (e.g., X-ray FELs). Space charge cannot be ignored for FELs operating in the Raman regime (e.g., THz FELs).

Introducing normalized variables

Normalized undulator coordinate

$$\tau = 2k_u\rho z$$

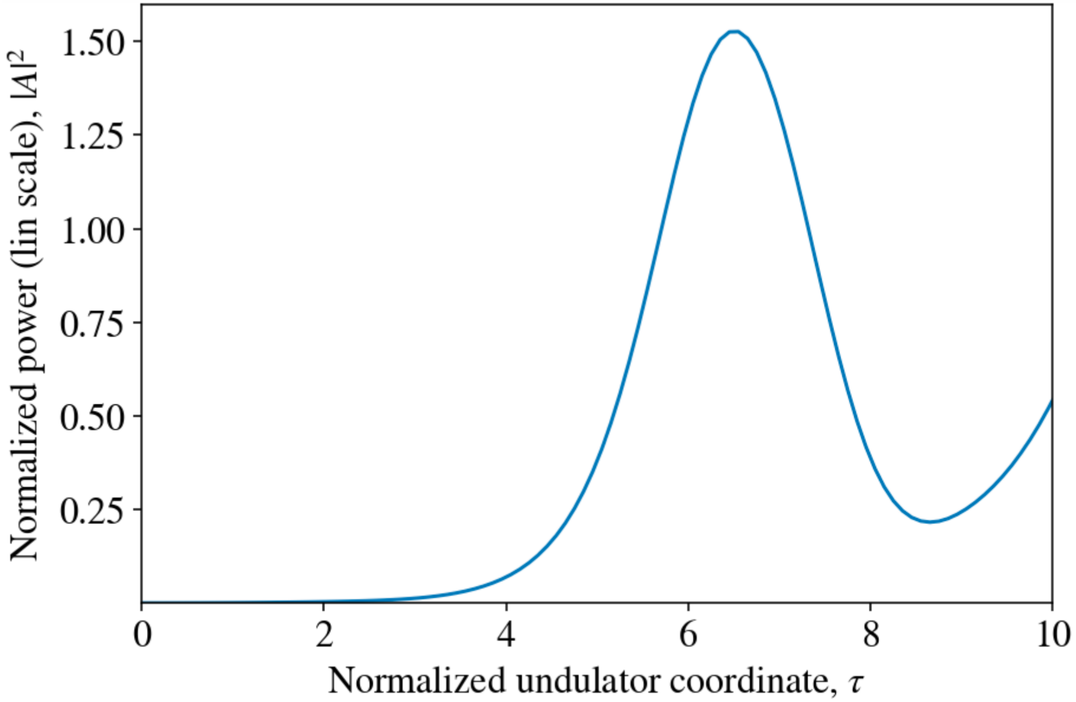
Normalized radiation field amplitude

$$A = \frac{E}{E_s}$$

Saturation electric field $$E_s = \sqrt{\frac{Z_o\rho P_b}{\pi\sigma_r^2}}$$

Normalized energy deviation from resonance

$$\bar{\eta}_n = \frac{\eta_n}{\rho}$$



Saturated normalized SASE power at zero initial energy detuning

$$|A|^2 \leq 1.5$$

1D FEL Equations

Normalized radiation field amplitude grows with electron microbunching

$$\frac{dA}{d\tau} = -\frac{1}{N}\sum_{n=1}^{N} \exp(-i\psi_n)$$

$$\frac{d\psi_n}{d\tau} = \eta_n$$

$$\frac{d\eta_n}{d\tau} = -2\mathrm{Re}\left(Ae^{i\psi_n}\right)$$

Electron microbunching grows with the normalized electron energy modulation

Energy modulation grows with radiation field amplitude correlated with electron phase

