Homework 2

2.1 Show that the equations of motion in a planar undulator can be written in the form of a Hill's equation below. Explain why this leads to the beam focusing in the y direction.

$$x'' + k_u^2 x = 0$$

$$y'' + \left(k_u^2 \frac{K^2}{\gamma^2} \cos^2 k_u z\right) y = 0$$

2.2 Show that we can write the matrix equation for x in a FODO lattice with period Lc and focal length f as below.

$$\begin{bmatrix} x \\ x' \end{bmatrix}_{z \to z + L_{cell}} = \begin{bmatrix} 1 - \frac{L_c^2}{8f^2} & L_c\left(1 + \frac{L_c}{4f}\right) \\ -\frac{L_c}{4f^2}\left(1 - \frac{L_c}{4f}\right) & 1 - \frac{L_c^2}{8f^2} \end{bmatrix}$$

2.3 What is the reason behind selecting large beta functions for short FEL wavelengths?

2.4 Rewrite the FEL gain (Pierce) parameter for the case of a fixed FEL wavelength and beam emittance. Show that the FEL gain increases with beam gamma.

2.5 Provide the physics limitations that prevent us from compressing the electron bunches to very short bunch length in order to maximize the FEL gain parameter.