Accelerator Physics

Practicum

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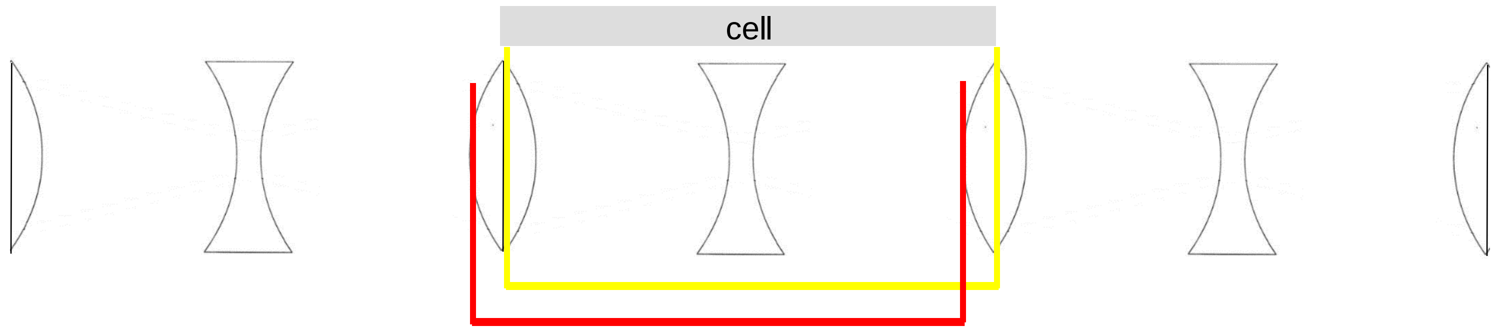
FODO Cell

- What is a FODO cell?

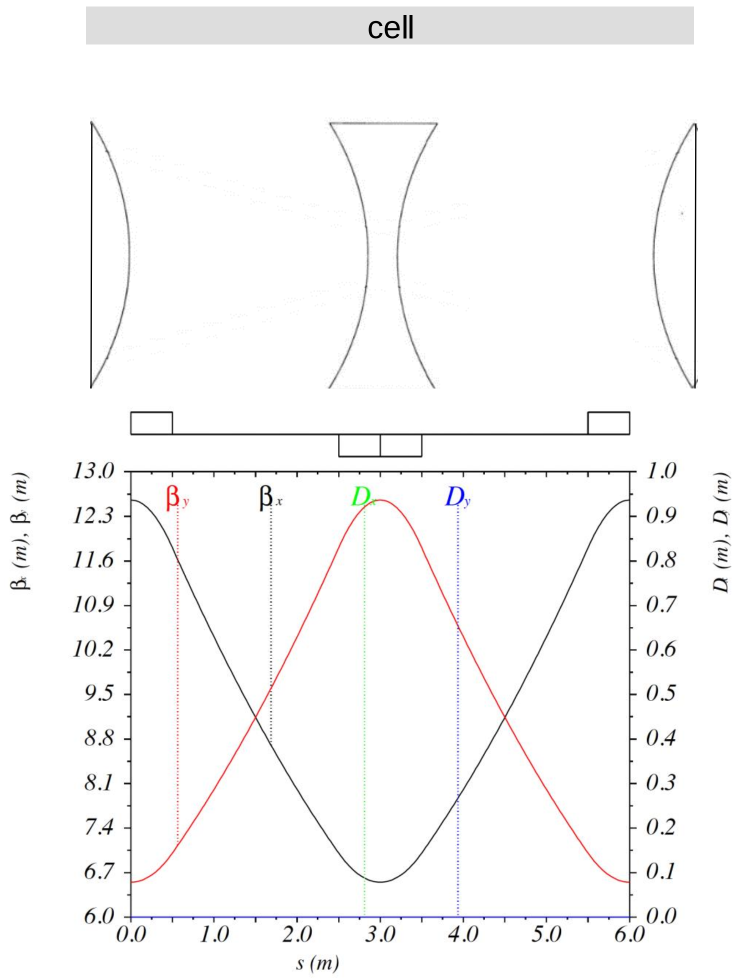


  - The simple periodic system consisting focusing and defocusing quadrupole magnets and drift sections
  - Uses **A**lternating **G**radient **S**trong focusing phenomena, and provides net focusing of a beam
- These elements can be arranged in few other ways, however maintaining periodicity is essential.