1D FEL Equations in Python

$$\frac{dA}{d\tau} = \frac{1}{N_e}\sum_{n=1}^{N_e} e^{-i\theta_n}$$

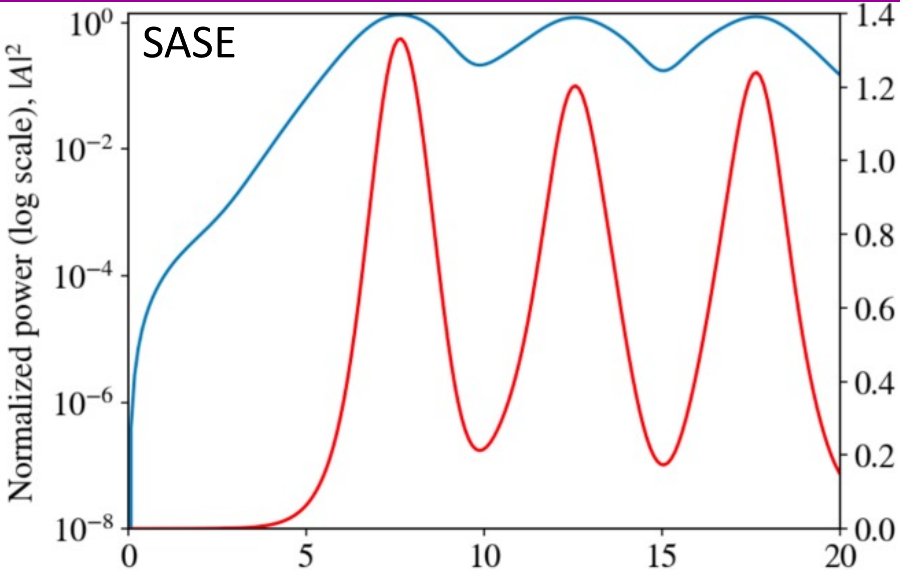
$$\frac{d\theta_n}{d\tau} = \eta_n$$

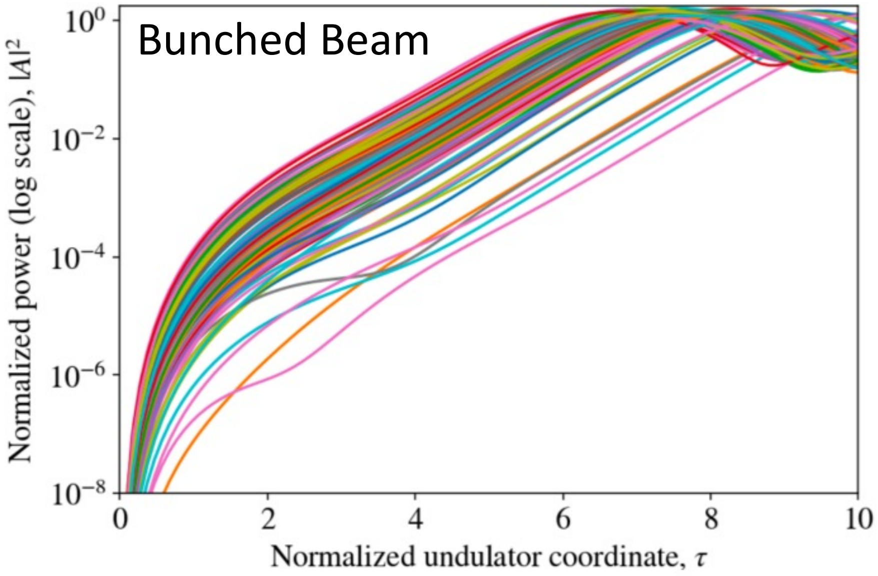
$$\frac{d\eta_n}{d\tau} = -2\,Re\left[A(\tau)e^{i\theta_n}\right]$$