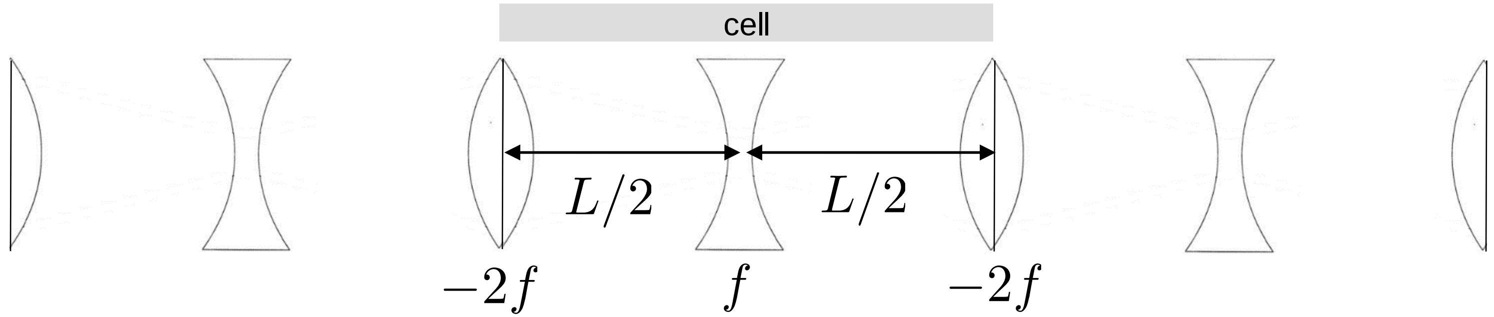
FODO Cell

- One of the most common modules is a FODO module
  - Beta-functions are periodic.
  - Horizontal beam size largest at centers of focusing quads
  - Vertical beam size largest at centers of defocusing quads

$$\textbf{Beam size} = \sqrt{\varepsilon.\beta(s) + \left(D(s)\frac{\delta E}{E}\right)^2}$$



FODO Cell: Transfer matrix



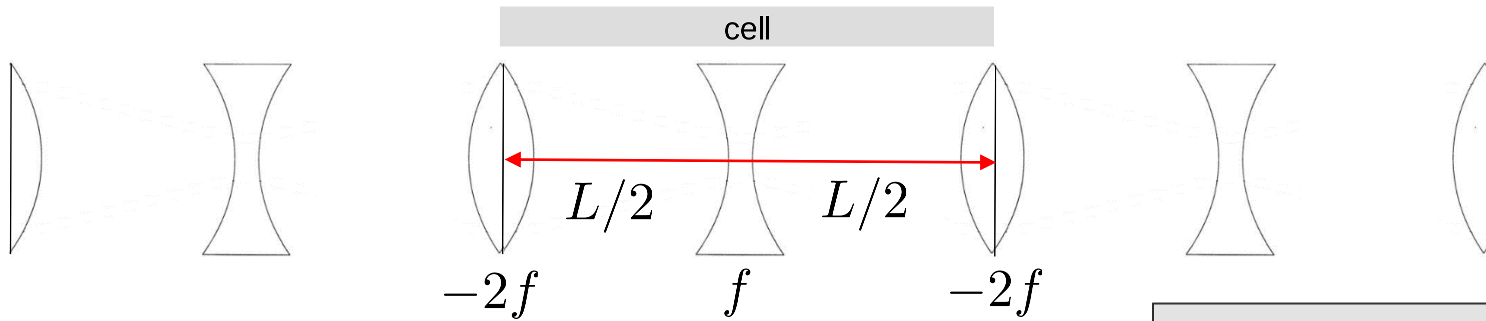
$$\textbf{Focal length } (f) = \frac{1}{k l_q}$$

- Beam transport matrices.

- Quadrupole : $M_{qF} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$, $M_{qD} = \begin{pmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{pmatrix}$

- Drift : $M_{qF} = \begin{pmatrix} 1 & \frac{L}{2} \\ 0 & 1 \end{pmatrix}$

FODO Cell: Transfer matrix



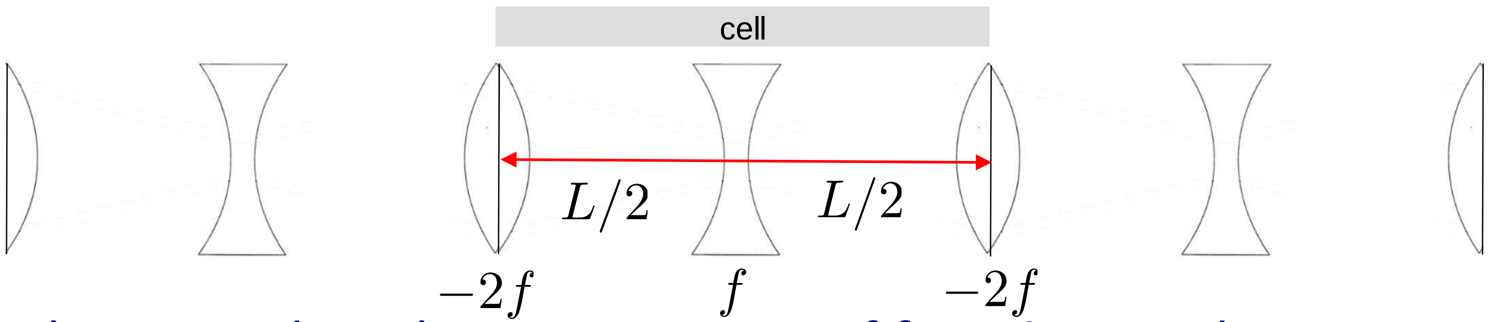
Focal length $(f) = \dfrac{1}{kl_q}$

- Select periodicity between centers of **focusing** quads.
- Both quads have same focusing strength, hence same ***'f '***

$$M = \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{L}{2} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{L}{2} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix}$$

$$M = \begin{pmatrix} 1 - \frac{L^2}{8f^2} & \frac{L^2}{4f} + L \\ \frac{L^2}{16f^3} - \frac{L}{4f^2} & 1 - \frac{L^2}{8f^2} \end{pmatrix}$$