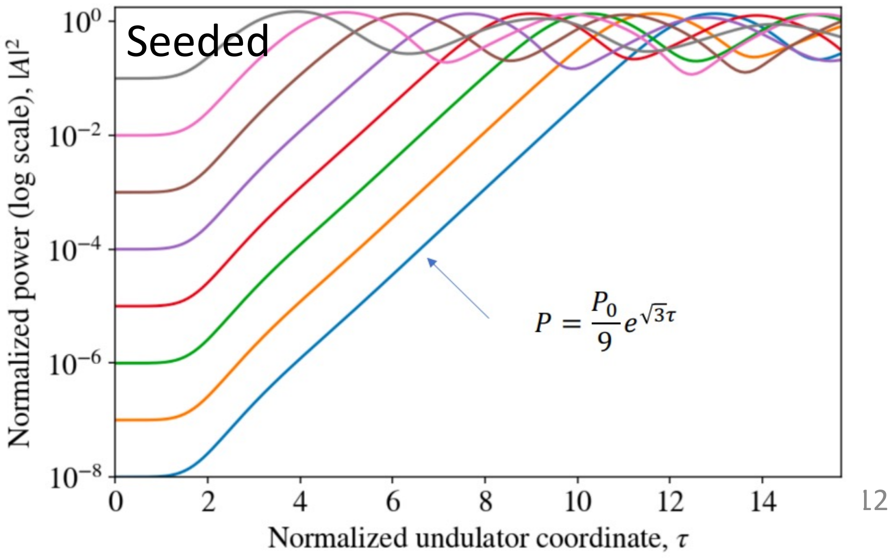
```python
from scipy.integrate import solve_ivp
import numpy as np
import matplotlib.pyplot as plt

def rhs(t, y):
    """
    The right-hand side of the 1D canonical FEL equations;
    t - the current time;
    y - array of [A, theta, eta]
    """
    n = len(y)//2
    A = y[0]
    theta = y[1:n+1]
    eta = y[n+1:]
    dA_dt = np.mean(np.exp(-1j*theta))
    dtheta_dt = eta
    deta_dt = -2*np.real(A*np.exp(1j*theta))
    return np.concatenate(([dA_dt],
                           dtheta_dt,
                           deta_dt))

sol = solve_ivp(rhs, [0, 4*np.pi],
                np.concatenate(([A0+0j], theta0, eta0)),
                max_step=0.1)
```

Initial Conditions

- SASE FEL $$A(0) = 0$$
- Seeded FEL $$A(0) = A_0$$
- Bunched Beam FEL $$|b(0)|^2 \neq 0$$



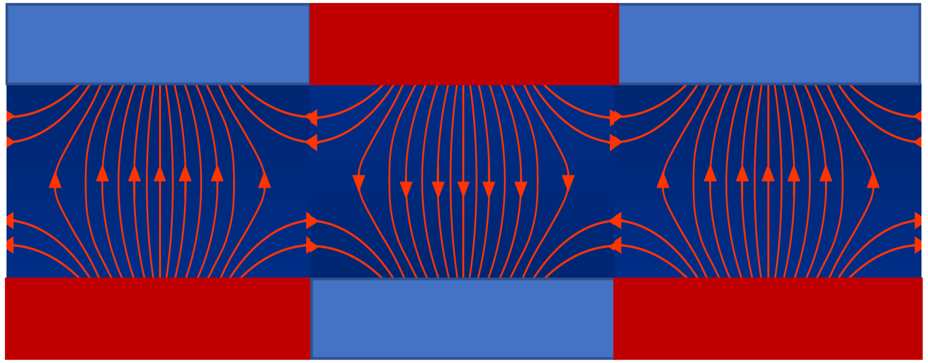




Planar Undulator Magnetic Field

Planar undulator magnetic field that satisfies Maxwell's equation

$$\boldsymbol{B} = B_0 \hat{\boldsymbol{y}} \sin k_u z \cosh k_u y + B_0 \hat{\boldsymbol{z}} \cos k_u z \sinh k_u y$$