FODO Cell: Phase advance



Select periodicity between centers of **focusing** quads

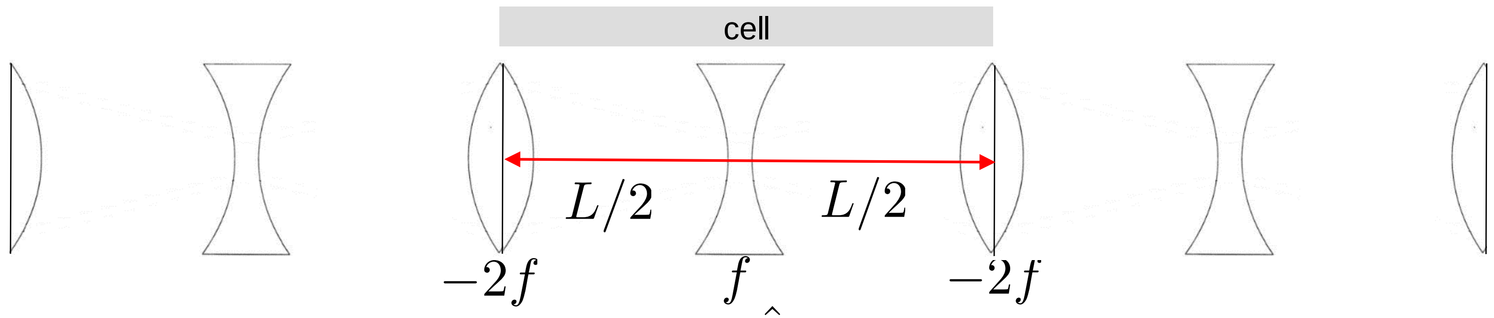
$$M = \begin{pmatrix} 1 - \frac{L^2}{8f^2} & \frac{L^2}{4f} + L \\ \frac{L^2}{16f^3} - \frac{L}{4f^2} & 1 - \frac{L^2}{8f^2} \end{pmatrix}$$

$$\mathrm{Tr}\, M = 2\cos\mu = 2 - \frac{L^2}{4f^2}$$

$$1 - \frac{L^2}{8f^2} = \cos\mu = 1 - 2\sin^2\frac{\mu}{2} \quad \Rightarrow \quad \sin\frac{\mu}{2} = \pm\frac{L}{4f}$$

μ only has real solutions (stability) if $\frac{L}{4} < f$

FODO Cell Beta Max/Min



What is the maximum beta function, $\hat{\beta}$?

$$M = \begin{pmatrix} 1 - \frac{L^2}{8f^2} & \frac{L^2}{4f} + L \\ \frac{L^2}{16f^3} - \frac{L}{4f^2} & 1 - \frac{L^2}{8f^2} \end{pmatrix} \Leftarrow \quad m_{12} = \beta \sin \mu$$

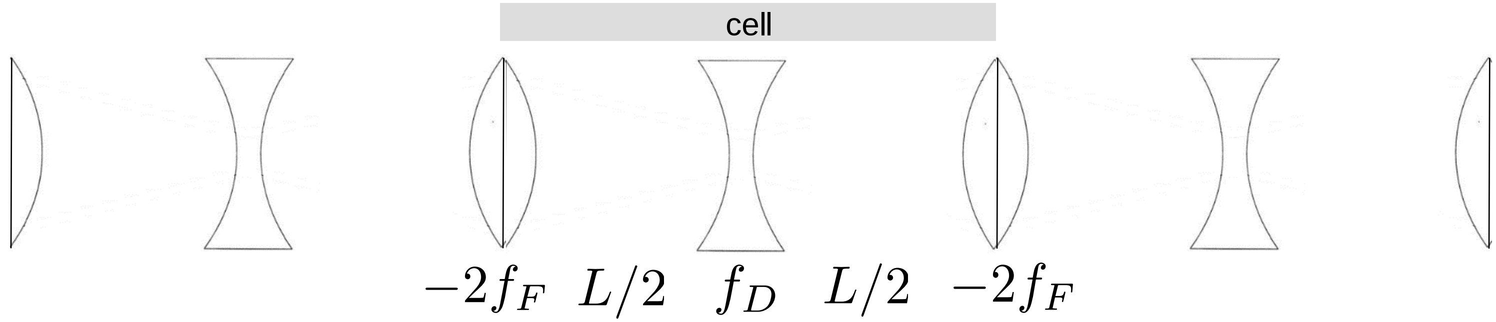
$$M = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cos(\mu) + \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix} \sin(\mu)$$

$$\hat{\beta} \sin \mu = \frac{L^2}{4f} + L = L\left(1 + \sin \frac{\mu}{2}\right)$$

$$\hat{\beta} = \frac{L}{\sin \mu}\left(1 + \sin \frac{\mu}{2}\right)$$

Follow a similar strategy reversing F/D quadrupoles to find the minimum b(s) within a FODO cell (center of D quad)

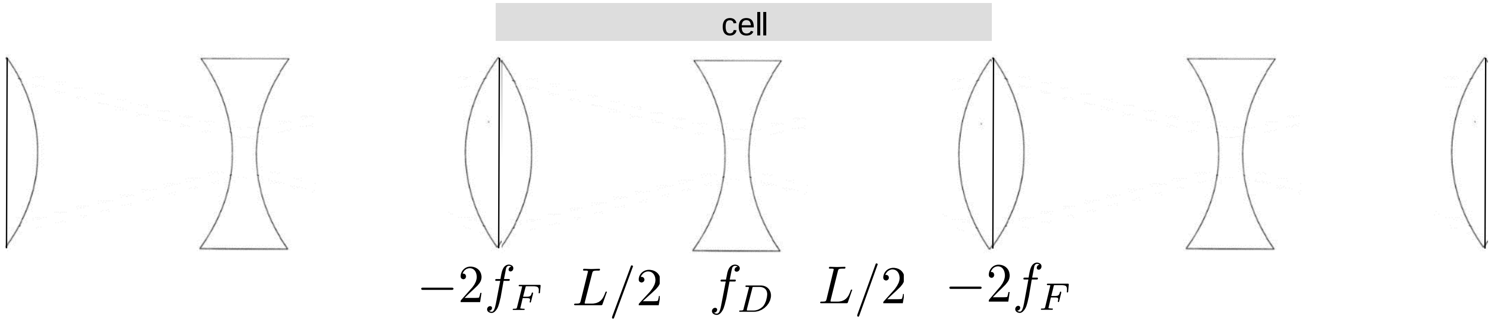
$$\check{\beta} = \frac{L}{\sin \mu}\left(1 - \sin \frac{\mu}{2}\right)$$



- Designers often want or need to change the focusing of the two transverse planes in a FODO structure
  - What happens if the focusing/defocusing strengths differ?

$$\mathbf{M} = \begin{pmatrix} 1 - \frac{L}{2}\left(\frac{1}{f_f} - \frac{1}{f_d} + \frac{L}{4f_f f_d}\right) & 2L + \frac{L^2}{4f_d} \\ \frac{1}{f_d} - \frac{1}{f_f}\left(1 - \frac{L}{4f_f} - \frac{L}{2f_d} - \frac{L^2}{16 f_f f_d}\right) & 1 - \frac{L}{2}\left(\frac{1}{f_f} - \frac{1}{f_d} + \frac{L}{4f_f f_d}\right) \end{pmatrix}$$

Stability Diagrams



- Use recalculated M matrix and these dimensionless quantities

$$M = \begin{pmatrix} 1 - \frac{L}{2}\left(\frac{1}{f_f} - \frac{1}{f_d} + \frac{L}{4f_f f_d}\right) & 2L + \frac{L^2}{4f_d} \\ \frac{1}{f_d} - \frac{1}{f_f}\left(1 - \frac{L}{4f_f} - \frac{L}{2f_d} - \frac{L^2}{16 f_f f_d}\right) & 1 - \frac{L}{2}\left(\frac{1}{f_f} - \frac{1}{f_d} + \frac{L}{4f_f f_d}\right) \end{pmatrix}$$

$$F \equiv \frac{L}{2f_F} \qquad D \equiv \frac{L}{2f_D}$$

- Then take the trace for stability conditions to find

$$\cos\mu = 1 + D - F - \frac{FD}{2}$$

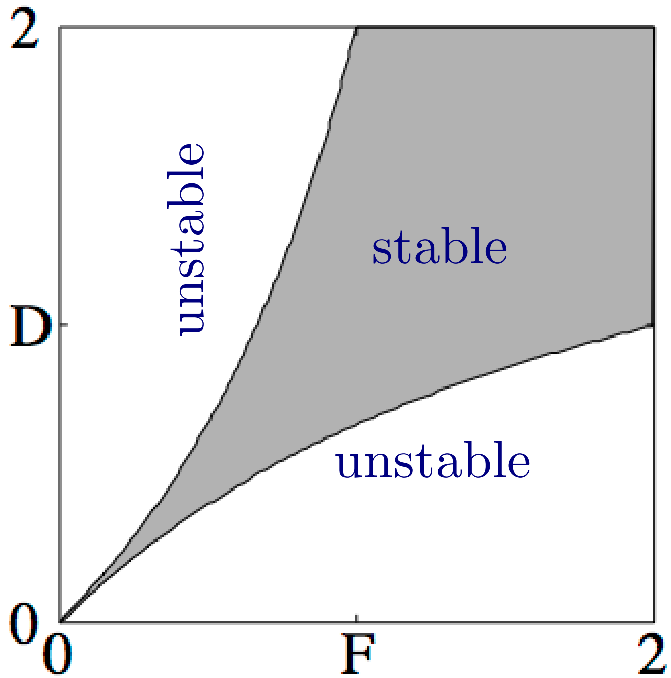
or

$$\sin^2\frac{\mu}{2} = \frac{FD}{4} + \frac{F-D}{2}$$

$$\cos\mu = 1 - 2\sin^2\frac{\mu}{2}$$

Stability Diagrams

- For stability, we must have $-1 < \cos\mu < 1$
- Using $\cos\mu = 1 - 2\sin^2\frac{\mu}{2}$, stability limits are where,