Lorentz force equation

$$\gamma m_e \frac{dv_x}{dt} = -e v_z B_0 sin(k_u z) \cosh k_u y$$

$$\gamma m_e \frac{dv_y}{dt} = -e v_x B_0 cos(k_u z) \sinh k_u y$$

Small $(k_u y)$ approximation

$$\cosh k_u y \cong 1 + \frac{(k_u y)^2}{2}$$

$$\sinh k_u y \cong k_u y$$

Planar Undulator Natural Focusing

Equations of motion

$$x'' + k_u^2 x = 0$$

$$y'' + \left(k_u^2 \frac{K^2}{\gamma^2} \cos^2 k_u z\right) y = 0$$

In homework problem 2.1, you'll be asked to derive these equations of motion

Averaging over one undulator period yields the following equation

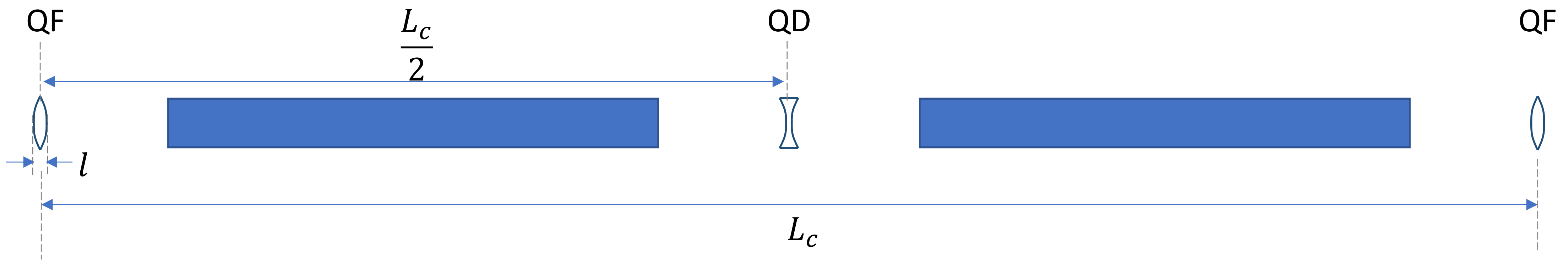
$$y'' + \left(k_u^2 \frac{K^2}{2\gamma^2}\right) y = 0 \qquad y'' + k_y^2 y = 0$$

The beam experiences natural focusing in y with focusing function

$$k_y = \frac{k_u K}{\sqrt{2}\,\gamma}$$

The natural focusing is considered weak focusing.

Strong Focusing in a FODO



QF focusing quad (in x)

$$\begin{bmatrix} x \\ x' \end{bmatrix}_{z\to z+l} = \begin{bmatrix} \cos\sqrt{k}l & \frac{\sin\sqrt{k}l}{\sqrt{k}} \\ -\sqrt{k}\sin\sqrt{k}l & \cos\sqrt{k}l \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix}_z$$

Drift

$$\begin{bmatrix} x \\ x' \end{bmatrix}_{z\to z+\frac{L_c}{2}} = \begin{bmatrix} 1 & \frac{L_c}{2} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix}_z$$

QF thin lens approximation

$$\begin{bmatrix} x \\ x' \end{bmatrix}_{z\to z+l} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix}_z$$

Quad focusing strength

$$k[m^{-2}] = 0.299 \frac{G_Q\left[\frac{T}{m}\right]}{E[GeV]}$$

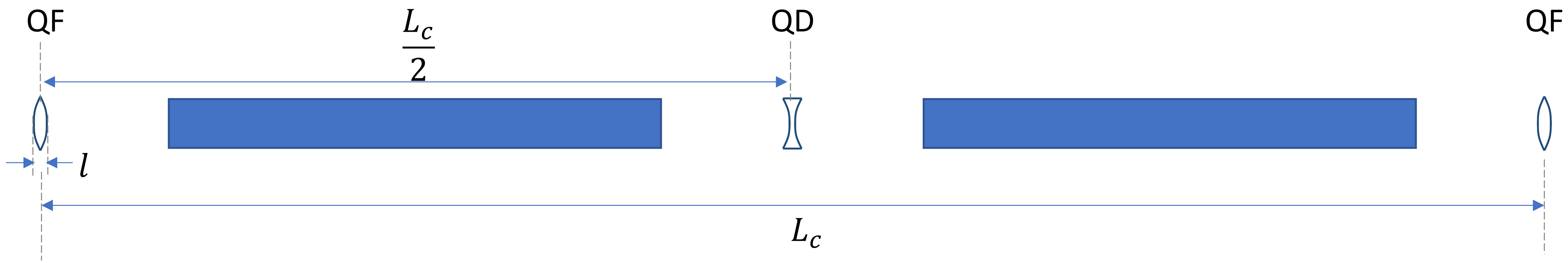
Quad focal length

$$f = \sqrt{k}l$$

QD thin lens approximation

$$\begin{bmatrix} x \\ x' \end{bmatrix}_{z\to z+l} = \begin{bmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix}_z$$

Thin Lens Model of a FODO



Half a QF

$$\begin{bmatrix} x \\ x' \end{bmatrix}_{z \to z+l} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix}_z$$

Full QD

$$\begin{bmatrix} x \\ x' \end{bmatrix}_{z \to z+l} = \begin{bmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix}_z$$

Half a QF

$$\begin{bmatrix} x \\ x' \end{bmatrix}_{z \to z+l} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix}_z$$

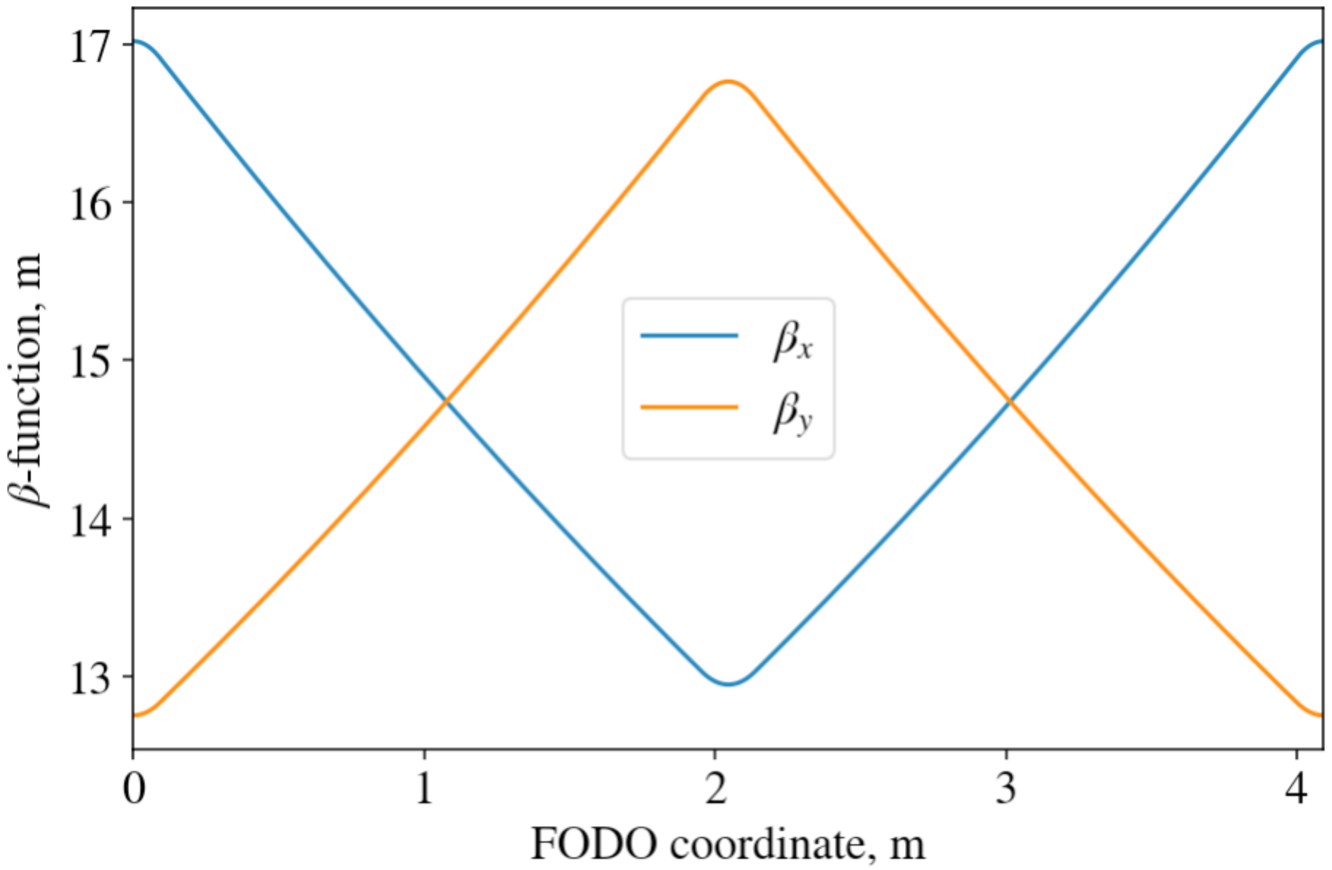
Drift

$$\begin{bmatrix} x \\ x' \end{bmatrix}_{z \to z+\frac{L_c}{2}} = \begin{bmatrix} 1 & \frac{L_c}{2} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix}_z$$

Drift

$$\begin{bmatrix} x \\ x' \end{bmatrix}_{z \to z+\frac{L_c}{2}} = \begin{bmatrix} 1 & \frac{L_c}{2} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix}_z$$

Beam Envelope in a FODO



Beam rms radius in x and y

$$\sigma_x = \sqrt{\frac{\beta_x \varepsilon_{n,x}}{\gamma}}$$

$$\sigma_y = \sqrt{\frac{\beta_y \varepsilon_{n,y}}{\gamma}}$$

Genesis 1.3 Simulation Code

Genesis 1.3 version 4

- We will focus on Genesis v4, written in C, for doing both time-independent and time-dependent FEL simulations.
- Genesis uses undulator averaged approximation and thus expresses all the distances in the units of undulator period XLAMD
- Electromagnetic fields are expressed on the Cartesian grid
- Electrons are represented by an equal number of macroparticles arranged in slices, one resonant wavelength XLAMDS long
- Slices are ≥ one wavelength apart
- Genesis v4 can be found at https://github.com/svenreiche/Genesis-1.3-Version4.