$$\sin^2\frac{\mu}{2} = 0 \qquad \sin^2\frac{\mu}{2} = 1$$

- These translate to an a "necktie" stability diagram for FODO

MAD-X **(Methodical Accelerator Design)**

- MAD-X: Developed by CERN Accelerator Beam Physics Group

MAD - Methodical Accelerator Design
CERN - BE/ABP Accelerator Beam Physics Group

- Uses for design, simulation and optimization of particle accelerator lattices
- Features: Lattice design, Beam tracking, Beam optics calculations, Beam matching, Errors and corrections
- Program help is in the following link:
  - https://madx.web.cern.ch/
- Hope everyone working MAD-X version in your computers!

MAD-X **(Methodical Accelerator Design)**

- Can use a single input file or more..
- Require:
  - Element definition
  - Line (lattice) definition
  - Initial parameters
  - Used more features for advanced lattice simulations (not covered here!)

- Input file needs to be saved in MAD-X format (i.e. file_name.madx)
- To run MAD-X use; ./madx file_name.madx
- Require PostScript reader to open the output plot files.

MAD-X (Methodical Accelerator Design)

- Input file; A simple FODO lattice

example_1.madx — Edited

example_1.madx

example_1.madx 〉 No Selection

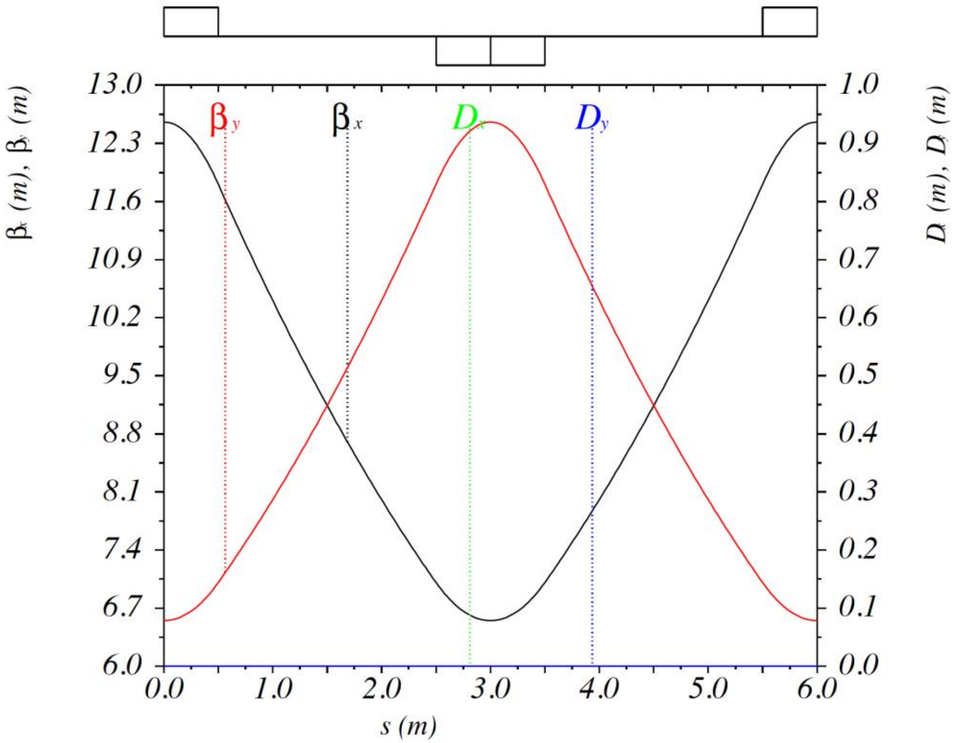
```
! Simple FODO Cell in MAD-X

! Define Beam Parameters
beam, PARTICLE=ELECTRON, PC=10.0;  ! Beam particle and energy (Electrons with E=10 GeV

! Define Elements
L_drift = 2.0;            ! Length of the drift (meters)
L_quad  = 0.5;            ! Length of the quadrupoles (meters)
K       = 0.25;            ! Quadrupole strength (1/m^2)

D1: drift, l=L_drift;     ! Drift
```

Refer MAD-X user manual for more details

```
! Beam Line
use, period=ex1;
twiss,save;

select,flag=twiss,clear;
plot,haxis=s,vaxis1=betx,bety,vaxis2=dx,dy, colour=100,noversion,notitle,interpolate,file=Ex1_twi;

!End of the file
```

MAD-X (Methodical Accelerator Design)

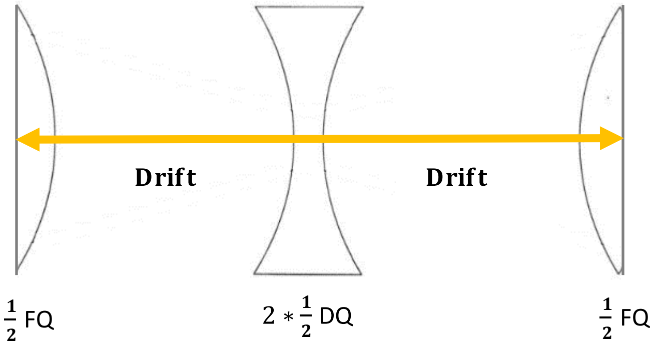
- Output file; Ex1_twi.ps (Twiss plot as defined)