LUME-Genesis

- LUME-Genesis is the Python interface to setup, run and analyze Genesis v4 simulations
- LUME-Genesis allows Genesis v4 to be run from a Jupyter Lab notebook
- Genesis v4 can also be run from a Linux command line
- Examples of Genesis4 JL notebooks:
  - intro-1-Quickstart.ipynb # Load an existing input file and run Genesis4
  - MingXie.ipynb # Analytic model of a high-gain FEL via Ming Xie parametrization

Input Files

- &setup
- rootname=Example1
- lattice=Example1.lat
- beamline=FEL
- lambda0=1e-10
- gamma0=11357.82
- delz=0.045000
- shotnoise=0
- nbins = 8
- &end

- &lattice
- zmatch=9.5
- &end

- &field
- power=5e3
- dgrid=2.00e-04
- ngrid=255
- waist_size=30e-6
- &end

- &beam
- current=3000
- delgam=1.00
- ex=4.00e-07
- ey=4.00e-07
- &end

Lattice Files

- D1: DRIFT = { l = 0.44};
- D2: DRIFT = { l = 0.24};
- QF: QUADRUPOLE = { l = 0.080000, k1= 2.000000 };
- QD: QUADRUPOLE = { l = 0.080000, k1= -2.000000 };
- UND: UNDULATOR = { lambdau=0.015000, nwig=266, aw=0.84853, helical= True};

- FODO: LINE={UND, D1, QF, D2, UND, D1, QD, D2};

- FEL: LINE={6*FODO};

Planar Undulators

$$a_w = 0.660\lambda_u B_0$$

Helical Undulators

$$a_w = 0.9338\lambda_u B_0$$

Quadrupole Strength

$$k_1\left(\frac{1}{m^2}\right) = \frac{0.29979}{E_b(GeV)} G\left(\frac{T}{m}\right)$$

Input Electron Beam Files

- &beam
  - @beamgamma (from beam_gamma.h5)
  - @beamcurrent (from beam_current.h5)

- &importbeam

  xxx.Beam

- &importdistribution

  xxx.distribution.h5

Output Files

- Output files (xxx.out.h5)

- Output field files (xxx.fld.h5)