FODO Lattice I

- Create a new file with a name fodo_1.madx and define following beam line; (take x-plane as the focusing plane)

  - $\frac{1}{2}$ FQ : L= 0.25 m, k1 = 0.015
  - $\frac{1}{2}$ DQ: L= 0.25 m, k1 =- 0.015
  - Drift: L= 5.0 m

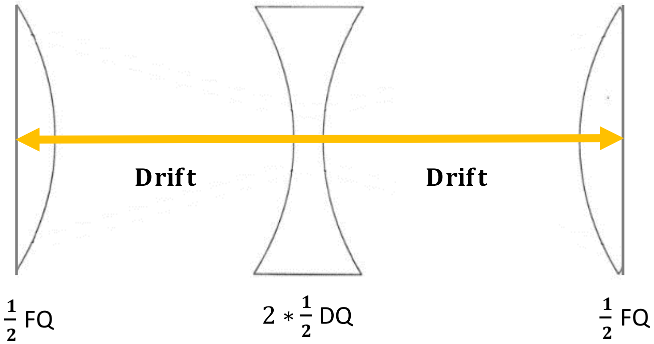


- Obtain periodic solution, in terms of Twiss output, and discuss the behavior of beta functions.

FODO Lattice II

- Create a new file with a name fodo_1.madx and define following beam line; (take x-plane as the focusing plane)

  - $\frac{1}{2}$ FQ : L= 0.50 m, k1 = 0.2
  - $\frac{1}{2}$ DQ: L= 0.50 m, k1 =- 0.2
  - Drift: L= 2.0 m



- Obtain periodic solution, in terms of Twiss output, and discuss the behavior of beta functions.
- Compare with previous plots

FODO Lattice II

- Get Twiss table output

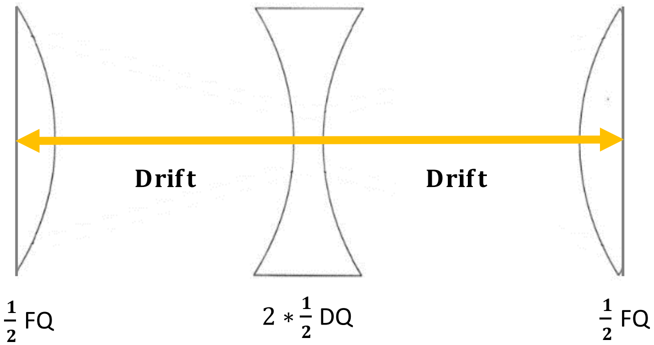
```
! Generate a Twiss Table:

select, flag = twiss, clear;
select, flag = twiss,column = name, s, betx, bety;
twiss, sequence = fodo, file = file_name.data;
```

- You can use these files and create plots using any other plotting tool you prefer!

FODO Lattice III

- As before, create a new file with a name fodo_2.madx and define following beam line; (take x-plane as the focusing plane)

  - $\frac{1}{2}$ FQ : L= 0.50 m, k1 = 0.15
  - $\frac{1}{2}$ DQ: L= 0.50 m, k1 =- 0.15
  - Drift: L= 1.0 m



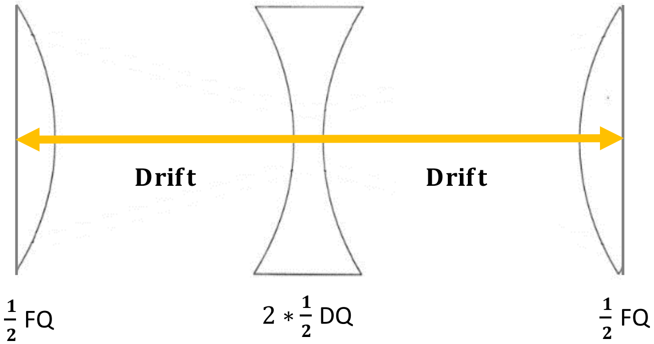
- Obtain periodic solution, in terms of Twiss output, and discuss the behavior of beta functions.

- Obtain the Phase advances in x & y planes. Calculate the phase advance of the cell and compare with the analytical value.

FODO Lattice IV

- As before, create a new file with a name fodo_3.madx and define following beam line; (take x-plane as the focusing plane)

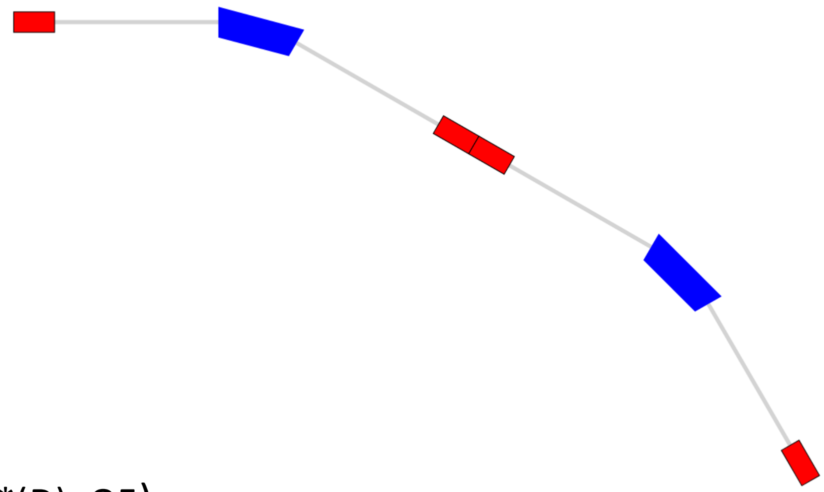
  - $\frac{1}{2}$ FQ : L= 0.25 m, k1 = 1.0
  - $\frac{1}{2}$ DQ: L= 0.25 m, k1 = -1.0
  - Drift: L= 5.0 m



- Discuss the outcome you get with this lattice.

FODO Lattice V

- Create create a new file with a name fodo_4.madx . Define dipoles with a 30^0 bend angle
  - $\frac{1}{2}$ FQ : L= 1.0 m, k1 = 0.025
  - $\frac{1}{2}$ DQ: L= 1.0 m, k1 = -0.025
  - Drift: L= 1.0 m
  - Dipole: L = 2.0 m, θ=50^0

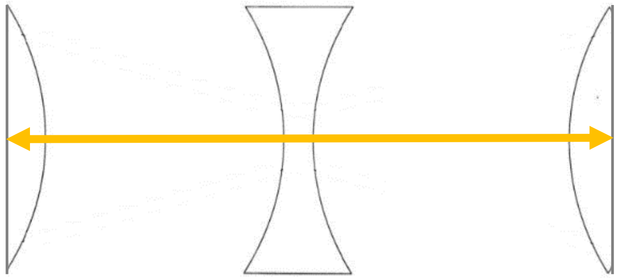


Line: (QF, 4*(D), B1, 4*(D), QD, QD, 4*(D), B1, 4*(D), QF)

- Obtain periodic solution, in terms of Twiss output, and discuss the behavior of beta functions.
- Obtain the dispersion plots, and discuss Twiss output.

FODO Lattice VI

- Matching using MAD-X
- Create a now MAD-X file with following elements. Elements are arranged as in 1st example
  - $\frac{1}{2}$ FQ : L= 0.25 m, k1 = 0.15
  - $\frac{1}{2}$ DQ: L= 0.25 m, k1 = -0.15
  - Drift: L= 0.5 m



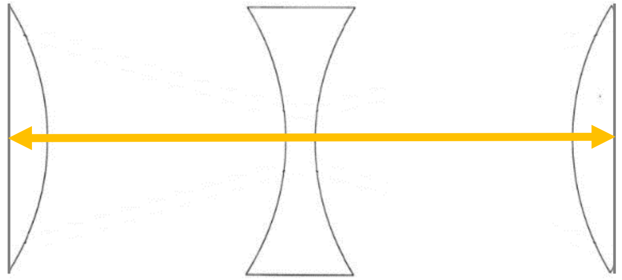
- Match the phase advance of the cell to be 90^0
  - What is the notation MAD-X uses for phase advance?
  - What are the elements we can use as variables?

FODO Lattice VI

- Match the phase advance of the cell to be 90^0.

- Define matching section as follow;

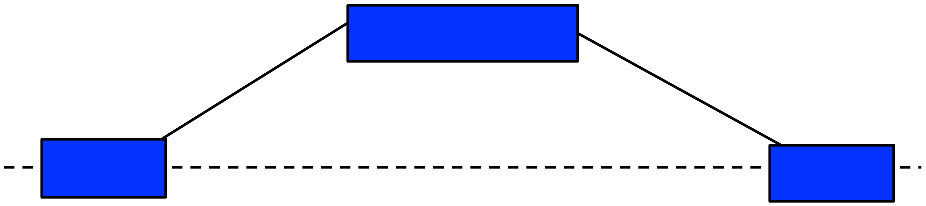
```
match, sequence=fodo;
constraint, sequence=fodo,range=#e, mux=**, muy=**;
vary, name = QF->k1, step = 1E-6;
vary, name = QD->k1, step = 1E-6;
simplex, calls = 100, tolerance = 1E-10;
endmatch;
value, qf->k1, qd->k1, tar;
```



Achromat

- Create a new MAD-X file and name it as achromat.madx
- Define following elements;