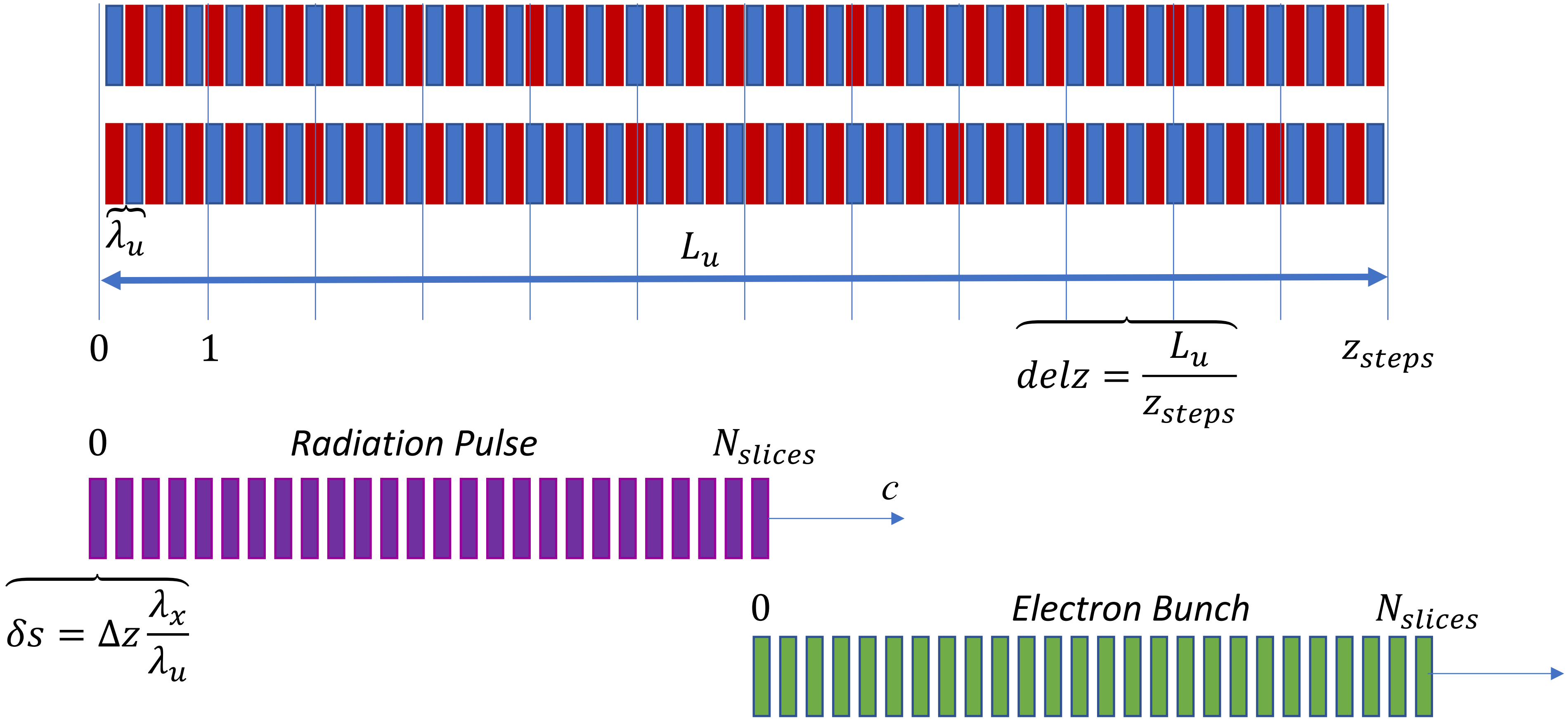
```
&write
field = end
&end
```

- Output beam files (xxx.par.h5)

```
&write
beam = end
&end
```

Time-Dependent FEL Simulations

Illustration of TD Genesis Simulation



Simulation Time Window

- **s0** (*double, 0*): Starting point of the time-window in meters.
- **slen** (*double, 0*): Length of the time window in meters. Note that for parallel jobs this might be adjusted towards larger values.
- **sample** (*int, 1)*): Sample rate in units of the reference wavelength from the `setup` namelist, so that the number of slices is given by `SLEN / LAMBDA0/SAMPLE` after `SLEN` has been adjusted to fit the cluster size.
- **time** (*bool, true*): Flag to indicate time-dependent run. Note that time-dependent simulations are enabled already by using this namelist. This flag has the functionality to differentiate between time-dependent run and scans, which disable the slippage in the tracking. To restrict the simulation to steady-state the `time` namelist has to be omitted from the input deck.

Profiles

profile_gauss

- **label** (`string, < empty >`): Name of the profile, which is used to refer to it in later calls of namelists
- **c0** (`double, 0`): Maximum function value of the Gaussian distribution
- **s0** (`double, 0`): Center point of the Gaussian distribution
- **sig** (`double, 0`): Standard deviation of the Gaussian distribution

profile_step

- **label** (`string, < empty >`): Name of the profile, which is used to refer to it in later calls of namelists
- **c0** (`double, 0`): Constant term
- **s_start** (`double, 0`): Starting point of the step function
- **s_end** (`double, 0`): Ending point of the step function

FEL Simulation Examples

- LEUTL Visible SASE FEL
- SPARC Visible SASE FEL
- FERMI HGHG FEL
- LCLS First-Lasing X-ray FEL

LEUTL Visible SASE FEL

Beam & FEL Parameters	Symbol	Value
Beam kinetic energy	E_b	217 MeV
Dimensionless energy	γ_0	426
Peak current	I_p	150 A
Relative energy spread (rms)	$\frac{\Delta\gamma}{\gamma_0}$	0.1%
Normalized x emittance	ε_{nx}	9 mm-mrad
Normalized y emittance	ε_{ny}	9 mm-mrad
FEL wavelength	λ	521 nm
FEL dimensionless gain	ρ	0.0017
1D gain length	L_G	0.9 m
Saturation length	L_s	36 m

Undulator Parameters	Symbol	Value
Undulator period	λ_u	0.033 m
Dimensionless parameter	K	
Undulator length	L_u	
Twiss beta function	β_T	
FODO period	λ_{FODO}	

SPARC Visible SASE FEL

Beam & FEL Parameters	Symbol	Value
Beam kinetic energy	E_b	152 MeV
Dimensionless energy	γ_0	298
Peak current	I_p	53 A
Relative energy spread (rms)	$\frac{\Delta\gamma}{\gamma_0}$	0.09%
Normalized x emittance	ε_{nx}	2.5 mm-mrad
Normalized y emittance	ε_{ny}	2.9 mm-mrad
FEL wavelength	λ	491.5 nm
FEL dimensionless gain	ρ	0.0021
1D gain length	L_G	0.62 m
Saturation length	L_s	20 m

Undulator Parameters	Symbol	Value
Undulator period	λ_u	0.028 m
Dimensionless parameter	K	
Undulator length	L_u	
Twiss beta function	β_T	
FODO period	λ_{FODO}	

FERMI VUV HGHG FEL

Beam & FEL Parameters	Symbol	Value
Beam kinetic energy	E_b	1,500 MeV
Dimensionless energy	γ_0	
Peak current	I_p	
Relative energy spread (rms)	$\frac{\Delta\gamma}{\gamma_0}$	
Normalized x emittance	ε_{nx}	
Normalized y emittance	ε_{ny}	
FEL wavelength	λ	260 nm
FEL dimensionless gain	ρ	
1D gain length	L_G	
Saturation length	L_s	

Undulator Parameters	Symbol	Value
Undulator period	λ_u	
Dimensionless parameter	K	
Undulator length	L_u	
Twiss beta function	β_T	
FODO period	λ_{FODO}	

LCLS First Lasing SASE FEL

Beam & FEL Parameters	Symbol	Value
Beam kinetic energy	E_b	13,640 MeV
Dimensionless energy	γ_0	26,694
Peak current	I_p	3,000 A
Relative energy spread (rms)	$\frac{\Delta\gamma}{\gamma_0}$	0.0001
Normalized x emittance	ε_{nx}	0.4 mm-mrad
Normalized y emittance	ε_{ny}	0.4 mm-mrad
FEL wavelength	λ	0.15 nm
FEL dimensionless gain	ρ	0.0009
1D gain length	L_G	1.6 m
Saturation length	L_s	40 m