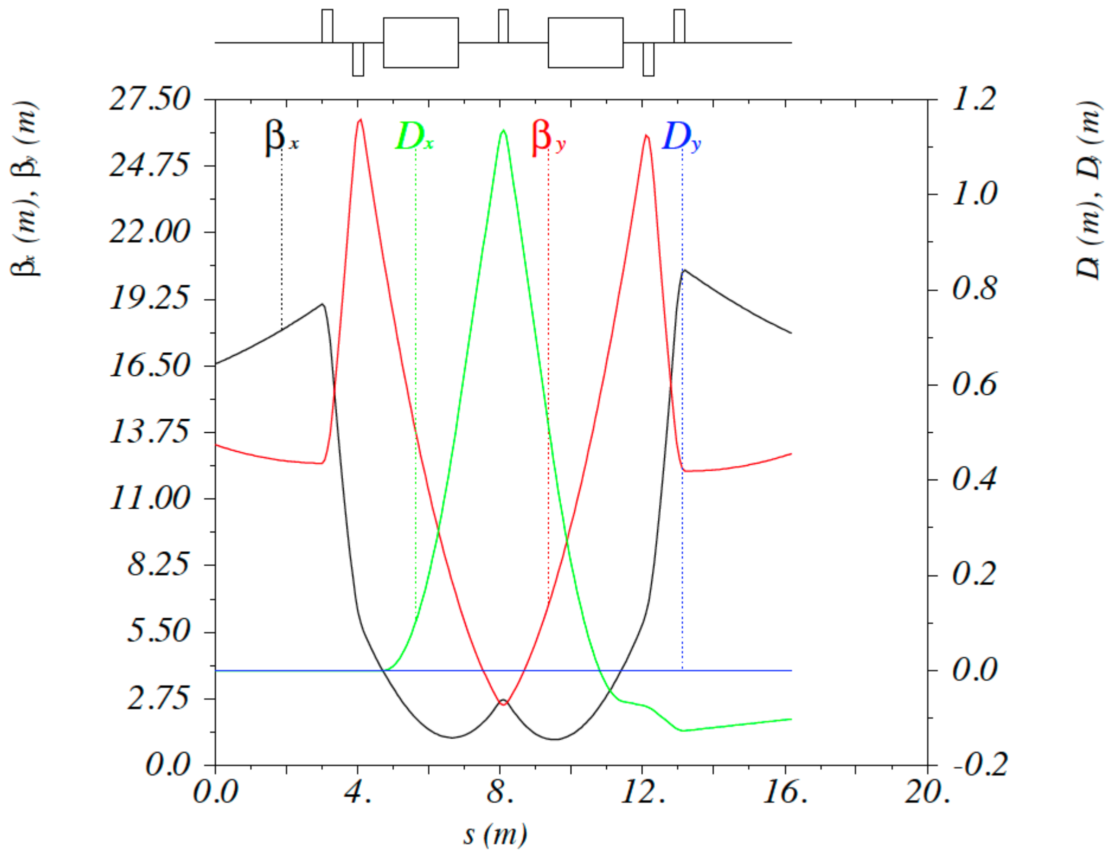
Drift: L = 1.0 m
Dipole1: L=2.0 m, θ=5^0
Dipole2: L=4.0 m, θ=10^0



- Obtain the D(s) and β(s) plot and discuss how they vary. Suggest a way to control β(s)

DBA Lattice

- Dispersion is suppressed by a single quad flanked by a pair of bends. Additional mirror-symmetric pair of doublets provides X/Y stability.



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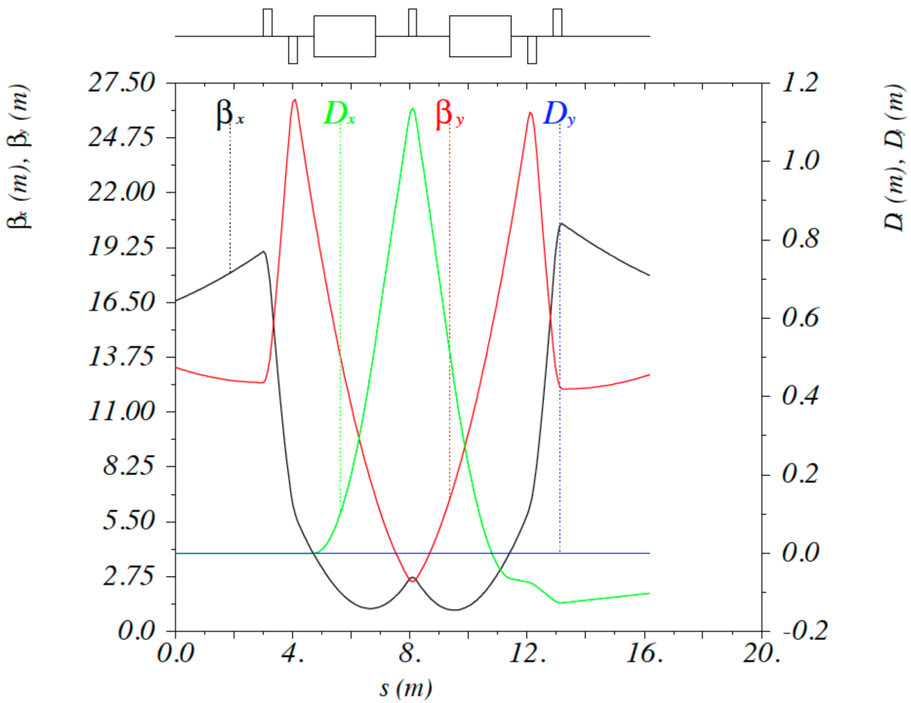

- Explore how far the cell length can be expanded?